A

MANUAL OF FORESTRY.

BY

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LATE INSPECTOR GENERAL OF FORESTS TO THE GOVERNMENT OF INDIA.

VOLUME III.

FOREST MANAGEMENT.

WITH 53 ILLUSTRATIONS.

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PREFACE.

In the Preface to the first volume of this Manual I intimated that the work would comprise the following parts:—

I. The Utility of Forests.
II. Principles of Sylviculture.
III. Formation, Regeneration, and Tending of Woods.
IV. Forest Finance.
V. Forest Working Plans.
VI. Forest Protection.
VII. Forest Utilization.

Parts I. to V. have been dealt with in this and the two previous volumes. My colleague, Mr. W. R. Fisher, has translated Dr. Hess' excellent work on "Forest Protection." That work will appear almost simultaneously with this volume, and take its place as Volume IV. of this series. Mr. Fisher has also commenced the translation of Dr. Gayer's classical work on "Forest Utilization," which will be published towards the end of the present year, and make the fifth and final volume of this Manual.

Under these circumstances I have now completed the task which I set myself at starting. That task consisted in providing, in the first place, text books on the various branches of forestry for the students at Coopers Hill College. At the same time I have endeavoured so to prepare the books

knots. Moreover, conifers generally grow too quickly in Britain, because the woods are too heavily thinned while young: hence the individual trees increase too rapidly, and produce timber inferior to that of the same species imported from the Baltic, and grown in crowded woods.

Secondly, the home-grown timber is brought into the market in fluctuating quantities, so that neither a regular timber trade, nor superior methods of working up the material, nor forest industries, have a chance of developing and thriving. In short, the whole business is far too haphazard.

Economic forestry, to be successful, must be conducted on true sylvicultural principles, and the yield must be so regulated, that, approximately, the same quantity of material may be brought into the market every year; in other words, the principle of a sustained and well-regulated yield must be recognized. Then, and then only, can adequate financial results be expected from forestry.

These are the principles which I have endeavoured to explain in this and the previous two volumes. Whether the student proposes to follow the profession of a forester in this country, in India, the Colonies, or in America, makes no difference; the principles are the same everywhere. Once they have been thoroughly assimilated, the student will without difficulty apply them to the special conditions with which he may have to deal in any part of the world.

While I was Inspector General of Forests to the Government of India, I was fortunate enough to obtain sanction to the establishment of that branch of the Indian Forest Department, which is known as "The Working Plan Branch." This, no doubt, was a very important step, because the measure provided that gradually working plans should be

own experience, extending over many years, utilized chiefly the following works:—

For Forest Mensuration:

(1) Schwappach, "Leitfaden der Holzmesskunde."
(2) Baur, "Holzmesskunde."

For Forest Valuation:

(1) G. Heyer, "Waldwerthrechnung."
(2) Wimmenauer, "Grundriss der Waldwerthrechnung."

In compiling the tables at pages 394 to 397, I have used those appended to the above-mentioned two works, and I desire to express herewith my best thanks to the family of the late Professor G. Heyer (under whom I studied forestry), and to Professor Wimmenauer, for the permission which they kindly gave me to do this.

For Forest Working Plans:

(1) Heyer, "Die Waldertrags-Regelung."
(2) Judeich, "Die Forsteinrichtung."

As Judeich's "Bestandswirthschaft" is the method of regulating the yield of forests which commends itself to me more than any other, I have naturally largely drawn upon Judeich. At the same time I have introduced certain modifications into the method where it seemed too rigid.

I have also made occasional use of,

(1) Puton, "Traité d'Economie Forestière."
(2) Hess, "Encyklopädie" und "Methodologie der Forstwissenschaft," Part III.

I am indebted to my colleague, Mr. A. Lodge, Professor

of Mathematics at this College, for his kind assistance in looking over the proof sheets of Parts I. and II. of this volume.

Herr Forstassessor Alwin Schenck has been good enough to obtain for me the working plan for the Krumbach Communal Forest, contained in Appendix A, and I herewith tender to him my best thanks.

The extract from the working plan for the Herrenwies Range, Appendix C, was prepared by me while I was on tour with the students of this College during the summer of 1893, the records having been kindly placed at my disposal by Herr Oberförster Ziegler of Herrenwies.

W. SCHLICH.

COOPERS HILL,
28th February, 1895.

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DR. SCHLICH'S MANUAL OF FORESTRY,

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ERRATA.

Page 49, line 14 from top, *for* $=\frac{(v_1'+\ldots}{s_1'+\ldots}$, *read* $V_1=\frac{(v_1'+\ldots}{s_1'+\ldots}$

,, 50, ,, 7 ,, foot, *for* $(v'+v'+v'''+\ldots)$, *read* $(v'+v''+v'''\ldots$

,, 64, ,, 12 ,, foot, *for* $F=\frac{S\times H}{V}$, *read* $F=\frac{V}{S\times H}$.

,, 97, between lines 14 and 15 *add* (2 a) The mean diameter of trees.

,, 120, for lines 5 & 6 from top, substitute "A rental *R* is due after *n* years, and again every *n* years, for ever; it is equal to an annual rental of—."

,, 140, line 4 from foot, *for* Under, *read* By.

,, 145, ,, 3 ,, foot, *for* $(1{\cdot}op-1)$, *read* $(1{\cdot}op^r-1)$.

,, 163, ,, 2 ,, foot, *for* ${}^{curr.}p$, *read* ${}^{curr.}p_f$.

,, 181, lines 14 and 15 from foot, strike out the words "the increment of."

,, 186, line 3 from foot, *for* $\sqrt[n]{\ }$, *read* $\sqrt[10]{\ }$.

,, 192, ,, 3 ,, foot, *for* $\sqrt{\ }$, *read* $\sqrt[n]{\ }$.

,, 194, ,, 4 ,, top, *for* $4{\cdot}37-$, *read* $4{\cdot}37=$.

,, 194, ,, 5 ,, top, *for* $\sqrt[n]{\ }$ *read* $\sqrt[10]{\ }$.

,, 196, ,, 2 ,, top, *for* $\frac{\log S-\log s}{10}$, *read* $\frac{\log S-\log s}{n}$.

,, 198, ,, 3 ,, top, *for* current, *read* correct.

,, 203, last column of table, *for* Indicatin, *read* Indicating.

,, 220, line 3 from top, *for* $r=20$,, *read* $r=20$ years.

Page 221, line 8 from foot, *for* $m \times r$, *read* $t \times r$.
,, 224, ,, 3 ,, top, *for* Coupe No. [illegible], *read* Coupe No. 0
,, 224, ,, 10 ,, top, *for* Coupe No. 20, *read* Coupe No. 1.
,, 227, ,, 12 ,, foot, *for* Under, *read* By.
,, 240, ,, 3 ,, top, *for* Under, *read* By.
,, 245, ,, 14 ,, top, *for* annual yield, *read* final annual yield.
,, 249, ,, 3 ,, top, *for* $\underline{(m-1)} \times \underline{I}$, *read* $\frac{(m-1)\ I}{2}$.
,, 267, ,, 17 ,, foot, *for* Under, *read* By.
,, 280, ,, 7 ,, foot, *add* the word "soil" after geology.
,, 342, ,, 13 ,, foot, *read* 6. Determination of the yield.
,, 349, line 2 from foot, *for* a lotment, *read* allotment.
,, 374, first column, *for* Hoher Ochsen, *read* Hoher Ochsenkopf.
,, 376, line 7 from foot, *after* single, *add* the word "trees."
,, 392, in heading, fill in figures 7 and 8.
,, 394, ,, ,, ,, ,, figure 4.

Note.—Some of the above mistakes do not occur in all copies of the book.

FOREST MANAGEMENT.

INTRODUCTION.

THE management of forests depends, apart from local conditions, on the objects which it is proposed to realise. These differ considerably according to circumstances, but whatever they may be, they can be brought under one of the following two headings:—

1. The realisation of indirect effects, such as landscape beauty, preservation or amelioration of the climate, regulation of moisture, prevention of erosion, landslips, avalanches, etc.
2. The management of the forest on economic principles, such as the production of a definite class of produce, or the greatest possible quantity of produce, or the best financial results.

It rests with the owner of the forest, in so far as his choice is not limited by the laws of the country, to determine in each case what the objects of management shall be, and it then becomes the duty of the forester to see that these objects are realised to the fullest extent.

In some cases the realisation of indirect effects requires a special and distinct management, but in the majority of cases they can be produced in combination with economic working. The present volume deals with the latter.

The economic working, whether it aims at the production of a special class, or the greatest quantity of produce, or the best

financial results, must be based on the yield of the forest. In order to determine this, the forester must study the laws which govern production; he must be able to measure the produce and the increment accruing annually or periodically, to determine the capital invested in the forest, to regulate the yield according to time and locality, and to organise the systematic conduct of the business.

Accordingly, forest management may be divided into the following parts:—

Part I.—Forest Mensuration, dealing with the determination of the dimensions of trees, the volume of trees and whole woods, their age and increment.

Part II.—Forest Valuation, dealing with the determination of the capital employed in forestry, and the financial results produced by it.

Part III.—Principles of Forest Working Plans.

Part IV.—Preparation of Forest Working Plans.

PART I.

FOREST MENSURATION.

FOREST MENSURATION.

Forest Mensuration deals with the determination of the dimensions, volume, age and increment of single trees and whole woods.

These determinations are required for the calculation of the material standing on a given area, the yield which a wood can give, and the value of single trees, whole woods and forests. They serve also as the basis for the calculation of the effects of different methods of treatment.

As a rule, the units of measurement employed in Britain and India are the foot, square foot, and cubic foot.

The subject has been divided into the following chapters :—

I. Of the Instruments used in Forest Mensuration.

II. Of the Measurement of Felled Trees.

III. " " " " Standing Trees.

IV. " " " " Whole Woods.

V. Determination of the Age of Single Trees and Whole Woods.

VI. Determination of the Increment of Single Trees and Whole Woods.

CHAPTER I.

INSTRUMENTS USED IN FOREST MENSURATION.

INSTRUMENTS are required to measure the circumference or diameter of logs and trees, the length of logs, the height of trees, and the increment. Such measurements have for their object, either to ascertain the various dimensions, or to calculate from them the volume; in the latter case the measurement of the girth or diameter is used to calculate the sectional area, on the assumption that it forms a circle.

The instruments may be classified as follows:—

1. *Instruments for the Measurement of the Girth.*

The girth may be measured with a tape, or with a string and tape.

The tape consists of a band, of about half an inch in breadth, so constructed that it alters its length as little as possible when moist. It is divided on one side into feet, inches, and, if necessary, decimals of inches; on the other side the sectional areas corresponding to the length of girth are sometimes noted. It is useful to have a small hook on one end, which can be pressed into the bark when the girth exceeds 5 feet. Long tapes are rolled up in cases, which are made of leather, wood or metal.

Of late years flexible steel tapes have come much into use.

The advantages of the tape are, that it is easy to handle and convenient to carry.

Measurements with the tape are subject to various sources

of inaccuracy, amongst which the following deserve to be mentioned:—

(*a*) The sections of most trees are not circles.

(*b*) Owing to the presence of a rough bark, the measured girth is too large.

(*c*) Irregularities in the tree are difficult to avoid.

(*d*) The tape is frequently not applied at right angles to the axis of the tree.

In order to avoid some of the disadvantages of tape measurements, a thin string is sometimes used, which is then held parallel to a graduated tape or rule. In this way more accurate results may be obtained, but the procedure takes more time, and is therefore not employed where large numbers of trees have to be measured.

2. *Instruments for the Measurement of the Diameter.*

The diameter of sections of trees is measured with an ordinary rule or a tape; in all other cases the calliper is used, or sometimes the tree compass.

a. The Calliper or Diameter Gauge.

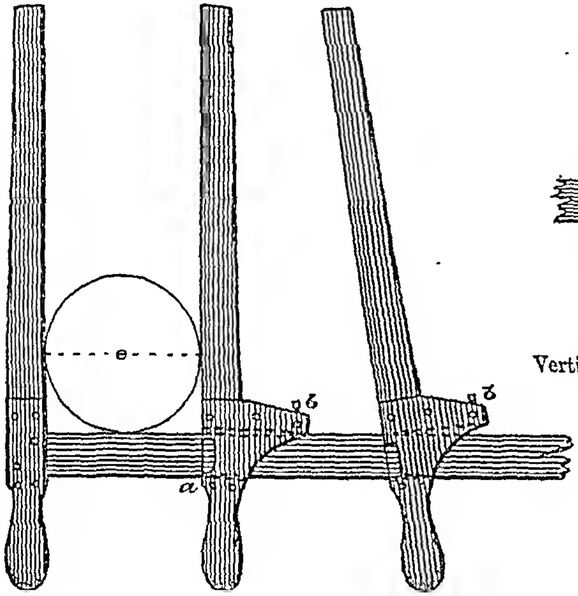
It consists of a graduated rule and two arms. Of the latter, one is fixed at one end at right angles to the rule, so that its inner plane lies in the starting point of the graduated scale; the other arm moves along the rule, parallel to the fixed arm.

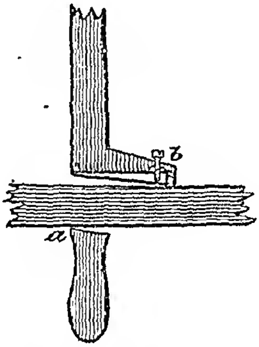
In using the calliper, the tree is brought between the two arms until it touches the rule, then the fixed arm is pressed against the tree on one side and the movable arm shifted until it touches the tree on the other side. The diameter can then be read off on the rule (see Fig. 1 on next page).

The length of the rule and of the arms depends on the size of the trees to be measured; each arm should be at least half the length of the rule. Callipers exceeding 4 feet in length are

rarely used. The rule is divided into units, which depend on the desired degree of accuracy. Ordinarily they will be inches or two inches; in some cases half inches, and for very accurate measurements decimals of inches.

Where large numbers of trees are to be measured, it is desirable to round off the limits of each unit; for instance, if





Vertical Section of Movable Arm.

Fig. 1.—Friedrich's Calliper.

the rule is divided into intervals of inches, the first division line is placed at $\frac{1}{2}$ inch from zero, the second at $1\frac{1}{2}$, the third at $2\frac{1}{2}$, and so on (Fig. 2). In this way all trees measuring from $\frac{1}{2}$ to $1\frac{1}{2}$ inches are recorded as having a diameter of 1 inch, those from $1\frac{1}{2}$ to $2\frac{1}{2}$ inches as 2 inches, and so on.

A good calliper must fulfil the following conditions:—

(1) It must be sufficiently light so as not to fatigue the labourer, and yet sufficiently strong to resist the wear and tear which it is likely to be subjected to.

(2) The two arms must be at right angles to the rule, or at least parallel to each other, when pressed on to the tree.

(3) The movable arm must move with sufficient ease along the rule.

Callipers of iron would be too heavy and too cold in winter, hence they are made of wood. As wood alters with the degree *of humidity, the movable arm is liable to jam at one*

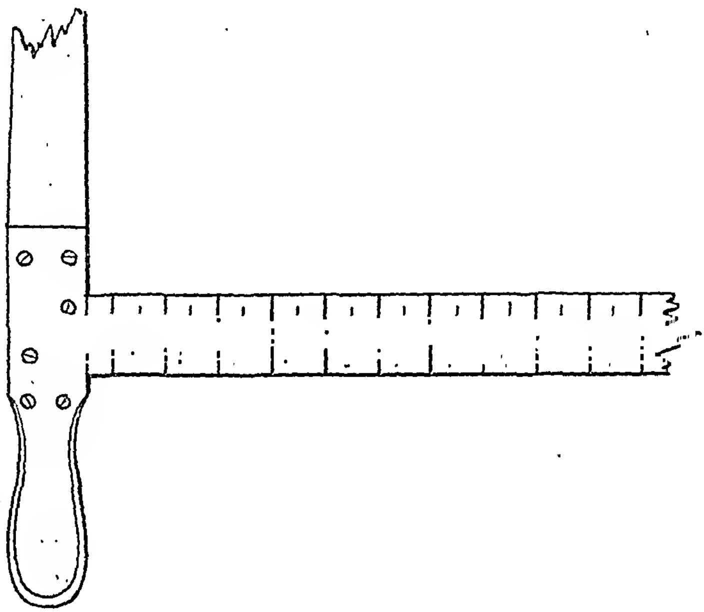


Fig. 2.

time, or to move too easily at others. To avoid this drawback, various constructions have been adopted, resulting in a number of callipers, of which the following two deserve to be specially mentioned :—

Gustav Heyer's Calliper.—The distinguishing feature of this instrument is that the rule is given, in section, the shape of a trapezium, and that it is pressed up or down in the movable arm by means of a wedge, so as to counteract the swelling or shrinking of the wood. In Fig. 3, *a* represents the cross section of the rule, *b* the wedge, and *c* the section of the movable arm. The wedge is fastened to a screw, which can be moved by a key at *d*. On moving the wedge from left to right, it presses the rule upwards and thus tightens it; on moving the

wedge from right to left, it releases the rule, and enables it to move more freely.

To force the rule to follow the backward movement of the wedge, a spring is fastened at *e*, which pushes it from right to left, so that it always must be in touch with the wedge.

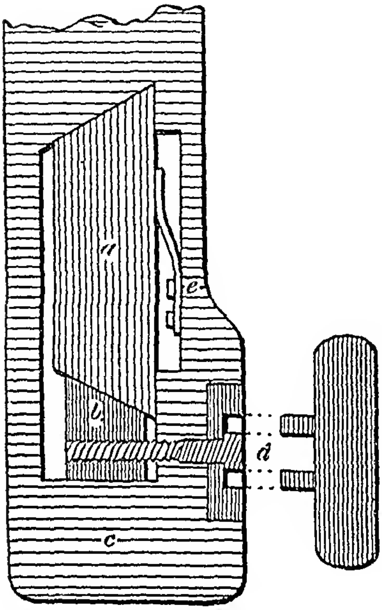


Fig. 3.—Heyer's Calliper.

Friedrich's Calliper.—In this instrument the section of the rule has the shape of a rectangle, while the opening of the movable arm is larger than the section of the rule, and placed slanting towards it. At the same time it is so shaped, that, on being pressed against the tree, it assumes a position which is at right angles to the rule (Fig. 1). In this position the arm rests on the two points, *a* (below) and *b* (above). As these points are liable to wear away, thus causing the arm to assume a position which is no longer at right angles to the rule, Böhmerle has added a spring at *b*, which can be moved by a screw, until the true position of the arm is established.

b. Accuracy of Measurements with the Calliper.

To insure the greatest possible accuracy, the following precautions must be taken :—

(1) Moss, creepers, etc., found on the tree must be removed before measurement.

(2) In the case of an abnormal swelling or indenture, the measurement must be taken above or below it, or both, and the average taken.

(3) In the case of excentric or elliptic trees, two diameters at right angles to each other must be measured and the mean taken.

(4) The height fixed for the measurement must be strictly adhered to.

(5) In the case of trees which are divided into two or more limbs below the fixed height of measurement, each limb must be measured and recorded as a separate tree.

(6) The calliper must be placed at right angles to the axis of the tree, and the rule must touch the tree.

(7) The reading must be taken while the calliper rests on the tree, and not after it has been withdrawn.

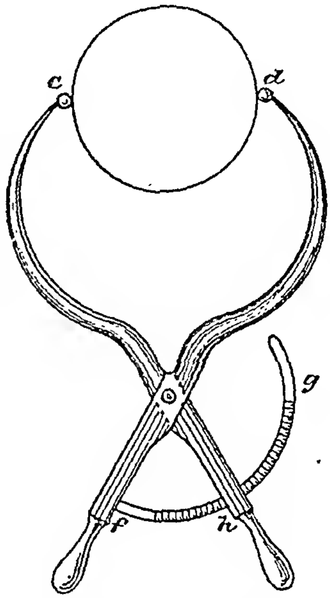


Fig. 4.—The Tree Compass.

c. The Tree Compass.

The shape of this instrument will be understood on reference to Fig. 4. The diameter of the tree or log is taken by the two points *c* and *d*, while it can be read off at *h* on the arc *f*, *g*. In order to produce sufficient stiffness in the arms of the compass, they have to be made of metal, which makes the instrument very heavy and unsuited for continued use.

d. Dendrometers.

In some cases certain dendrometers are used to measure the diameter of trees at some height from the ground. The theory is this:—

The angle which is formed by two rays running to the two sides of the tree is measured, as well as the distance of the eye

of the observer from the tree. From these data the diameter is calculated. Instead of the angle, the distance $a\,b$ between the two lines of sight can be measured, in which case the diameter is obtained in the following way (Fig. 5) :—

$$C\,A : C\,a = A\,B : a\,b,$$

and

$$A\,B = \frac{C\,A}{C\,a} \times a\,b.$$

If, therefore, the instrument gives $a\,b$ and Ca, and the distance $C\,A$ has been measured, the diameter can be calculated.

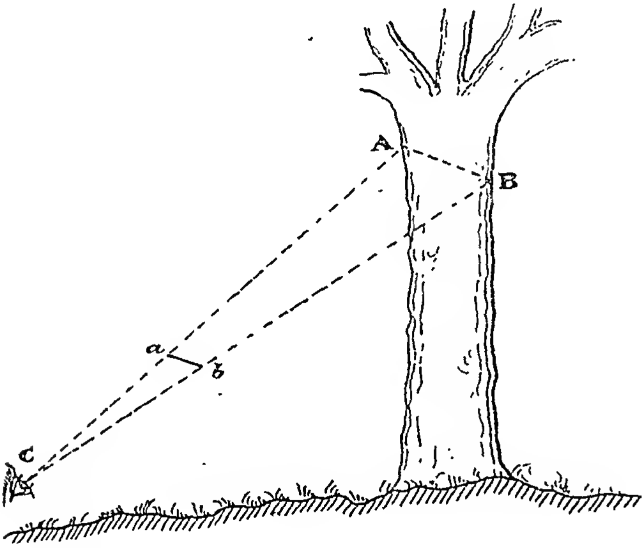


Fig. 5.

So far, instruments of this class have not obtained a footing in practice, because those at present available do not work with sufficient accuracy and $C\,A$ is difficult to ascertain.

3. *Instruments for the Measurement of the Diameter Increment.*

The diameter increment of prepared sections is measured with an ordinary rule, or with a pair of compasses and a rule. Such rules are made of metal or wood, and are sufficiently sub-divided.

If no section is available, as in the case of standing trees, the measurements are made with Pressler's Increment Borer (Fig. 6). This instrument extracts a cylinder of wood from the stem, and it consists of the following parts:—

(*a*) A hollow borer, *A*, which is slightly conical from the handle towards the point.

(*b*) A handle, *B*, which is hollow and serves to receive the borer, wedge and cradle, when the instrument is not in use (see *E* in figure).

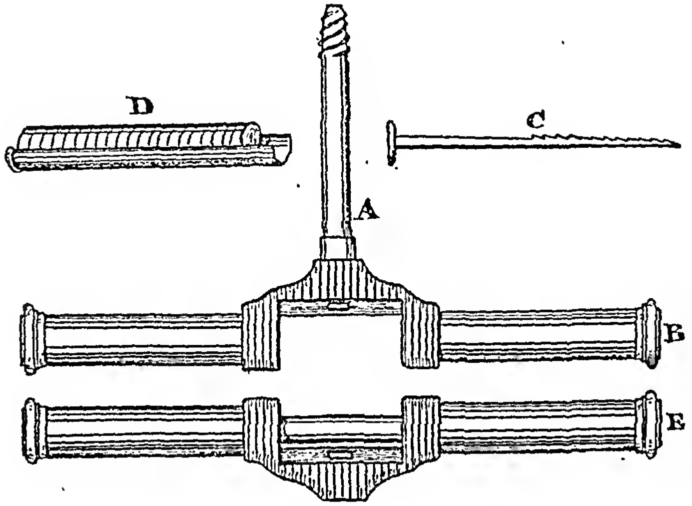


Fig. 6.—Pressler's Increment Borer.

(*c*) A wedge, *C*, which has a scale marked on one side wherewith to measure the breadth of the concentric rings, and is roughly toothed on the other side to assist in extracting the cylinder of wood.

(*d*) A cradle, *D*, into which the cylinder of wood is placed, after extraction, to prevent its breaking.

The borer is used in the following way:—

It is screwed in a radial direction into the tree, at right angles to its axis, to the desired depth, whereby a cylindrical column of wood enters the hollow borer, which is severed from the tree except at its base; then the wedge is inserted between

the column of wood and the inner wall of the borer, with its toothed side towards the former, and firmly pressed in. This prevents the cylinder from turning round inside the borer during the following operation. The borer is now screwed backward one or two turns, whereby the cylinder of wood is severed at its base from the tree. The borer is now screwed further in, which causes the severed cylinder of wood to be pushed back, until it can easily be withdrawn and placed into the cradle. In this way a column of wood is obtained of about ·2 inches diameter and from 2 to 6 inches long according to the length of the borer. The breadth of the concentric rings is then measured. If the rings are not distinct, a smooth surface may be prepared with a sharp knife.

4. *Instruments for the Measurement of the Length of Felled Trees and Logs.*

The length of felled trees and logs is measured with the tape or measuring staff. The former has already been described. The staff varies in length up to about 15 feet; it should be made of hard, straight-grained, well-seasoned wood, and well varnished to protect it against moisture. The ends may usefully be capped with metal plates.

5. *Instruments for the Measurement of the Height of Standing Trees.*

The instruments which have been designed for measuring the height of standing trees are very numerous, but they are all based upon one of two principles: either they determine the height by means of similar triangles (geometrical height measuring), or they serve to measure the angles of elevation and depression (trigonometrical height measuring).

a. Geometrical Height Measuring.

If a horizontal plane is drawn from the eye of the observer to a tree, it will hit the same, according to the position of the observer, either between the top and the foot of the tree, thus dividing it into two parts, one of which is situated above, and the other below the horizontal plane; or above the top; or below the foot. If the observer holds a plumb-line at some distance from his eye, it may be considered parallel to the axis of the tree; hence by looking at the top and foot of the tree

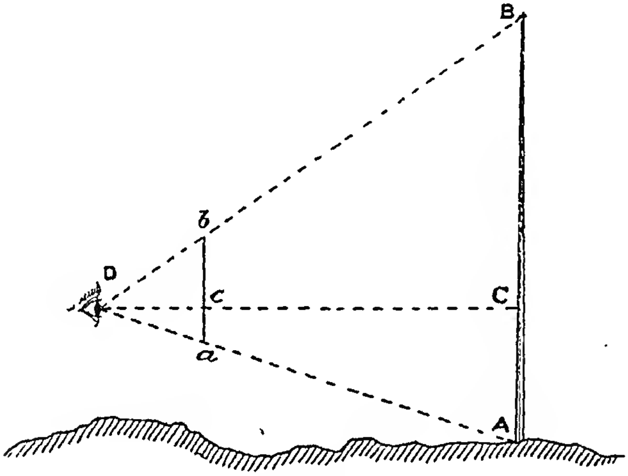


Fig. 7.

similar triangles are formed, which are used for the determination of the height of the tree.

Let $A\ B$ (Fig. 7) be the height of the tree.

$D\ B$ a ray from the eye of the observer to the top of the tree.

$D\ A$, ditto to the foot of the tree.

$D\ C$ a horizontal line.

a, b, and c the points where the three rays hit the plumb-line. Then the height is determined as follows:—

(1) The horizontal line hits the tree between the top and foot. Here the following equation holds good:—

$$B\,C : b\,c = D\,C : D\,c$$

and

$$BC = \frac{bc \times DC}{Dc}$$

Again,

$$AC : ac = DC : Dc$$

and

$$AC = \frac{ac \times DC}{Dc}$$

Hence,

$$BC + AC = AB = \text{height of tree} = \frac{(bc + ac) \times DC}{Dc}$$

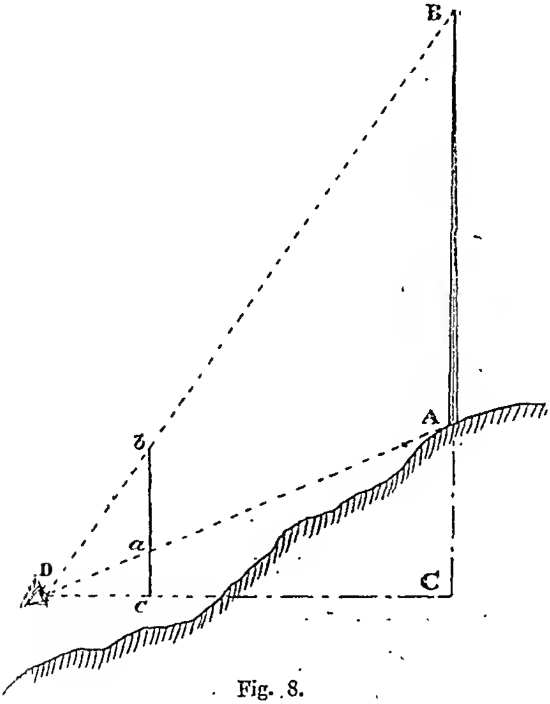


Fig. 8.

(2) The horizontal plane passes below the foot of the tree. In that case (Fig. 8) :—

$$H = BC - AC = \frac{(bc - ac)\,DC}{Dc}$$

(3) The horizontal plane passes above the top of the tree (Fig. 9). Then :—

$$H = AC - BC = \frac{(ac - bc)\,DC}{Dc}$$

In each of the above three cases two measurements are

required, unless the foot of the tree happens to be at the same level with the eye of the observer. The horizontal distance $D\ C$ must be measured, and $a\ c$, $b\ c$ and $D\ c$ are read off upon the instrument in the same units in which $D\ C$ has been measured.

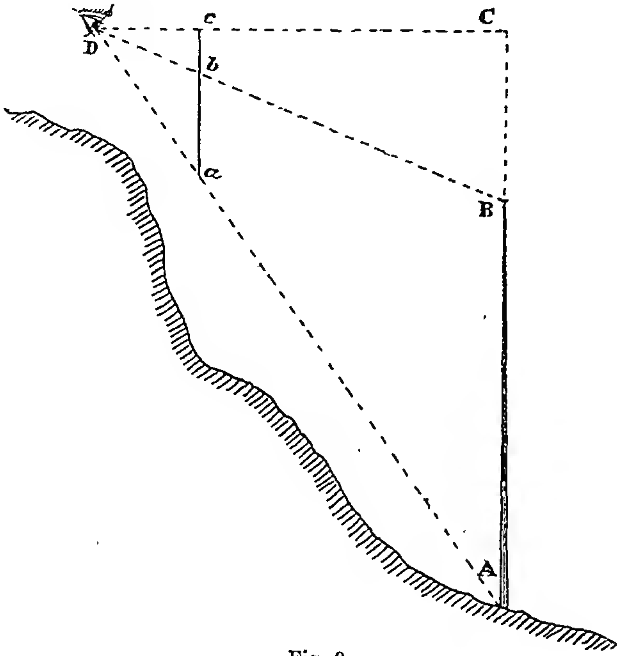


Fig. 9.

The measurement of $D\ C$ can be avoided in the following manner (Fig. 10 on next page):—

A staff $M\ N$, of a known length $= l$, is placed alongside the tree, so that both its ends can be seen. In this way the plumb line gives two further points, m and n, and the similar triangles $D\ C\ M$ and $D\ c\ m$, as well as $D\ C\ N$ and $D\ c\ n$, so that the following equations hold good:—

$$D\,C : D\,c = A\,B : a\,b$$

and

$$D\,C : D\,c = M\,N : mn\,;$$

hence,

$$A\,B : a\,b = M\,N : mn$$

and

$$A\,B = H = \frac{a\,b \times M\,N}{mn} = \frac{a\,b \times l}{mn}$$

This equation can easily be modified for the cases when the horizontal plane lies above the top or below the foot of the tree.

The indirect determination of the distance by means of a staff is less accurate than measuring it on the ground, as it is difficult to read off $m\ n$ with sufficient accuracy, owing to its smallness and the necessarily primitive arrangement of the height measuring instruments.

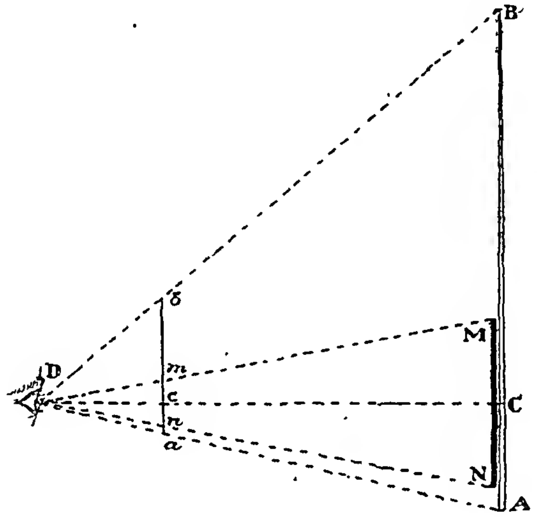


Fig. 10.

If the length of $a\ b = h$ can be read off at once, the business becomes more simple, and may be expressed as follows :—

If two parallel objects are cut by diverging rays, then the portions of the parallel objects lying between the said rays are proportionate to the lengths of the rays.

Let $D\ A = L$ (Fig. 11) be the length of ray from the eye of the observer to the foot of the tree, $D\ a = l$ that from the eye to the plumb line, $A\ B = H$ the height of the tree, and $a\ b = h$ the length of the plumb line between two rays going from the eye of the observer to the top and foot of the tree, then,

$$l : L = h : H$$

and

$$H = \frac{L}{l} \times h.$$

In this case L may be measured along the surface of the soil, whether it be level or slanting, while l and h are read off on the instrument.

The number of hypsometers based upon the above theories is very large; some being used with stands, others without. Only the latter are really useful for forest operations. Most in use are those by Faustman, Weise, and Christen. Others are those by Hossfeld, Winkler, Bose, and Klaussner. A very simple instrument is the measuring board by König.

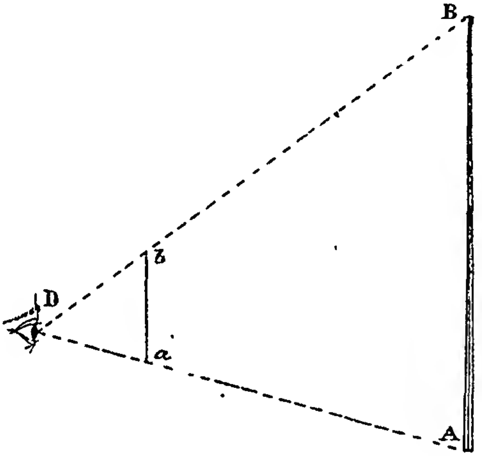


Fig. 11.

Measurements made with the above mentioned hypsometers are liable to yield inaccurate results, owing to the following causes:—

(1) Inaccurate reading owing to the unsteadiness of the plumb line in windy weather, or in consequence of a shaky hand.
(2) Inaccurate measurement of the base line.
(3) Slanting position of the tree.

Other things being equal, the most accurate results are obtained if the distance of the observer from the tree equals the height of the tree.

The inaccuracy of the better hypsometers does not exceed 2 per cent. of the height of the tree.

b. Trigonometrical Height Measuring.

This is based upon the measurement of the angles of elevation and depression indicated by rays running from the eye

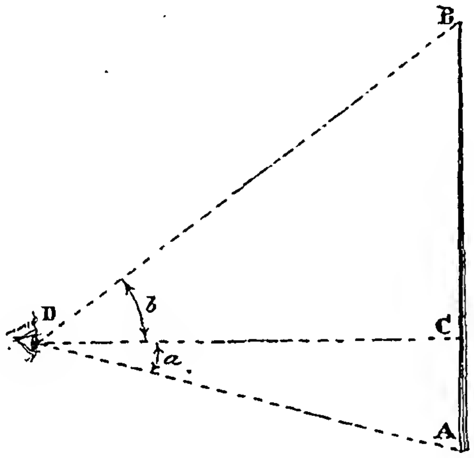


Fig. 12.

of the observer to the top and foot of the tree. In $\Delta\ B\ C\ D$ (Fig. 12):—

$$B\ C = D\ C \times tan.\ b$$

and in $\Delta\ D\ C\ A$,

$$A\ C = D\ C \times tan.\ a$$

hence,

$$A\ C + B\ C = H = D\ C\ (tan.\ a + tan.\ b).$$

If the horizontal line of vision passes below the foot of the tree, the above formula becomes :—

$$H = D\ C\ (tan.\ b - tan.\ a).$$

If it passes above the tree,

$$H = D\ C\ (tan.\ a - tan.\ b).$$

In each of these cases, the measuring of the horizontal line $D\ C$ can be avoided by placing a staff of known length alongside the tree. In that case (Fig. 13) :

$$M\,C = D\,C \times tan.\,m\,;\; N\,C = D\,C \times tan.\,n$$

and

$$M\,N = l = D\,C\,(tan.\,m + tan.\,n)$$

hence,

$$D\,C = \frac{l}{tan.\,m + tan.\,n}$$

By introducing this value into the former equation, the height is obtained as :—

$$H = \frac{l \times (tan.\,a + tan.\,b)}{tan.\,m + tan.\,n}$$

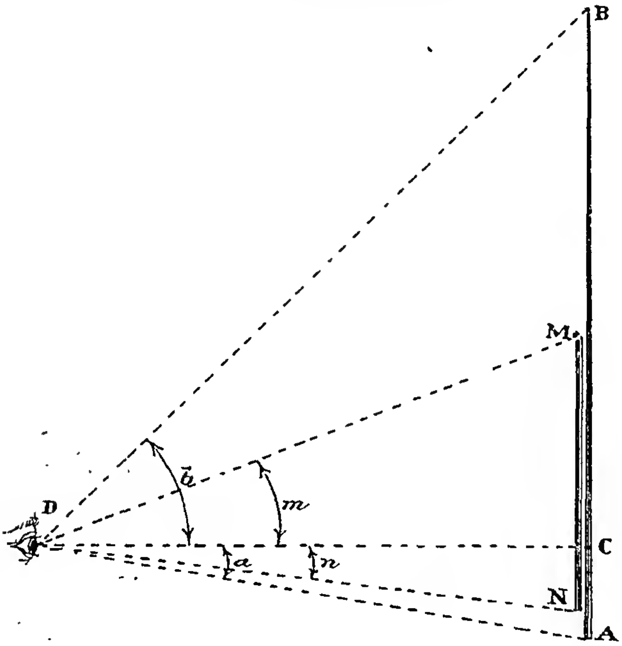


Fig. 13.

All instruments which measure vertical angles are suited for trigonometrical height measuring. For practical purposes it is desirable that the instrument should not require a stand, and that, besides the angles, the corresponding tangents should be marked on it.

c. Description of some of the more useful Instruments.

Weise's Instrument.—It consists of (1) a tube (T) with an objective in the shape of a cross at one end (O) and an eye-

piece (E) at the other. (2) A scale fastened longitudinally to the tube (called the height scale, H, Fig. 14); it is toothed on one side, and has the zero point some distance from its end. (3) A second scale, D, moving at the zero point of the height scale and at right angles to it (called the distance scale). From the upper or zero point of this scale depends a plumb line P.

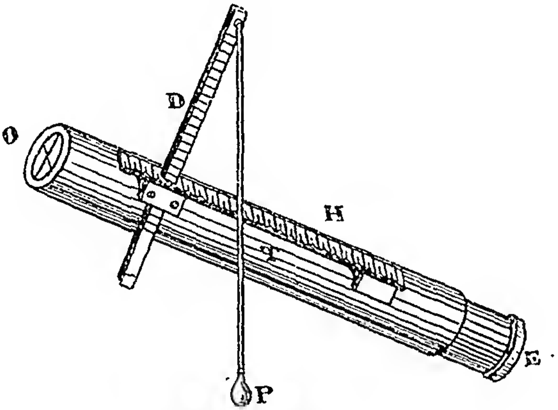


Fig. 14.—Weise's Hypsometer.

When not used, the distance scale and plumb line are kept in the tube.

In using the instrument, a position is chosen from which both the top and foot of the tree can be seen; then the horizontal distance from the point of observation to the tree is measured, and the distance scale drawn out until it indicates at the zero point of the height scale the number of units in the distance; then the tube is raised and directed towards the top of the tree, taking care that the up and down line of the objective keeps a vertical position. As soon as the horizontal line of the cross covers the top of the tree, the tube is gently turned from left to right, thereby causing the plumb line, which hitherto swung free, to be caught by the toothed edge of the height scale. The instrument is then taken down and the number of units, from the zero point to the point where the plumb line was arrested, read off. This number gives the

number of feet (or yards as the case may be) from the horizontal of the eye of the observer to the top of the tree. To this must be added (or deducted) the difference in height between the eye of the observer and the foot of the tree, which is obtained in the same way, by directing the tube towards the foot of the tree, reading the height on the prolongation of the scale towards O.

The theory of the instrument rests upon the similarity of

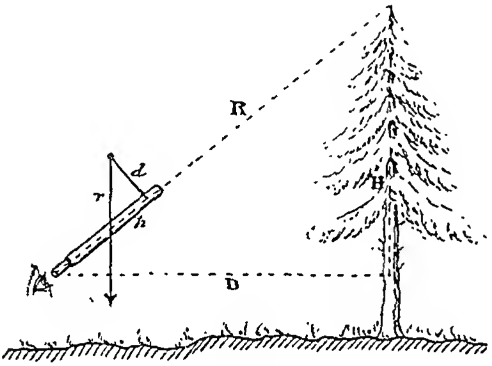


Fig. 15.

the triangles with the sides $R\ H\ D$ and $r\ h\ d$; that is to say, the following equation holds good :—

$$d : h = D : H$$

and

$$H = D \times \frac{h}{d}$$

If therefore the units of the scales, which give h and d, are of the same size, and d is so fixed that its units are the same number as the units of the measured distance D, it follows that the above formula gives the height.

Christen's Instrument.—It consists of a piece of metal (see Fig. 16) with protruding upper and lower edges (see a and b). The instrument is based upon the theory explained on page 17, which avoids the measurement of a base line. A staff of known length$=l$, say 4 yards, is placed alongside the foot of the tree. The instrument is then held in a vertical position

at some distance from the observer, and moved backward and forward until the top of the tree is seen along the upper edge a, and the foot along the lower edge b; then the point is marked

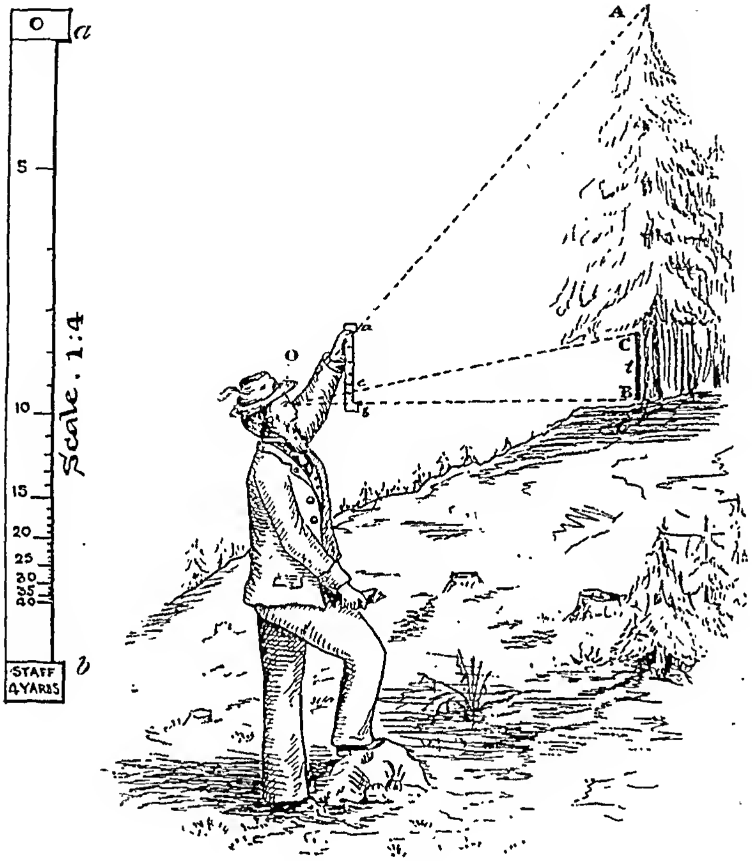


Fig. 16.—Christen's Hypsometer.

on the instrument, where a ray from the eye to the top of the staff hits the instrument at c. In this way similar triangles are formed, in which the following equation holds good :—

$$A\,B = \frac{a\,b}{b\,c} \times l, \text{ or } b\,c = \frac{a\,b \times l}{A\,B}$$

If now $a\,b = 12$ inches, and $l = 4$ yards, and successive values for $A\,B$ = height of tree, are introduced, corresponding

values of *b c* are obtained and can be marked on the instrument. In this way the heights can be read off straight on the instrument. For convenience sake the marks on the instrument are cuts, so that the top of the staff may be more easily seen.

The instrument has the disadvantage that the marks are very close one to another for heights over 30 yards. This might be obviated to some extent by lengthening the instrument and making it with a clasp in the middle, so that it could be folded together when out of use.

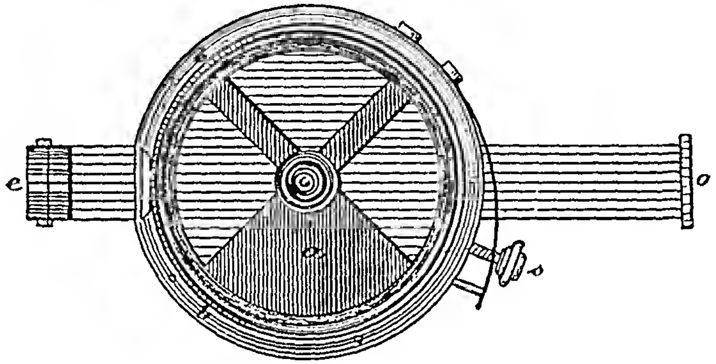


Fig. 17.—Brandis' Hypsometer and Clinometer.
(The front lid removed, so as to show the wheel.)

It is evident that, instead of using a staff 4 yards long, one of, say, 2 yards can be used. In that case the height read off on the instrument must be divided by 2.

The instrument works well up to 25, or at the outside up to heights of 30 yards; for higher trees it cannot be recommended in its present shape.

Brandis' Instrument * (Fig. 17).—This instrument is based on the trigonometrical method of height measuring. It consists of a tube with an objective *o* at one end and an eye-piece, *e*, in the shape of a horizontal slit, at the other. Attached to this tube is a wheel, which is weighted on one side and swings between two pivots, so that it always maintains the

* To be obtained from Herr Max Wolz, Mechaniker, Bonn, Germany.

same position when at rest. Oscillations can be arrested by a stop (see at *s* in figure). That point of the wheel which corresponds with the horizontal line of vision is marked as zero, and from this point the wheel is graduated to 60° up and down. A lens is fastened alongside the eye-piece, to facilitate the reading of the angle on the wheel. By directing the tube to any point the angle can be easily read off on the wheel, which preserves the same position while the instrument is being raised or lowered. The wheel is placed in a firm metal case.

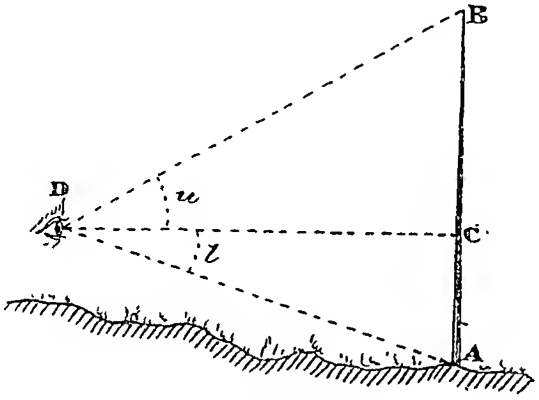


Fig. 18.

In using the instrument, any convenient position, where the top and foot of the tree can be seen, is chosen, the angles to the top and foot of the tree read off, and the distance from the eye of the observer to the foot of the tree measured. The height is then found by the formula (see Fig. 18) :—

$$H = \frac{DA \times \sin(u + l)}{\cos. u}$$

For convenience sake a little table accompanies the instrument, in which the heights corresponding to various distances and upper plus lower angles are given. In order to reduce this table as much as possible, it gives only upper angles from 40° to 50° in intervals of 2°, and lower angles from 0° to 25° in intervals of 5°. This necessitates placing a staff, on which

feet are marked by alternate colours, alongside the tree, so as to read off the distance between the lower ray of the lower angle and the foot of the tree, a distance which has to be added to the height taken from the table.

The instrument is at the same time an admirable clinometer, with which the angles of slopes can be measured and roads laid out.

The author has used the instrument extensively both for

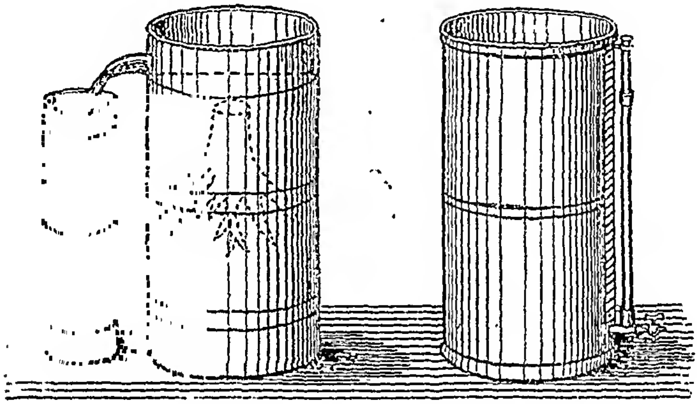


Fig. 20. Fig. 19.

the measurement of the height of trees, and for the laying out of forest roads; he has arrived at the conclusion that it is decidedly the most useful of similar instruments which are known to him.

The instrument works accurately, and much quicker than the reader would imagine; besides, its strong construction renders it admirably adapted for forest work.

6. *Instruments for the Direct Measurement of the Volume.*

For this purpose the xylometer, either alone or in combination with a scale, is used. The method is based upon the fact that a submerged body displaces a volume of water equal to the volume of the body, and the instrument used is called a xylometer. It consists of a graduated vessel, Fig. 19, in

which the wood is submerged. Before and after immersion the position of the water is noted, and the difference gives directly the volume. The method is employed for the measurement of irregular pieces, such as root wood and fagots. To obviate the necessity of submerging large quantities of wood, the whole is first weighed, and only a fraction immersed. Let the weight of the whole be $= W$, that of the immersed portion $= w$, the volume of the former $= V$, of the latter v, then :—

$$W : w = V : v$$

and

$$V = \frac{v}{w} \times W.$$

Instead of having a graduated vessel, the latter may be filled up to an opening, then the wood is immersed, the outflowing water caught in a separate vessel and measured (Fig. 20).

CHAPTER II.

MEASUREMENT OF FELLED TREES.

THE methods of measuring the various dimensions of felled trees have been explained in Chapter I. In this place the measurement of the volume will be dealt with.

Each tree consists of a stem or trunk, branches and roots. These have peculiar shapes of their own, which differ considerably; hence they must be considered separately.

1. *Volume of the Stem.*

If the stem, or trunk, of a tree had a regular or distinct shape, its volume could be calculated direct by means of a formula corresponding to that particular shape. As a matter of fact the stem shows different shapes in different parts of the tree.

Again, the shape of trees differs widely according to species, the ages of the trees, and the conditions under which they have grown up, whether in the open or in a crowded wood. At the same time, trees of the same species and age, which have grown under the same conditions, generally show shapes which are nearly identical. Moreover, experience has shown that each part of the stem shows approximately a constant form.

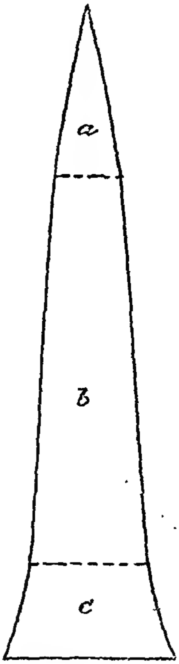


Fig. 21.

Thus the uppermost part, *a*, of an undivided stem has generally the shape of a cone, the lowest part, *c*, that of a truncated semicubical paraboloid, while the

bulk between these extremes approaches in shape a truncated Apollonian paraboloid or a cylinder.

If h = the height, or length,
S = the lower section,
s = the upper section, and
s_m = the middle section (Fig. 22),

the volume of each of the above-mentioned truncated solids is, according to Simpson's rule:—

$$V = \frac{S + 4 \times s_m + s}{6} \times h.$$

This formula reduces:—

For the cylinder to $V = S \times h$

For the cone to $V = \frac{S \times h}{3}$

And for the Ap. paraboloid to $V = \frac{S + s}{2} \times h$ or $= S_m \times h$.

By means of these formulæ it would be possible to calculate the volume of each part of the stem, provided its particular shape had first been ascertained. This, however, would be a tedious business, and it is necessary to search for a more simple procedure.

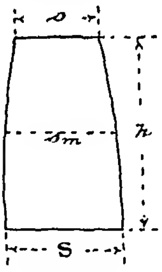


Fig. 22.

It has been found that by far the greater portion of the stem approaches in shape that of a paraboloid, and that, if the stem is divided into a number of sections of moderate length, each can, without committing any appreciable error, be considered as a truncated paraboloid, the volume of which is—

$$V = \frac{S + s}{2} \times h.$$

Or,

$$V = s_m \times h.$$

Of these two formulæ the latter is the more convenient, and experience has shown that it is even more accurate than the former.

According to this method the volume of the whole stem is obtained by means of the following formula (see Fig. 23) :—

$$\text{Volume of stem} = s_1 h_1 + s_2 h_2 + s_3 h_3 + \ldots,$$

where s_1, s_2, s_3 . . . are the sectional areas taken in the middle of successive paraboloids, and h_1, h_2, h_3 . . . the corresponding heights or lengths. If the pieces are made of equal length, the above formula changes into the following :—

$$\text{Volume of stem} = (s_1 + s_2 + s_3 + \ldots)\, h.$$

This formula is used in all scientific investigations, and the

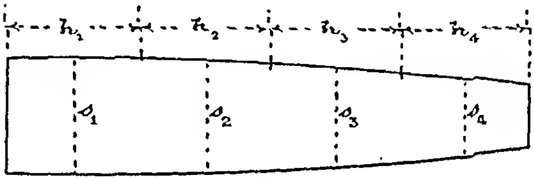


Fig. 23.

degree of accuracy with which it works depends on the length of the pieces.

For the purposes of determining the yield of woods, and for the sale of logs, the formula is further simplified by considering each log as one paraboloid, in other words, the volume is calculated from the middle section of the log, multiplied by its length, according to the formula

$$V = S_m \times H,$$

where S_m represents the sectional area in the middle and H the total length of the log. Experience has shown this formula to give sufficiently accurate results for all practical purposes.

The sectional area is obtained either by measuring the girth or the diameter. If g = girth, and d = diameter, the section is :—

$$S = \frac{g^2}{4\pi} = \cdot 0796 \times g^2,$$

or

$$S = \frac{\pi\, d^2}{4} = \cdot 785 \times d^2$$

and

$$V = \cdot 0796 \times g^2 \times H,$$

or

$$V = \cdot 785 \times d^2 \times H.$$

In practical work the sectional areas are taken from specially prepared tables; there are also tables which give directly the volume of logs according to their mean girth and length, or their mean diameter and length. (See App. D. pp. 390 to 393.)

All these calculations are made on the assumption that the section represents a circle. This is, however, rarely the case. As a rule the degree of divergence from the circular shape depends on:—

(1) Part of the stem; the lowest and uppermost parts differ most.

(2) Age of tree; young trees are more regularly shaped than old trees.

(3) Species.

(4) Conditions under which the tree has grown up; in crowded woods the shape is more regular than in the case of trees grown in the open; exposure to strong winds, slanting position, and the nature of the soil also affect the shape.

Generally, the sections of trees approach the shape of an ellipse, the great axis of which lies, in the same locality, as a rule in a constant direction. Where trees are much exposed to wind, the great axis lies generally in the direction of the prevailing wind; in Western Europe, therefore, from west to east or from south-west to north-east.

The inaccuracy caused by measuring the girth and calculating therefrom the sectional area has been found to amount to about 7 per cent. on an average; where only one diameter is measured the error may be the same or even more; where two diameters at right angles are measured and the mean taken, the error generally does not exceed 2 per cent. of the true value.

In Britain and in India, the sectional area in the middle is calculated by the method of the quarter girth, that is to say, by the formula—

$$S=\left(\frac{g}{4}\right)^2=\cdot 0625\times g^2.$$

In comparing this with the real sectional area $=\cdot 0796\times g^2$, it is found that the quarter girth method gives only $78\frac{1}{2}\%$ of the true volume, omitting $21\frac{1}{2}$ per cent. The method is based upon the assumption that this amount represents the waste incurred in squaring the timber. Quantities calculated by the exact method can be converted into the quantities corresponding to the other, by deducting $21\frac{1}{2}$ per cent. of the volume.

2. *Volume of Branch and Root Wood.*

In some cases special pieces of branch and root wood are of a sufficiently regular shape to measure and calculate their volume separately in the manner given above. As a general rule, however, such wood requires a different treatment. Its volume is ascertained by shaping it according to custom, and stacking it in a space of regular geometrical form. The volume of this space is ascertained, as well as the quantity of solid wood of a particular description which can be stacked in it.

Taking, for instance, a space of 100 cubic feet, the quantity of solid wood which can be stacked in it is ascertained according to whether the material consists of split wood, branch wood, fagots or root wood. This can be done by measuring each piece separately, an operation of considerable difficulty, and one which takes much time. A more expeditious way is to submerge the material in a xylometer and ascertain the volume by measuring the quantity of the displaced water. From the data thus obtained, average coefficients are calculated.

It is evident that different descriptions of wood give different coefficients. The solid contents of stacked wood depend on many things, amongst which may be mentioned:—

(1) Shape and nature of the pieces; thick, smooth, and straight pieces give more solid contents than thin, bent, uneven pieces.
(2) Length of pieces; short pieces pack better than long ones, hence they give a higher percentage of solid contents.
(3) Method of stacking; careful stacking causes the percentage of solid wood to be considerably increased.

It is evident from the above remarks that no absolutely average data can be given. By way of illustration, it may be mentioned that the coefficients which are officially recognized in Hesse-Darmstadt as representing averages, are the following:—

Split firewood	= ·7
Round firewood billets under 5″ diameter	= ·6
Root and stump wood	= ·5
Fagot wood stacked (not bound) . .	= ·2

That is to say, 100 cubic feet of stacked split firewood contain 70 cubic feet of solid wood and 30 cubic feet of air, etc.

3. *Volume of the Bark.*

In many cases it is desirable to ascertain the volume of the bark, especially when it is sold separately, as in the case of tanning bark. This can be done stereometrically or xylometrically. In the former case the pieces of wood are measured before and after barking, the difference giving the volume of the bark. If a xylometer is used, the bark can be measured separately, or the pieces of wood are measured before and after barking.

According to species, age, and locality, the bark comprises from 6 to 20 per cent. of the total volume. Schwappach found on a limited number of trees the following results:—

Oak . . .	= 15—20%	Alder . .	= 16—19
Ash . . .	= 12—14	Lime . . .	= 16—19
Elm . . .	= 9—11	Aspen . .	= 9—13
Birch . . .	= 13—17	Scotch Pine . .	= 10—16

CHAPTER III.

MEASUREMENT OF STANDING TREES.

1. *Ocular Estimate.*

Originally, the volume of standing trees was estimated, either merely by observation, or after having measured such dimensions as could easily be ascertained, namely the diameter and height. Such an estimate takes into consideration the special shape or form of each tree, and fixes the volume accordingly.

The accuracy of purely ocular estimates depends entirely on the person who makes them. To be only approximately correct, the estimator requires great practice, and opportunities to compare his estimates with actual measurements after the trees have been felled. Even then the results are subject to considerable errors, unless the estimator practises his art constantly. Mistakes of 25 per cent. are of common occurrence, and they may reach up to 100 per cent. in the case of an inexperienced estimator.

The uncertainty of absolutely ocular estimates led to the measuring of diameter (or girth) and height; this done, the basal area near the ground can be calculated, multiplied by the height, and an estimate made of the actual volume of the tree. It stands to reason that such an estimate is less dependent on the individuality of the estimator than that mentioned above, since he has only to estimate the proportion which exists between the actual volume and that of an imaginary body constructed out of the height and the sectional area at the base, a matter which he must decide according to the peculiar form of the tree. By degrees it was considered desirable to collect data regarding the form of various trees,

which might be utilized in subsequent estimates, and thus foresters arrived at the following method.

2. *Estimate of Volume by means of Form factors.*

a. Definition and Classification of Form factors.

Under "form factor" is understood the proportion which exists between the volume of a tree and that of a regularly-shaped body which has the same base and height as the tree. The form factor means, therefore, a coefficient with which the volume of the regularly-shaped (geometrical) body must be multiplied in order to obtain the volume of the tree.

Any regularly-shaped body, the volume of which can easily be calculated by means of a mathematical formula, is suited for the above purpose. In practice only the cone and cylinder have been employed, and at the present time only the latter is used. Let s be the area of the basal section of the tree, h its height, f the form factor, and v the volume, then—

$$\text{Volume of cylinder} = s \times h,$$
$$\text{Volume of tree} = v = s \times h \times f,$$

and

$$\text{Form factor} = f = \frac{v}{s \times h}.$$

The volume of the stem of a tree by itself is always smaller than that of the corresponding cylinder; hence the form factor for the stem only is always smaller than 1. If the volume of the branches is added, the form factor is sometimes greater than 1, especially during early youth.

Various kinds of form factors are used in forestry, of which the following may be mentioned :—

(1) Stem form factors, which refer only to the volume of the stem above ground.

(2) Tree form factors, which refer to stem and branches, omitting root wood.

(3) Timber form factors, which refer only to those parts of the tree which are classed as timber, whether they are

taken from the stem or branches, omitting all other material.

Form factors for branch wood, fagots, or root wood only are, as a rule, not used; their volume is ascertained by utilizing the results of actual fellings and determining their proportion to the volume of timber.

As it would be highly inconvenient to measure the diameter, or girth, of the tree close to the surface of the ground, where it is usually cut, it has been agreed to take the measurement at a convenient height. According as to whether that point is fixed or variable, the following kinds of form factors may be distinguished:—

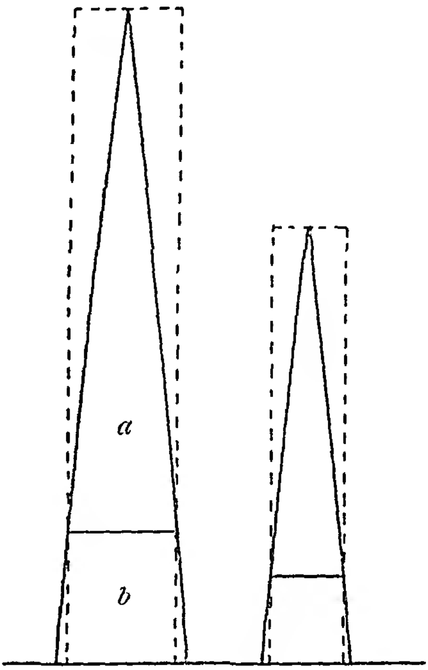


Fig. 24. Fig. 25.

(1) *Absolute Form factors.*—The diameter (or girth) is measured at any convenient height above the ground, and the form factor refers only to the part *a* of the tree above that point (Fig. 24), while the volume of the piece *b* below it, is ascertained by separate measurement and added to the rest. This is evidently troublesome and takes extra time.

(2) *True or Normal Form factors.*—The diameter (or girth) is measured at a constant proportion of the height of the tree, say $\frac{1}{10}$th, $\frac{1}{20}$th, etc. (Figs. 24 and 25). In this case the height of the ideal cylinder is equal to the height of the tree. Such form factors, it was believed, would have the advantage that all trees of the same shape would have the same form factor, since they have been measured at a height which bears in all cases the

same proportion to the total height. There are, however, various drawbacks to the employment of these form factors. In the first place the height of the tree must be determined before the point of measurement can be fixed; secondly, the latter may be very inconvenient in the case of very tall, as well as very short trees; thirdly, it has been found from actual measurements, that the factors thus obtained are by no means so regular as had been supposed, that is to say, trees of different heights show by no means the same form factor if measured at a constant proportion of the height.

(3) *Form factors based on Measurements made at Height of Chest, called Artificial Form factors.*—The diameter, or girth, is measured at the most convenient height from the ground, namely at chest-height of an ordinary man. (In Germany now generally fixed at 1·3 metres = to about 4′ 3″.) The height of the ideal cylinder is equal to the height of the tree. Owing to the measurements being taken at an absolutely constant height, the form factors of two trees, which show the same shape but differ in height, cannot be the same. It follows that, in using such form factors for calculating the volume of trees, the height of the latter must be taken into consideration. Nevertheless, in practice, these are the only form factors now used.

b. Determination of Form factors.

Formerly form factors were estimated, taking into consideration all points which affect them, such as species of tree, height, age, free or crowded position, etc. Such an operation requires much skill and practice, and in fact it comes pretty much to the same thing as estimating the volume direct. To eliminate such uncertainty, tables have been prepared which give the form factors for different species, heights and ages, such tables being based upon the results obtained by the measurement of numerous felled trees. Of late years it has been recognized that the variations due to age can be omitted, except in scientific investigations.

The following table shows the form factors for a number of species:—

FORM FACTORS FOR THE **SCOTCH PINE** ACCORDING TO KUNZE, **SPRUCE** AND **BEECH** ACCORDING TO BAUR, AND **SILVER FIR** ACCORDING TO LOREY.

Height (or Length). Feet.	Timber only down to 3 inches diameter.				Timber and Firewood (exclusive of Root wood).			
	Scotch Pine.	Spruce.	Silver Fir.	Beech.	Scotch Pine.	Spruce.	Silver Fir.	Beech.
20	·14	·18	·27	·13	·83	·88	·83	·73
30	·32	·31	·38	·21	·68	·77	·77	·67
40	·45	·41	·51	·30	·62	·69	·68	·62
50	·48	·47	·53	·40	·57	·64	·65	·59
60	·47	·48	·53	·45	·53	·61	·63	·57
70	·46	·49	·52	·47	·51	·59	·60	·56
80	·46	·49	·52	·48	·50	·57	·59	·56
90	·45	·48	·50	·50	·50	·55	·56	·57
100	·45	·47	·49	·51	·49	·53	·55	·58
110	·44	·46	·48	·52	·49	·51	·52	·59
120	·43	·44	·47	·52	·48	·49	·51	·60
130	...	·43	...	...	...	·48	...	...
140	...	·42	...	...	...	·48	...	...
150	...	·41	...	...	...	·47	...	...

In using these tables, it must not be forgotten that they give the averages of numerous measurements; hence they do not give reliable results in calculating the volume of a single tree. Their application should be restricted to the calculation of the volume of a number of trees, in other words of whole woods, where the differences between the several trees are likely to compensate each other.

3. *Estimate of Volume by means of Volume Tables.*

If, instead of giving the form factors only, they are multiplied by the corresponding heights and basal areas, the volumes of the trees are obtained, which can be arranged into so-called "volume tables." The latter can be defined as tables which give the volume of single trees arranged according to species, age, diameter, and height of tree.

These tables rest upon the assumption that trees of the same species, which have reached in the same time an equal height and diameter, show also an equal volume, and that trees of the same species, diameter, and height show volumes which differ with the age of the trees; in other words, the volume becomes greater with advancing age, although the height and the diameter at chest-height may be the same. Foresters say the trees become less tapering, or more full-bodied, or the point where the real tapering commences moves higher up the tree with advancing age.

In order to use such tables, it is necessary to ascertain the diameter at chest-height, the total height and the approximate age of the tree, when the volume corresponding to these data can be obtained from the tables. It must, however, not be forgotten that the tables give only averages, and consequently only true results if used for determining the volume of a number of trees, or of whole woods.

Note.—The Bavarian volume tables are based upon the measurement of 40,000 trees; they give the volumes for spruce, silver fir, larch, oak, beech and birch, arranged into two age classes (up to 90 years, and above 90 years old), the trees having been measured at a height of 4′ 3″ above the ground.

Volume tables are now-a-days less used than form factors, as the latter are more handy.

4. *Measurement of Standing Trees by Sections.*

Analogous to the measurement of felled trees by sections, the volume of standing trees can be ascertained by determining the diameter (or girth) at various heights from the ground. For this purpose a man must be sent up the tree, which is a cumbrous procedure, or the several diameters must be determined indirectly. The latter, as has been explained in Chapter I. (p. 12), is subject to great inaccuracies; hence the method is without practical value.

5. *Pressler's Method of ascertaining the Volume of Standing Trees.*

In order to avoid the determination and application of average figures, Pressler devised the following method of determining the volume by direct measurement on the tree: He determines on the stem of the tree the point where the diameter is one-half of that at chest-height; let the distance between the two points be h (Fig. 26). He found that the volume of the stem above height of chest is equal to $\frac{2}{3}$ of the basal area multiplied by h.

If s = basal area at height of chest and v = volume, then

$$v=\frac{2}{3}\times s\times h.$$

The above formula agrees very well with the results of actual measurements. To the volume thus ascertained has to be added that of the piece below height of chest, which Pressler assumes to have also a cross section = s. If its length is equal to l, the volume of the whole stem comes to:

$$v=\frac{2}{3}\times s\times h+s\times l=\frac{2}{3}s\left(h+\frac{3}{2}l\right)$$

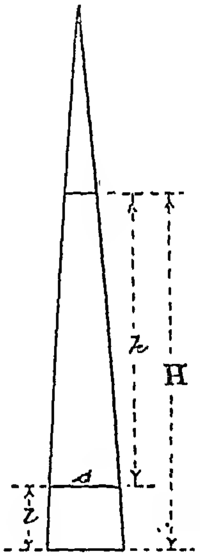


Fig. 26.

If now $h + l$ is put = H, the formula becomes

$$v=\frac{2}{3}s\left(H+\frac{l}{2}\right)$$

This formula gives the volume near enough for all practical purposes; to its amount has to be added the volume of branches, whenever required. The drawback of the method lies in the difficulty of ascertaining the point of the stem where the diameter is equal to $\frac{1}{2}$ of the diameter at chest-height. Pressler ascertains it with an instrument specially constructed

for the purpose; but neither this nor any other instrument works with sufficient accuracy. Moreover, the stems of many trees are irregularly shaped, so that two or more places may have the desired diameter, apart from the fact that in many cases trees divide into two or more branches before the diameter has fallen to $\frac{1}{2}$ of the diameter at chest-height.

CHAPTER IV.

DETERMINATION OF THE VOLUME OF WHOLE WOODS.

This chapter may be divided into three sections, according to whether the measurements extend over the whole wood, or over only a selected portion of it, or whether the volume is estimated.

Section I.—Measurements extending over the Whole Wood.

The method demands a uniform treatment of the whole wood, but a distinction may be drawn between the measurement of all trees, or that of selected trees, called sample trees.

I. *MEASUREMENT OF ALL TREES.*

Each tree is measured separately, and its volume ascertained in one of the ways described in Chapter III. By adding up the volumes of the several trees, that of the whole wood is obtained.

As the method takes much time, it is, in practice, only employed when the total number of trees is small, or when the wood is of an irregular description. As a rule, the following system is chosen, as it works more rapidly.

II. *DETERMINATION OF VOLUME BY MEANS OF SAMPLE TREES.*

The volume of a wood consists of the sum of the volumes of the individual trees. The volume of each tree is calculated according to the formula

$$v = s \times h \times f,$$

where s represents the basal area at a certain height, h the total height, and f the form factor of the tree. In all cases where s, h, and f differ from tree to tree, nothing remains but to ascertain them separately for each tree. In the case of regularly-grown woods, however, there are always a number of

trees which show, at any rate approximately, the same basal area, height, and form factor, so that they can be thrown together and dealt with in an uniform manner; in other words all trees of a wood which show the same base, height, and form factor, are joined into one class; the volume of one tree (or of a few trees) is ascertained, and the volume of the whole class obtained by multiplying the former by the number of trees in the class. If every class is dealt with in the same way, the volume of the whole wood is obtained by adding together the volumes of the several classes.

So far, however, little or no advantage is gained, because it would be necessary to ascertain the base, height, and form factor of each tree in order to put it into its proper class, and when this has once been done, the volume of each tree may just as well be calculated separately. Moreover, in crowded woods the height is not always easy to measure, and the form factor could only be estimated, unless it is taken from a table. Only the basal area is easily ascertainable by measuring either the diameter or the girth.

Here, experience had to be called in, which fortunately showed that in regularly-grown crowded woods, the height and form factor are approximately functions of the diameter of the tree; in other words, trees of the same diameter have approximately the same height and form factor. At any rate this is found to hold good to a sufficient extent, so as to justify a classification according to diameter classes only.

In open woods, however, the height and form factor vary within much wider limits, so that, besides diameter classes, at any rate also height classes must be formed. Hence, the two cases must be dealt with separately.

A. The Height is a Function of the Diameter.

1. *Description of the General Method.*

a. Formation of Diameter Classes.

The number of classes depends on the difference between the largest and smallest trees of a wood, and the desired degree

of accuracy. As a rule all classes are given the same extent, that is to say, either one inch, two inches, three inches, etc., or part of an inch. For the purpose of forest working plans in Europe each class comprises one or two inches; in India frequently as yet 6 inches.

The calliper used in measuring the diameters should have a rounded-off scale, as described in Chapter I., that is to say, in the case of inch classes, the first should comprise the space from $\frac{1}{2}''$ to $1\frac{1}{2}$ inch; the second that from $1\frac{1}{2}$ to $2\frac{1}{2}$ inches, etc.

For scientific investigations the classes may be further reduced to a part of an inch.

b. Height and Manner of Measurement.

All trees must be measured at the same height, the latter being so chosen, that the place of measurement falls above the irregular swelling frequently observed near the foot of the tree; at the same time the height should not be so great that it becomes difficult for an ordinary-sized man to measure accurately. Whenever practicable, the height should be the height of chest of an average man.

In executing the measurement all the precautions indicated in Chapter I. must be duly taken, so as to obtain as accurate results as possible. More especially any irregularity in the shape of the sections must be duly considered. Where the section differs systematically from that of a circle, either two diameters at right angles must be measured, or the direction of measurement changed from time to time. For instance, after a certain number of stems have been measured with the face of the measurer to the east, an equal number must then be measured at right angles, that is to say, with the face of the measurer towards the north or south. Or the change can be made at alternate trees. In this manner average diameters are obtained.

c. The Booking of the Measurements.

In measuring the diameter, the gaugers call out each measurement, and in mixed woods also the species; the book-keeper enters each announcement, repeating it at the time, so as to prevent mistakes.

A book-keeper may work with one or two gaugers, the party taking a narrow strip of the wood at the time; each tree is marked as soon as measured, preferably with chalk.*

The booking can be done in a variety of ways, as the following samples will show:—

Diameter in Inches measured at 4′ 3″ from the Ground.	Species.			Number of Trees.			
	Beech.	Oak.	Ash.	Beech.	Oak.	Ash.	Total
8				18	9	3	30
9				23	13	8	44
10				37	13	9	59
11				27	14	7	48
			Grand Total	105	49	27	181

The first method of booking is least liable to errors.

d. Selection and Number of Sample Trees.

As the volume of the whole class is to be calculated from that of the sample tree, it is necessary to select for the latter

* In some parts of India it is customary to paste small pieces of paper on the trees, each different colour representing a distinct class; this method has for its object to facilitate the control exercised by the supervising officer. The method was introduced by Mr. R. Ellis, Deputy-Conservator of Forests.

a tree which represents the average of the class; in other words, the sample tree should have the mean height, as near as possible a circular section, a fairly straight and not a forked stem, and an average extent of crown. Even with the greatest care it is not always possible to avoid errors in the selection; hence it is generally advisable to take several sample trees for each class, and to ascertain the average for the calculation of the volume of the class. The actual number of sample trees depends on the desired degree of accuracy, and the total number of trees in the class. At the same time the felling of many sample trees is undesirable, hence their number should be kept within reasonable limits.

A further requirement is, that the sample tree should show a basal area which corresponds exactly to the mean section of the class. Such a tree is only in exceptional cases found, hence it is necessary to take a tree as near as possible to the true section and to modify the volume in proportion of the basal areas of the true and false sample trees. Let v be = volume of true sample tree, v' that of the false sample tree, s and s' the corresponding basal areas, then v is found by the formula:

$$v : v' = s : s'$$

and

$$v = v' \times \frac{s}{s'},$$

e. Determination of the Volume of Sample Trees.

The volume of the sample trees is determined, either by felling and measuring them on the ground, or by means of form factors or volume tables.

If the trees are felled, the stem and all straight pieces of branches, in fact all regularly shaped parts, are divided into sections of moderate length, from 3 to 10 feet, according to the desired degree of accuracy, and the volume of each section is ascertained separately according to the formula :—

Volume = section in the middle × by the length.

The volume of all irregular pieces, including root and branch wood, is ascertained:

either by the xylometric method,
or by proportionate figures,
or by measuring their volume stacked, and multiplying it by known reducing factors, if such are available.

The xylometric method has been explained in Chapter I.

Proportionate figures are obtained from actual fellings. If it has been found that in the felling of a wood, every 100 cubic feet of timber are accompanied by, say, 20 cubic feet of firewood, that proportion can be applied to other woods of a similar description.

The determination of the volume of sample trees by means of form factors or volume tables can be highly recommended whenever suitable data are available, because they give averages, and that is just what is wanted in this case. Experience has shown that form factors and volume tables are applicable for a considerable distance outside the locality for which they have been prepared.

f. Calculation of the Volumes of the Classes and of the whole Wood.

Here several cases may occur:

(1) One sample tree has been measured, the dimensions of which are exactly the average of the class. In that case the volume of the class is obtained by multiplying the volume of the sample tree by the number of trees in the class.

If

V = volume of whole wood,
$V_1, V_2, V_3 \ldots$ = volumes of classes 1, 2, 3 ...
$v_1, v_2, v_3 \ldots$ = volumes of mean sample trees of successive classes,
$n_1, n_2, n_3 \ldots$ = numbers of trees in successive classes,

then

$$V_1 = v_1 \times n_1, \; V_2 = v_2 \times n_2 \ldots$$

and

$$V = V_1 + V_2 + V_3 + \ldots = v_1 \times n_1 + v_2 \times n_2 + v_3 \times n_3 + \ldots$$

(2) The sample trees in the several classes differ in basal area somewhat from the mean basal areas.

If the volumes of the approximate sample trees are v_1'; v_2'; v_3' . . . and the corresponding basal areas $= s_1'$; s_2'; s_3'; . . . . then

$$V_1 = \frac{v_1' \times s_1}{s_1'} \times n_1; \quad V_2 = \frac{v_2' \times s_2}{s_2'} \times n_2; \quad V_3 = \frac{v_3' \times s_3}{s_3'} \times n_3 \ldots$$

As

$s_1 \times n_1 = S_1 =$ basal area of the first class,
$s_2 \times n_2 = S_2 =$ basal area of the second class, etc.

the volume of the wood is:

$$V = \frac{v_1' \times S_1}{s_1'} + \frac{v_2' \times S_2}{s_2'} + \frac{v_3' \times S_3}{s_3'} + \ldots$$

(3) Several sample trees are measured in each class. In that case:—

$$V_1 = \frac{(v_1' + v_1'' + v_1''' + \ldots) \times S_1}{s_1' + s_1'' + s_1''' + \ldots}; \quad V_2 = \frac{(v_2' + v_2'' + v_2''' + \ldots) \times S_2}{s_2' + s_2'' + s_2''' + \ldots}$$

etc., and

$$V = \frac{(v_1' + v_1'' + v_1''' + \ldots) \times S_1}{s_1' + s_1'' + s_1''' + \ldots} + \frac{(v_2' + v_2'' + v_2''' + \ldots) \times S_2}{s_2' + s_2'' + s_2''' + \ldots} + \ldots$$

g. Clubbing together several Classes, leading to the Method of the Arithmetical mean Sample Tree.

In order to shorten the method described above, and to reduce the number of sample trees to be felled, several, or all, classes may be clubbed together into a group.

Let

$n_1, n_2, n_3 \ldots$ be the numbers of trees in the several classes
$s_1, s_2, s_3 \ldots$,, basal area ,, ,, ,, ,,
$h_1, h_2, h_3 \ldots$,, heights ,, ,, ,, ,,
$f_1, f_2, f_3 \ldots$,, form factors ,, ,, ,, ,,

and

s, h, f the basal area, height and form factor of the mean tree of all classes thrown together.

then the following equation holds good :—

$$V = n_1 \times s_1 \times h_1 \times f_1 + n_2 \times s_2 \times h_2 \times f_2 + \ldots$$
$$= (n_1 + n_2 + \ldots) \times s \times h \times f.$$

If it is now assumed that $h_1 \times f_1 = h_2 \times f_2 = h_3 \times f_3 = \ldots = h \times f$ then the above equation becomes:

$$n_1 \times s_1 + n_2 \times s_2 + \ldots = (n_1 + n_2 + \ldots)\ s,$$

and

$$s = \frac{n_1 \times s_1 + n_2 \times s_2 + \ldots}{n_1 + n_2 + \ldots} = \frac{S}{N}$$

where S = basal area of all trees of the group, and
N = total number of trees ,, ,,

In other words, the basal area of the average tree is equal to the arithmetical mean of the basal area of all trees contained in the group.

The volume of the group is then:

$$V = v \times N,$$

where v represents the volume of the arithmetical mean sample tree, with a basal area $= s$.

If no tree can be found with the basal area s, another as near as possible to it is chosen of a section s', and the volume of the group is obtained by the formula :

$$\mathrm{V} = \frac{\mathrm{v}' \times \mathrm{s}}{\mathrm{s}'} \times \mathrm{N} = \frac{\mathrm{v}' \times \mathrm{S}}{\mathrm{s}'},$$

since $s \times N = S$ = the basal area of all trees in the group.

If several approximately mean sample trees are taken, the formula changes into the following:—

$$\mathrm{V} = \frac{(\mathrm{v}' + \mathrm{v}'' + \mathrm{v}''' + \ldots) \times \mathrm{S}}{\mathrm{s}' + \mathrm{s}'' + \mathrm{s}''' + \ldots}$$

The above method rests on the assumption that $h_1 f_1 = h_2 f_2 = h_3 f_3 = \ \ldots\ = h f$.

This, however, is not absolutely correct, though it holds good approximately in all regularly-grown woods. It follows that the degree of accuracy decreases with the increase in the number of classes which are clubbed together into one group, the least

accuracy being obtained by joining all classes into one group. In this latter case the method is known as "the method of the arithmetical mean sample tree."

Example.—In order to illustrate this and the methods to be described hereafter, one acre of Scotch pine wood, 70 years old, was measured, and twenty-three sample trees of various diameters felled and measured. Only timber down to 3″ diameter at the small end has been included in the account. The wood is situated in the Cooper's Hill School forest at Cæsar's Camp, on gravelly sand, with a fair layer of humus, showing a quality between III. and IV. according to Weise's yield tables. The statement on pp. 52 and 53 illustrates the procedure which has just been described.

2. *Modifications of the Method.*

It has been shown above that the volume of a wood is represented by the formula :—

$$V = V_1 + V_2 + V_3 + \ldots = v_1 \times \frac{S_1}{s_1} + v_2 \times \frac{S_2}{s_2} + v_3 \times \frac{S_3}{s_3} + \ldots$$

It is obvious that, as long as the fractions $\frac{S_1}{s_1}, \frac{S_2}{s_2}, \frac{S_3}{s_3} \ldots$ differ, the volumes of the sample trees in the several classes must be measured separately. In order to avoid this inconvenience, it has been proposed to fix the number of sample trees in each class so, that

$$\frac{S_1}{s_1} = \frac{S_2}{s_2} = \frac{S_3}{s_3} = \ldots = \text{a constant} = c,$$

when the above formula reduces to :—

$$V = (v_1 + v_2 + v_3 + \ldots) \times c.$$

By following this method it is not necessary to keep the sample trees separate; they can be thrown together and measured in one lot. This is a great convenience, which saves also much time. The volume of all sample trees multiplied by the constant c gives the volume of the wood. In this way a modification of the general method has been elaborated, which is known by the name of Draudt's method.

a. Draudt's Method.

Draudt selects in each class the same percentage of sample trees, thus ensuring that—

$$\frac{S_1}{s_1}=\frac{S_2}{s_2}=\frac{S_3}{s_3}=\ldots$$

Let be

$$V=v_1\times n_1+v_2\times n_2+v_3\times n_3+\ldots$$

If p per cent. of the trees in each class are taken as sample

CALCULATION OF VOLUME BY INCH CLASSES, FOUR GROUPS, AND FOR ONE ACRE OF SCOTCH

Diameter in Inches.	Number of Trees.	Basal Area in Square Feet.	Method of Inch Classes. Diameter of real Sample-Tree.	Basal Area of Sample-Tree.	Volume of Sample-Tree in Cubic Feet.	Volume of Class in Cubic Feet.	Method of Number of Group.	Number of Trees in Group.	Basal Area of Group.	Mean Sample Tree. Basal Area.	Diameter
4	4	·349	4·0	·087	1·80	7					
5	12	1·637	5·25	·150	3·27	36	I.	83	18·048	·218	6·3
6	26	5·105	6·4	·223	4·41	101					
7	41	10·957	7·5	·307	6·92	247					
8	42	14·661	8·5	·394	7·26	270					
9	51	22·532	9·25	·467	12·13	585	II.	144	65·010	·451	9·1
10	51	27·817	9·6	·503	11·63	643					
11	34	22·439	11·0	·660	16·19	550					
12	16	12·566	12·1	·799	20·41	321	III.	59	43·301	·734	11·6
13	9	8·296	13·1	·936	25·93	230					
14	6	6·414	13·6	1·009	21·87	139					
15	5	6·136	15·0	1·227	28·40	142	IV.	16	19·892	1·243	15·1
16	3	4·189	16·4	1·467	38·53	110					
17	2	3·153	17·0	1·576	39·50	79					
Total...	302	146·251				3460					

trees, the proportion of these to the total number of trees is $=\frac{p}{100}=\cdot op$. By multiplying each term in the above equation by this coefficient, the following equation is obtained :—

$$V\times\cdot op=v_1\times n_1\times\cdot op+v_2\times n_2\times\cdot op+v_3\times n_3\times\cdot op+\ldots$$

Here, $v_1\times n_1\times\cdot op$ represents the volume of the sample trees in the first class, $v_2\times n_2\times\cdot op$ that of second class, &c. If the

THE ARITHMETICAL MEAN SAMPLE TREE OF THE WHOLE WOOD PINE WOOD, 70 YEARS OLD.

FOUR GROUPS.				METHOD OF ARITHMETICAL MEAN SAMPLE-TREE.							
REAL SAMPLE TREES.			Volume of Group.	Number of Trees.	Basal Area of all Trees.	Mean Sample Tree of Wood.		REAL SAMPLE TREES.			Volume of Wood.
Diameter	Basal Area.	Volume.				Basal Area.	Diameter	Diameter	Basal Area.	Volume.	
6·4	·223	4·41									
6·5	·230	4·52									
Total	·453	8·93	356								
9·25	·467	12·13									
9·40	·482	10·81									
Total	·949	22·94	1571					9·25	·467	12·13	
								9·40	·482	10·81	
								9·60	·503	11·63	
11·5	·721	17·43		302	146·251	·484	9·4	Total	1·452	34·57	3482
11·6	·734	19·17									
Total	1·455	36·60	1089								
15·0	1·227	27·79									
15·0	1·227	28·40									
Total	2·454	56·19	455								
		Total...	3471								

$$V=\frac{34\cdot57\times146\cdot251}{1\cdot452}=3482 \text{ cubic feet.}$$

volume of all the sample trees, $v_1 \times n_1 \times {\cdot}op + v_2 \times n_2 \times {\cdot}op + \ldots = v$, then

$$V \times {\cdot}op = v$$

and

$$V = \frac{v}{{\cdot}op} = \frac{v \times 100}{p}.$$

It happens generally that the number of sample trees in each class contains a fraction of one. Such fractions are eliminated by considering ·51 as a full sample tree, and by neglecting ·50 and under, taking care that the total number of sample trees is as nearly as possible correct. The result of this operation is, that the original proposition is no longer absolutely maintained; in other words, the volume of the actual sample trees does no longer represent the true value $\frac{V \times p}{100}$. To avoid this inaccuracy, Draudt introduces the basal area of the whole wood $=S$, and of the sample trees $=s$, by saying:—

$$s : S = v : V,$$

and

$$V = v \times \frac{S}{s}.$$

This formula is used to calculate not only the whole volume, but also that of timber and firewood separately.

The advantages of Draudt's method are:—

(1) That the sample trees can all be worked up together; and

(2) That it yields a high degree of accuracy.

Its drawbacks are:—

(1) That in rounding off the number of sample trees in each class, inaccuracies are likely to be introduced; and

(2) That frequently no sample trees at all are taken from classes which contain only a small number of trees.

The larger the wood, that is to say, the greater the number of trees in the several classes, the more accurately the method works.

Example.—The following example will further explain the

method. The data are the same as those used in the previous example, but they have been multiplied by ten all round, in order to obtain a larger number of trees.

CALCULATION OF VOLUME ACCORDING TO DRAUDT'S METHOD.
Area = 10 Acres.

Diameter	No. of Trees.	Basal Area.	Percentage = 1% Multiplied by Number of Trees.	Sample Trees.				Volume of Wood.
				Number Rounded Off.	Basal Area should be	Basal Area really is	Volume.	
4	40	3·49	·40					
5	120	16·37	1·20	1	·136	·150		
6	260	51·05	2·60	3	·588	·669		
7	410	109·57	4·10	4	1·068	1·228		
8	420	146·61	4·20	4	1·396	1·576		
9	510	225·32	5·10	5	2·210	2·335		
10	510	278·17	5·10	5	2·725	2·515		
11	340	224·39	3·40	3	1·980	1·980		
12	160	125·66	1·60	2	1·570	1·598		
13	90	82·96	·90	1	·922	·936		
14	60	64·14	·60	1	1·069	1·009		
15	50	61·36	·50	1	1·227	1·227		
16	30	41·89	·30					
17	20	31·53	·20					
	3020	1462·51	30·20	30	14·891	15·223	357·60	34357

Per Acre = 3436

$$V = \frac{357{\cdot}60 \times 1462{\cdot}51}{15{\cdot}223} = 34357.$$

b. Urich's Method.

In order to avoid the slight inaccuracy caused by rounding off fractions of sample trees, Urich proposes to arrange the trees in such groups, that each sample tree represents the same number of trees. For this purpose he arranges the trees in groups, so that each contains the same number of trees; he then calculates the arithmetical mean sample tree for each group. For the rest he proceeds on the same lines as Draudt, so that he obtains the volume of the whole wood by the same formula, namely:—

$$V = v \times \frac{S}{s}.$$

Urich's method is thus a combination of Draudt's method with that of the arithmetical mean sample tree for each group.

The degree of accuracy depends on the number of groups which are formed, and on the number of sample trees measured in each. Too large a number of groups is however inconvenient, as it involves repeated separation of the original diameter classes.

CALCULATION OF VOLUME,

Diameter.	Number of Trees.	Groups. No.	Groups. Diameter.	Number of Trees. Detailed.	Number of Trees. Total.	Basal Area. Detailed.	Basal Area. Total.
4	40	I.	4	40		3·49	
5	120		5	120		16·37	
6	260		6	260		51·05	
7	410		7	335	755	89·53	160·44
		II.	7	75		20·04	
8	420		8	420		146·61	
9	510		9	260	755	114·87	281·52
		III.	9	250		110·45	
10	510		10	505	755	275·44	385·89
		IV.	10	5		2·73	
11	340		11	340		224·39	
12	160		12	160		125·66	
13	90		13	90		82·96	
14	60		14	60		64·14	
15	50		15	50		61·36	
16	30		16	30		41·89	
17	20		17	20	755	31·53	634·66
	3020				3020		1462·51

The method has the disadvantage that the basal areas must be calculated before the sample trees can be selected. Urich proposes to avoid this by estimating the sizes of the sample trees in the several groups, a procedure which may lead to inaccuracies.

Example.—

ACCORDING TO URICH'S METHOD.

Mean Sample Tree.		Real Sample Trees.					Volume of Wood, Cubic feet.
Basal Area.	Diameter.	Number.	Diameter.	Basal Area. Detailed.	Basal Area. Total.	Volume. Cubic feet.	
		1	6·4	·223			
·213	6·25	2	6·5	·230	·453		
		3	7·5	·307			
·373	8·25	4	8·5	·394	·701		
		5	9·4	·482			
·511	9·7	6	9·6	·503	·985		
		7	12·1	·799			
·841	12·4	8	12·1	·799	1·598		
				Total...	3·737	88·36	34581
						Per Acre	=3458

$$V = \frac{88{\cdot}36 \times 1462{\cdot}51}{3{\cdot}737} = 34581.$$

c. Robert Hartig's Method.

In Draudt's and Urich's methods each sample tree represents the same number of trees. As the volume increases rapidly with the diameter, it follows that a sample tree in a class or group of small diameter represents a much smaller volume than one in a class or group with a large diameter. For this reason Robert Hartig argues that the number of

MEASUREMENT OF VOLUME ACCORDING

Diameter	Number of Trees.	Basal Area.	Groups. No.	Groups. Diameter	Number of Trees. Detailed.	Number of Trees. Total.	Basal Area. Detailed.	Basal Area. Total.
4	40	3·49	I.	4	40	1337	3·49	365·53
5	120	16·37		5	120		16·37	
6	260	51·05		6	260		51·05	
7	410	109·57		7	410		109·57	
8	420	146·61		8	420		146·61	
9	510	225·32		9	87		38·44	
			II.	9	423	751	186·88	365·78
10	510	278·17		10	328		178·90	
			III.	10	182	575	99·27	365·29
11	340	224·39		11	340		224·39	
12	160	125·66		12	53		41·63	
			IV.	12	107	357	84·03	365·91
13	90	82·96		13	90		82·96	
14	60	64·14		14	60		64·14	
15	50	61·36		15	50		61·36	
16	30	41·89		16	30		41·89	
17	20	31·53		17	20		31·53	
Total...	3020	1462·51				3020		1462·51

sample trees in each class or group should be proportionate to the volume which it contains, and not to the number of trees. As the volume is fairly proportionate to the basal area, Robert Hartig forms groups which contain equal basal areas. He divides the total basal area of the wood by the number of groups which he proposes to form; this gives the basal area to be allotted to each group. After having placed in each a sufficient number of trees to give that basal area, he calculates

TO ROBERT HARTIG'S METHOD.

Mean Sample Trees.		Real Sample Trees.					Volume of Groups and of Whole Wood.	Remarks.
				Basal Area.				
Basal Area.	Diameter	No.	Diameter	Detailed.	Total.	Volume.		
		1	7·5	·307				
·273	7·1	2	7·5	·307	·614	13·54	8061	$\frac{13·54 \times 365·53}{·614} =$ 8061 cubic feet.
		3	9·4	·482				
·487	9·4	4	9·6	·503	·985	22·44	8334	
		5	10·7	·624				
·635	10·8	6	10·7	·624	1·248	29·51	8638	
		7	13·1	·936				
1·025	13·7	8	13·6	1·009	1·945	47·80	8992	
						Total...	34025	Per Acre = 3402 cubic feet.

the mean sample tree for each group, and selects an equal number of these for each.

The formula for R. Hartig's method is as follows :—

$$V = v_1 \times \frac{S_1}{s_1} + v_2 \times \frac{S_2}{s_2} + v_3 \times \frac{S_3}{s_3} + \ldots$$

As $\frac{S_1}{s_1}, \frac{S_2}{s_2}, \frac{S_3}{s_3}, \ldots$ are not equal the one to the other, it follows that the sample trees must be measured and the volume of each group calculated separately. By adding together the volumes of the groups the volume of the whole wood is obtained. This makes the method more laborious than those of Draudt and Urich.

For example, see pp. 58 and 59.

3. *Comparative Accuracy of the several Methods.*

Guided by investigations made by various authors, it may be said that in the majority of cases the difference between the calculation made according to any one of the above-mentioned methods and the results of actual fellings keeps within 2 per cent., that the maximum error in the case of the method of the arithmetical mean sample tree may be placed at 10 per cent., and in the case of all the other methods at 5 per cent.

If the results obtained in the examples used above are put together, the following data are obtained :—

	Volume in c'.	Difference in %.
Method of inch classes :		
Each inch class being calculated separately	= 3460 . . .	0
All sample trees being thrown together	= 3554 . . .	+ 2·7
Method of four groups :		
Each group being calculated separately	= 3471 . . .	+ ·3
All sample trees being thrown together	= 3433 . . .	− ·8
Method of arithmetical mean sample tree	= 3482 . . .	+ ·6
Draudt's method	= 3436 . . .	− ·7
Urich's method	= 3458 . . .	− ·06
Hartig's method :		
Each group being calculated separately	= 3402 . . .	− 1·7
All sample trees being thrown together	= 3457 . . .	− ·09

It will be seen that for the methods as described above the greatest difference amounts to 1·7 per cent. Even if, in the case of ordinary inch classes, all sample trees are thrown together, the difference amounts only to 2·7 per cent. Under these circumstances it appears that any of the methods meets the requirements of measurements for the preparation of working plans, and that much more depends on the care bestowed upon the operation than on the particular method followed. If the actual fellings show greater differences than the calculations justify, they are frequently due to extraneous causes, such as the felling of the trees at some distance above the ground, careless working up of the material, inaccurate measurement of the fall, theft of material, etc.

In all cases where special accuracy is required, as for instance for scientific investigation or for the determination of the sale value of woods, the classes or groups should be small and the number of sample trees large. In this way greater accuracy is likely to be obtained than by making a distinction between the different methods which have been described. This conclusion appears justified by the facts that in regularly-grown woods the basal area is a most powerful factor, and that the height is to a sufficient extent a function of the diameter or girth.

4. *Determination of Volume by means of Form Factors and Volume Tables.*

Instead of felling and measuring sample trees, their volume can be ascertained by means of form factors or taken from volume tables. This applies to all the methods given above. In all these cases the volume is ascertained according to the general formula:—

$$V = S \times H \times F.$$

How the basal areas of the trees of a class or a wood are obtained has already been explained. The mean height of a

number of trees, or of a whole wood, is ascertained in the following way:—

$$V=S\times H\times F=s_1\times h_1\times f_1+s_2\times h_2\times f_2+s_3\times h_3\times f_3+\ldots$$

hence

$$H=\frac{s_1\times h_1\times f_1+s_2\times h_2\times f_2+\ldots}{S\times F.}$$

If it is assumed that the form factors are the same throughout, the above formula reduces to the following:—

CALCULATION OF VOLUME BY

Diameter	Number of Trees.	Basal Area.	Height ascertained from 23 Sample Trees.	Basal Area Multiplied by Height $c\times d$.	ACCORDING			
					By Inch-Classes.		By Four	
					Form Factors.	Volume.	Basal Area.	Basal Area Multiplied by Height.
a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*
4	4	·349	38	13·26	·43	6	18·048	775·26
5	12	1·637	40	65·48	·45	29		
6	26	5·105	42	214·41	·45	96		
7	41	10·957	44	482·11	·46	222		
8	42	14·661	46	674·41	·47	317	65·010	3146·82
9	51	22·532	48	1081·56	·47	508		
10	51	27·817	50	1390·85	·48	668		
11	34	22·439	52	1166·83	·48	560	43·301	2309·97
12	16	12·566	54	678·56	·48	326		
13	9	8·296	56	464·58	·47	218		
14	6	6·414	58	372·01	·47	175	19·892	1201·68
15	5	6·136	60	368·16	·47	173		
16	3	4·189	62	259·72	·47	122		
17	2	3·153	64	201·79	·47	95		
					Total...	3515		

$$H = \frac{s_1 \times h_1 + s_2 \times h_2 + \ldots}{S}.$$

in words: "the mean height is equal to the total volume of cylinders erected over the spot where the measurement of the diameters is taken, divided by the total basal area." This formula holds good in the case of the several trees of one class, as well as for calculating the mean height of several classes, or of a whole wood.

MEANS OF FORM FACTORS.

TO KUNZE'S FORM FACTORS.							
Groups.			By One Group.				
Mean Height $\frac{i}{h}$	Form Factors.	Volume $h \times k \times l$.	Basal Area.	Basal Area Multiplied by Height.	Mean Height $\frac{o}{n}$	Form Factor.	Volume $n \times p \times q$
k	l	m	n	o	p	q	r
42·95	·45 =	349					
48·45	·47 =	1479					
			146·25	7433·73	50·83	·48 =	3568
53·35	·48 =	1109					
60·42	·47 =	565					
	Total......	3502					

If a somewhat smaller accuracy suffices, the following method may be followed :—A number of trees are selected which show about an average diameter and height, their heights accurately measured and the mean taken, which represents the mean height of the class, or wood. In some cases the height of the arithmetical mean sample tree is taken as the mean height of the wood. Good results are obtained by ascertaining the mean height by graphic interpolation. In that case the diameters are plotted as abscissæ and the heights as ordinates; an average line is then drawn between the various points which gives the mean heights for successive diameters.

It remains to be noted, that the heights obtained by means of these simplified methods are generally a foot or two smaller than that obtained according to the formula :—

$$H = \frac{s_1 \times h_1 + s_2 \times h_2 + s_3 \times h_3}{S.}$$

How the form factors of single trees are ascertained has been described above. Similarly form factors for whole woods can be determined according to the formula :—

$$V = S \times H \times F,$$

or

$$F = \frac{S \times H}{V}$$

If volume tables are used, the calculation is made according to the formula :—

$$V = n \times s \times h \times f.$$

Here $s \times h \times f$, equal to the volume of the mean tree, is taken direct from the tables.

The example on pp. 62 and 63 shows :—		Volume.	Difference in % compared with the ordinary inch class method.
Volume calculated with Kunze's form factors according to inch classes	=	3515	+ 1·6
Volume calculated with mean height of trees arranged in four groups	=	3502	+ 1·2
Volume calculated with mean height of whole wood	=	3568	+ 3·1

These data show that form factors for Scotch pine obtained from measurements in Germany are well applicable to woods in England.

B. The Height is not a Function of the Diameter.

If it is found that in the case of equal diameters the heights differ considerably, then height classes must be formed in addition to diameter classes.

In some cases it happens that the different height classes are separated according to area, for instance where a marked change in the quality of the locality occurs, due to change in the soil or subsoil, aspect, etc. In such cases the wood is divided into as many parts as different height classes appear, and each is treated as a separate wood.

If the several height classes are mixed over the whole area, a case which is comparatively rare, as in irregular selection forests, then the diameter and height must be measured in each case. Where only two height classes are adopted, the height may be estimated while the diameter is measured, and the tree placed in the one or the other height class. Each of the latter is then considered as a separate wood, and its volume ascertained according to one of the methods described under A.

Cases where more than two height classes, in addition to diameter classes, are called for, are very rare. Generally, the distinction of height classes is a matter of considerable difficulty, unless the heights are measured. They are only necessary where a very high degree of accuracy is aimed at.

The following example will show the manner of booking in the case of two height classes :—

Beech.

Diameter in Inches.	I. Height-Class.	II. Height-Class.	Totals. I. Height-Class.	Totals. II. Height-Class.	Totals. Total.
8	𝍸 𝍸 𝍸 |||	𝍸 𝍸 ||	18	12	30
9	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 ||||	𝍸 𝍸 𝍸 |||	34	18	52
10	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 ||	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |||	47	33	80
11	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 ||||	𝍸 𝍸 𝍸 𝍸	39	20	59
12	𝍸 𝍸 𝍸 ||	𝍸 𝍸 |	17	11	28
		Grand Total	155	94	249

Section II.—Determination of Volume by means of Sample Plots.

1. *Definition.*

Instead of measuring all trees in a wood, a certain part of the area may be selected, the volume on it ascertained, and from it the volume of the whole wood calculated. Such a part is called a sample plot. It may be defined as a portion of a wood which contains an average volume of material per unit of area.

Having ascertained the volume of the sample plot, that of the whole wood can be calculated in two ways: either according to area, or according to the number of trees on the sample plot and in the wood.

Let A = area of wood,

a = area of sample plot,

V = volume of wood,

v = volume of sample plot,

then the following proportion is assumed to exist:—

$$v : V = a : A$$

and

$$V = \frac{v \times A}{a}.$$

Again, if

N = number of trees in the wood,
n = number of trees on the sample plot,

then

$$v : V = n : N,$$

and

$$V = \frac{v \times N}{n}.$$

In the first case it is necessary to ascertain the areas, and in the latter the number of trees in both sample plot and wood. As, however, the counting of all trees gives hardly less trouble than measuring them, it yields only a small saving of labour, and it can only come into consideration when the area of the wood is not known, or cannot readily be ascertained.

2. *Selection of Sample Plots.*

The proportion given above will hold good only if the sample plot represents a fair average of the whole wood, so that it can be considered as a model of it; in other words, if a measurement of the trees on it yields an average basal area of stems per unit of area, an average height and the same form factors. Hence, the sample plots must be selected accordingly.

Here several cases must be distinguished:—

(a) The quality of the wood is the same throughout the area. In this case the sample plot may be selected anywhere, as long as the density of stocking represents an average. In very large woods it may become desirable to take several sample plots and calculate the mean.

(b) Several qualities occur, which are clearly separated according to area. Here each quality is treated separately, and one or more sample plot taken in each part (Fig. 27).

(c) Several qualities exist, which change gradually from one to the other. In this case the sample plot may take the shape of a strip, which runs through the whole wood, so as to include a due proportion of each quality (Fig. 28). As this is difficult to accomplish, it is generally better to follow the method given under (b), to divide the wood into several parts, and to take a sample plot in each.

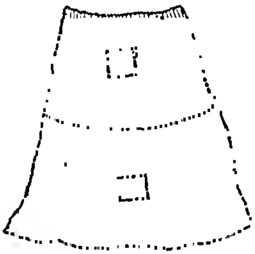


Fig. 27.

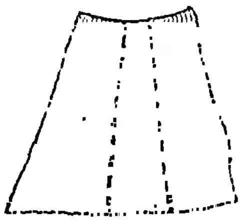


Fig. 28.

(d) Several qualities prevail irregularly over the whole wood. Here a sample plot of average stocking must be selected, a matter frequently beset by great difficulties.

3. *Extent and Shape of Sample Plots.*

The sample plot must be of sufficient extent to contain the different classes of trees in the same proportion as the wood. Hence, its size depends on the degree of regularity of the stocking; the more uniform this is, the smaller may be the sample plot. It follows that they may be made smaller in young fully-stocked woods, than in old irregularly stocked areas.

Very small sample plots have the disadvantage that proportionately too many trees fall into the boundary lines.

The absolute extent of the sample plot depends on the desired degree of accuracy. In mature woods it should not be less than 5 °/₀ of the whole area, but in young woods it may be much less, down to a quarter of an acre in very young woods.

The best shape would be that which includes the greatest area, as compared with the boundary, in other words a circle. As this is impracticable, it is usual to give to the sample plot the shape of a square, or of a rectangle approaching a square.

4. *Measurement of Volume on Sample Plots.*

This can be done according to any one of the methods described above. As here a conclusion is drawn from the volume of a small area to that of the whole wood, it is desirable to measure the volume on the sample plot as accurately as possible.

5. *Merits of the Method of Sample Plots.*

The method of sample plots works quickly, and it affords a great saving of time and expense as compared with the measurement of whole woods. On the other hand, its accuracy depends on the degree to which the sample plot represents an average of the whole wood. Hence, it only yields accurate results in regular grown young and middle-aged woods, less so in old irregularly stocked areas, or where the quality changes frequently. The method is chiefly useful where very extensive areas have to be assessed, or where the value of produce is small, in fact where a high degree of accuracy is either impossible to attain, or not required. Where only small areas have to be measured, or where the value of a forest has to be ascertained for the purpose of sale, in fact where a high degree of accuracy is wanted, the whole wood should be measured.

Section III.—Determination of Volume by Estimate.

Instead of measuring the trees on the whole or a part of the area, the volume can be estimated in various ways, of which the following deserve to be mentioned:—

1. *Estimating the Volume of the Wood as a whole.*

This method, being the oldest and roughest of all, consists in going through the wood and estimating the volume either of the whole wood, or per unit of area, if the total area is known. The estimator must consider differences in the density of stocking, the average volume per tree, the differences in the quality of the locality, and, if for the whole wood at once, its area. It stands to reason that the method requires great experience and practice on the part of the estimator, and eveu then considerable mistakes may be made.

2. *Estimating by Trees.*

Under this method each tree is estimated separately, the volume of the wood being obtained by adding together the volumes of the several trees. With great care an experienced estimator can obtain fairly accurate results, but if done carefully the operation takes almost as much time as if the diameters of all trees and the height of some of them are measured; in the latter case the volume can be calculated by means of form factors or volume tables, a procedure which yields far more reliable results.

The method is only justified in open woods, consisting chiefly of old trees, such as standards in high forest or in coppice with standards, or where a low degree of accuracy meets the requirements of the case. In such cases the estimate may extend over the whole area, or over a sample plot only.

3. *Estimating according to the results of Past Fellings.*

Where fellings have been made and the fall accurately measured, the results can be used to estimate the standing crop in similar woods. In such cases it is necessary to take into consideration any differences in the age, density of stocking, height, etc.

Frequently fellings made in clearing strips for roads or rides

give useful data for estimating the crop of the adjoining woods. In all such cases the estimate is based on the volume per unit of area.

4. *Estimating the Volume by means of Yield Tables.*

In the same way as volume tables of single trees are constructed, which give the average volume of trees arranged according to diameter, height, form factor and age, so tables can be compiled on the basis of extensive measurements in cut woods, which show the volume of woods according to species, age, quality of locality, etc.

If tables are available which are suited to a particular part of a country, it is necessary to ascertain in the wood to be estimated—

(1) The quality class of the locality.
(2) The density of the crop.
(3) The age of the crop.

The first is best judged by the height of the trees; the second by ascertaining the basal area of the trees on a sample plot; the third by counting the concentric rings on stumps or on a few trees cut very close to the ground, unless the age is known from records.

Based upon these data the volume can be taken from the yield table. If for a certain age the basal area given in the table differs from that of the wood, the volume of the table must be modified accordingly; a second correction may be necessary owing to a difference in the height.

The method just indicated is, however, not much used, because, if basal area and mean height of the wood have been ascertained, it is much shorter to calculate the volume by means of form factors or volume tables. Yield tables are better adapted for ascertaining the increment of woods; hence the method of preparing them will be described in Chapter VI.

CHAPTER V.

THE AGE OF TREES AND WOODS.

It is of importance to know, not only the actual dimensions of the trees and their volume, but also the time which has been necessary to produce them. To solve this question the age of single trees as well as that of whole woods must be ascertained.

1. *Determination of the Age of Single Trees.*

a. Standing Trees.

All trees increase annually in diameter and also by the elongation of the leading shoots and branches, at any rate up to a certain age. The diameter increment produces every year an additional concentric ring, and the new leading shoot leaves marks, which are more or less distinguishable, according to species and age. These facts yield data by which the age can be determined in the majority of cases, but not in all, when no records are available which give the age. Accordingly, the following methods of determining the age may be distinguished:—

i. Determination from Existing Records.

Reliable records yield the best results, if they refer to individual trees. In the case of trees which form part of the wood they are not always accurate, as many woods are not altogether even-aged.

ii. Determination by Estimate.

As a general rule, it may be assumed that the larger the tree the older it is. Taking, therefore, into consideration the

conditions under which a tree has grown up, its age can be estimated within 10 or 20 years, at any rate as long as height-growth continues. In the case of very old trees the limit of accuracy is much wider. At all times this method requires much practice and experience, and even then it yields only approximately correct results.

iii. Determination by the Number of Annual Shoots.

In the case of species which leave clear marks of the successive annual shoots, the age can be ascertained by counting these shoots from the top downwards and by adding a proportionate number of years for the lowest part of the stem, where the marks are no longer distinguishable. This method is, in Europe, only applicable to the various species of pine up to a certain age, less so in the case of firs, and not at all in that of larch or of the ordinary broad-leaved species.

iv. Determination by means of Pressler's Increment Borer.

As explained in Chapter I., with this instrument a narrow cylinder of wood can be extracted from the stem on which the concentric rings may be counted. The instrument does, however, not work satisfactorily beyond a depth of 6 inches, so that the centre can only be reached if the diameter of the tree does not exceed 12 inches. Even then it is frequently difficult to hit off the centre, as the trees grow generally more or less excentric.

b. Felled Trees.

It is by far the best method to fell a tree and count the concentric rings on the stump. At the same time this is not always an easy operation, and in some cases it is altogether impracticable. It is easiest in the so-called ring-porey broad-leaved species, and in conifers, which produce a darker coloured summer, or autumn, wood than that formed in spring. Frequently false rings appear. These may be distinguished from true rings by finding that they do not run right round the

tree (Hornbeam, Alder). In the case of suppressed trees the true rings are frequently so narrow, either all round or in parts, that they are difficult to distinguish.

The business may be facilitated by smoothing the surface, by making a slanting cut, or by applying colouring matters (as indigo, alizarine ink, Prussian blue, alcohol coloured with aniline, sulphuric acid, etc.). Such colouring does, however, not always facilitate the counting.

The number of rings thus counted represents only the age of the tree above the place where it has been cut. To the number so obtained the number of years which the tree took to reach that height must be added. If it is desirable to avoid mistakes in this respect, the stool must be split open along the centre and the rings counted to the starting point.

In this way the physical age of the tree can be ascertained, provided that each concentric ring represents a year's growth. It is, however, by no means certain whether this is always the case, as temporary interruptions of growth may cause two rings to be formed in one year. (For instance, the destruction of the leaves by insects and the subsequent sending forth of a second crop of leaves, fire running through a wood, or even late frost.) Moreover, there are trees in the tropics on which the concentric rings do not exist, or cannot be distinguished.

Another point is, that a distinction must be made between the physical and economic age of a tree. By the latter is understood the actual growing age, leaving out of consideration any years during which the tree may have been at a stand-still, owing for instance to heavy shade from above.

2. *Determination of the Age of Whole Woods.*

a. Even-aged Woods.

If the age of such woods is not known from authentic records, it can be ascertained by determining the age of a tree by one of the methods indicated above. If a tree is felled for the purpose of counting the concentric rings, it is desirable

to avoid exceptionally thick trees, as such trees may represent former advance growth.

As whole woods are rarely established in one year, owing to failures and subsequent repairing, or, in the case of natural regenerations, owing to two or more seed years being necessary to the complete stocking of the area, it is generally desirable to examine several trees and take the mean.

b. Uneven-aged Woods.

In many cases woods are less even-aged than has been indicated above. The differences in the age of the several component parts of the wood may be very considerable, as regeneration may have extended over a long period. In such cases the mean age must be ascertained.

By the "mean age" of an uneven-aged wood is understood that period which an even-aged wood requires to produce the same volume as the uneven-aged wood.

Let

$a_1, a_2, a_3, \ldots$ be the ages of the several age classes;
$v_1, v_2, v_3, \ldots$ be the volumes of the several age classes;
I, the mean annual increment of an even-aged wood of the same volume as the uneven-aged one;
A, the mean age, or the age of an even-aged wood of the same volume as the uneven-aged one;

Then, according to the above definition, the following equation holds good:—

$$v_1+v_2+v_3+\ldots=I\times A,$$

and

$$A=\frac{v_1+v_2+v_3+\ldots}{I}.$$

As the even-aged and uneven-aged woods are assumed to have the same volume, it follows that I must be equal to the sum of the mean increments of the several age classes of the uneven-aged wood, that is to say:—

$$I=\frac{v_1}{a_1}+\frac{v_2}{a_2}+\frac{v_3}{a_3}+\ldots$$

By substituting this expression for I in the above equation, the latter becomes:—

$$A = \frac{v_1 + v_2 + v_3 + \ldots}{\frac{v_1}{a_1} + \frac{v_2}{a_2} + \frac{v_3}{a_3} + \ldots} \qquad \ldots \qquad (1)$$

This formula is known as that of Smalian and C. Heyer. It says in words: The mean age of a wood is obtained by dividing the volume of the whole wood by the sum of the mean annual increments of the several age classes. The method may be simplified by assuming that the age is approximately proportionate to the diameter; hence the diameter classes may be taken as the age classes. The above formula is chiefly used when the age classes are irregularly mixed over the area.

If the areas of the several age classes are represented by m_1; m_2; m_3; . . . . and the average annual increment per acre by i_1; i_2; i_3; . . . . then formula (1) can be written in this way (after Gustav Heyer):—

$$A = \frac{m_1 \times i_1 \times a_1 + m_2 \times i_2 \times a_2 + m_3 \times i_3 \times a_3 + \ldots}{\frac{m_1 \times i_1 \times a_1}{a_1} + \frac{m_2 \times i_2 \times a_2}{a_2} + \frac{m_3 \times i_3 \times a_3}{a_3}}$$

or

$$A = \frac{m_1 \times i_1 \times a_1 + m_2 \times i_2 \times a_2 + m_3 \times i_3 \times a_3 + \ldots}{m_1 \times i_1 + m_2 \times i_2 + m_3 \times i_3 + \ldots} \qquad . \qquad (2)$$

If it is now assumed that:

$$i_1 = i_2 = i_3 = \ldots$$

the above formula reduces to the following:

$$A = \frac{m_1 \times a_1 + m_2 \times a_2 + m_3 \times a_3 + \ldots}{m_1 + m_2 + m_3 + \ldots} \qquad \ldots \qquad (3)$$

This formula was first given by Gümpel. It holds good only if the differences in age are small, and the age itself is close to that at which the increment culminates, as it then changes but slowly.

André follows yet a different method. He bases the calculation upon the number of trees in the several age classes. If they are n_1; n_2; n_3; . . . . his formula would be:

$$A = \frac{n_1 \times a_1 + n_2 \times a_2 + n_3 \times a_3 + \ldots}{n_1 + n_2 + n_3 + \ldots} \quad . \quad . \quad (4)$$

All these formulæ are somewhat troublesome. Formulæ (1) and (2) demand a knowledge of the volume, and (2) besides of the areas occupied by each age class. Formula (3) necessitates also a knowledge of the areas, while for formula (4) the numbers of trees in each age class must be ascertained. In practice the mean age is frequently taken as equal to the average age of the sample trees, or of the age classes, according to the formula:

$$A = \frac{a_1 + a_2 + a_3 + \ldots}{n} \quad . \quad . \quad . \quad . \quad (5)$$

where n represents the number of sample trees or age classes.

Finally, the age of the arithmetical mean sample tree can be taken as the mean age of the wood.

Example:—

Let

$v_1 = 4000$	$a_1 = 50$	$m_1 = 2$ acres	$n_1 = 1500$
$v_2 = 9000$	$a_2 = 60$	$m_2 = 3$,,	$n_2 = 1600$
$v_3 = 7000$	$a_3 = 70$	$m_3 = 2$,,	$n_3 = 800$
$v_4 = 4000$	$a_4 = 80$	$m_4 = 1$,,	$n_4 = 300$

Mean age according to formula:

$$(1) \quad A = \quad . \quad \frac{4000 + 9000 + 7000 + 4000}{\frac{4000}{50} + \frac{9000}{60} + \frac{7000}{70} + \frac{4000}{80}} = \frac{24000}{380} \quad . \quad = 63{\cdot}2$$

$$(2) \quad A = \frac{2 \times 80 \times 50 + 3 \times 150 \times 60 + 2 \times 100 \times 70 + 1 \times 50 \times 80}{2 \times 80 + 3 \times 150 + 2 \times 100 + 1 \times 50} . \quad = 61{\cdot}6$$

$$(3) \quad A = \quad . \quad \frac{2 \times 50 + 3 \times 60 + 2 \times 70 + 1 \times 80}{2 + 3 + 2 + 1} \quad . \quad = 62{\cdot}5$$

$$(4) \quad A = \frac{1500 \times 50 + 1600 \times 60 + 800 \times 70 + 300 \times 80}{1500 + 1600 + 800 + 300} = 59{\cdot}8$$

$$(5) \quad A = \quad . \quad . \quad . \quad \frac{50 + 60 + 70 + 80}{4} \quad . \quad . \quad . \quad = 65$$

CHAPTER VI.

DETERMINATION OF THE INCREMENT.

During every growing season a tree increases by the elongation of the top shoot, side branches and roots, and by the laying on of a new layer of wood and bark throughout its extent. Thus the height and diameter (or basal area) as well as the spread of the crown increase constantly, up to a certain age, producing an increase of volume called the increment. By adding up the increase of the several trees in a wood, that of the whole is obtained.

The increment may refer to one or more growing seasons, and accordingly a distinction must be made between :—

(1) The current annual increment, or that laid on in the course of one year.

(2) The periodic increment, or that laid on during a number of years or period.

(3) The total increment, or that laid on from the origin of a tree or wood, up to a certain age, frequently that when the tree, or wood, is cut over.

(4) The mean annual increment, or that which is obtained by dividing the increment laid on during a given period by the number of years in the period. If the mean annual increment is calculated for a portion of the total age, it is called the periodic mean annual increment, if for the total or final age of the tree, or wood, it is called the final mean annual increment.

In determining the increment of whole woods, it must be remembered that a certain number of trees disappear from time to time owing to thinnings and natural causes. All such

removals must be taken into account, in determining the total increment laid on.

The determination of the increment may refer to the past (backward), or to the future (forward). As the former deals with actually existing quantities, the determination can be made with a comparatively high degree of accuracy; the latter, on the other hand, is to a considerable extent based on speculation, hence less reliable.

Section I.—Determination of the Increment of Single Trees.

1. *Height Increment.*

a. Of the Past.

The height increment of standing trees can in some cases be ascertained by the whorls formed in successive years. This, however, refers to a limited number of species. In the majority of cases it is necessary to cut down a tree for the purpose of investigating the height increment.

The height increment of the last few, say n, years, can be ascertained, in the case of some conifers, by measuring the length of the last n shoots. In the case of all other species the height increment of the tree during the last n years is ascertained by cutting off a certain length and counting the rings; if they are less than n in number, an additional piece must be cut off, and so on until that spot has been found where the section shows n rings.

If the section of the first cut shows more than n rings, then another cut higher up is made, until again the section shows n rings. The length above that point gives the height growth of the last n years.

In all cases where a complete knowledge of the height increment during the several periods of life is required, the tree should be divided into a series of sections, the length of which depends on the desired degree of accuracy. The con-

centric rings are then counted at the end of each section, and from the data thus obtained, the height of the tree at successive periods of life can be ascertained, either by calculation, or interpolation.

Generally, graphic interpolation gives the better results, as it equalizes accidental irregularities. In this case the abscissæ represent the ages and the ordinates the corresponding heights. By connecting the points thus indicated by a steady curve, the height at successive ages can easily be read off.

Example.—See analysis of a Scotch pine tree, at p. 83.

b. Height Increment of the Future.

The expected height increment for a number of years to come can be estimated from the increment of the immediate past. In doing this the rate of increment during the past must be studied, and especially the time ascertained when the current annual increment of the species usually culminates. If the increment immediately before the time of inquiry was still rising, it may continue to do so or not, according to whether the maximum has been reached or not. If it is falling already, it will continue to do so, and in that case the rate at which it is likely to fall must be estimated. In this way the probable increment for a limited number of years (10) can be estimated with satisfactory accuracy. This is best done by constructing a height curve of the past, and elongating it for the required period so as to form a continuous curve.

2. *Diameter Increment.*

a. Of the Past.

This can refer to wood and bark, or to wood only.

The increment of wood and bark laid on by standing trees can be ascertained by repeated measurements of the same tree, a certain number of years being allowed to pass between every

two measurements. The latter are made with the calliper, care being taken to mark the place of measurement without causing an unusual swelling at that part of the tree. Where immediate results are required, the increment can be ascertained with Pressler's increment borer. The number of years for which it can be ascertained depends on the length of the cylinder which can be extracted, and on the rate of growth. As most trees grow irregularly, it is necessary to ascertain the increment at opposite sides, or at four sides, and to take the mean. These investigations rest on the assumption that the concentric rings are distinguishable, and that each ring represents one year's growth.

The increment can be ascertained with much greater accuracy by felling a tree and measuring the breadth of the desired number of rings in the section, the latter being laid at right angles to the axis of the stem. The measurements are made with a scale subdivided to a sufficient degree. This is either laid on the section and the breadths read off, or the latter are taken off with a pair of compasses, and the dimensions then taken from the scale. In either case care must be taken to obtain averages by measuring along two, four, or more radii, equally arranged over the section, and then taking the mean of the several readings.

In the case of standing trees the increment can only be ascertained for a limited number of years. If a tree is felled, the increment can be ascertained for the several periods of its life, say for every five, ten, or more years. The result can be graphically represented, and a mean curve of increment constructed, from which the increment for any desired intervals can be easily determined.

By repeating the above operation at successive heights from the ground, the increment can be ascertained in the several parts of the stem.

b. Diameter Increment of the Future.

This is estimated from the increment of the immediately preceding period, taking into consideration how far the future diameter increment may be affected by the method of treatment, more especially the proposed degree of thinning.

3. *Area Increment.*

The increment in basal area is calculated from that of the diameter.

Let D be the mean diameter of the whole section, *d* the diameter of the same section *n* years ago, then

$$\left.\begin{array}{r}\text{Basal increment}\\ \text{during } n \text{ years}\end{array}\right\} = \frac{D^2 \times \pi}{4} - \frac{d^2 \times \pi}{4}$$

$$= \frac{(D^2 - d^2) \times \pi}{4} = (D^2 - d^2) \times {\cdot}785.$$

The basal increment can be ascertained for a limited number of years only, or for the several periods of the life of a tree. An estimate of the future increment is based upon that of the immediate past, taking into consideration the proposed treatment, as in the case of the diameter increment.

4. *Volume Increment.*

a. Of the Past.

The past volume increment of a tree during a certain period of years, *n*, is equal to the difference of volumes at the commencement and end of the period. These volumes can be ascertained by examining a series of sections at various heights of the tree, or by basing the calculation upon measurements made at the middle section, or by using form factors.

i. Determination of the Increment by Sections.

If the increment of only a limited number of years, *n*, is desired, it can be ascertained by means of the increment borer.

The breadth of n rings is ascertained at regular intervals, and the difference between the present volume and that n years ago calculated.

The investigation of the progress of increment throughout the life of a tree is called a stem analysis. It consists of a combination of a height and a diameter analysis.

The tree having been divided into a suitable number of sections, each is cut through in the middle, the number of concentric rings counted on the basal area, and the size of the diameter at the several ages measured. The measurements are best plotted, so that a representation of a longitudinal section through the tree is obtained. For this purpose the heights of the several cross sections from the ground are marked on a vertical line, which represents the axis of the stem; also the heights which the tree had obtained at successive periods of its life. Next the radii of each cross section are marked on horizontal lines, and the points thus obtained connected by a series of lines, which represent the stem curves of the several stages during the life of the tree. From the data thus obtained the increment throughout the several periods of the life of the tree can be calculated. As the thickness of the bark at former periods cannot be ascertained, these investigations can only refer to the increment in wood, exclusive of bark.

The following example will explain the procedure:

Analysis of a Scotch Pine Tree.

The tree was cut up into nine pieces, which gave the following cross sections:—

Section	I. taken at foot of tree				showing 97 concentric rings.			
,,	II.	,,	5 feet above ground		,,	95	,,	,,
,,	III.	,,	15 ,,	,, ,,	,,	89	,,	,,
,,	IV.	,,	25 ,,	,, ,,	,,	85	,,	,,
,,	V.	,,	35 ,,	,, ,,	,,	80	,,	,,
,,	VI.	,,	45 ,,	,, ,,	,,	72	,,	,,
,,	VII.	,,	55 ,,	,, ,,	,,	64	,,	,,
,,	VIII.	,,	64 ,,	,, ,,	,,	34	,,	,,
,,	IX.	,,	68 ,,	,, ,,	,,	26	,,	,,

Top = 9 feet long.

Total height = 77 feet.

Height of Section.	Number of Rings.	Number of Years which the Tree took to reach that Height.
0	97	0
5	95	2
15	89	8
25	85	12
35	80	17
45	72	25
55	64	33
64	34	63
68	26	71
77	0	97

Radius of Section I. at 0′ *from the* ground in inches.	Radius of Section II. at 5′ from the ground in inches.	Radius of Section III. at 15′ from the ground in inches.	Radius of Section IV. at 25′ from the ground in inches.
Total = 11·50	Total = 8·82	Total = 6·92	Total = 6·38
97 = 10·56	95 = 8·32	89 = 6·78	85 = 6·21
87 = 9·88	85 = 7·86	79 = 6·45	75 = 5·94
77 = 9·22	75 = 7·34	69 = 6·16	65 = 5·62
67 = 8·50	65 = 6·77	59 = 6·04	55 = 5·30
57 = 7·65	55 = 6·25	49 = 5·46	45 = 4·99
47 = 6·71	45 = 5·74	39 = 4·95	35 = 4·41
37 = 5·74	35 = 5·06	29 = 4·24	25 = 3·80
27 = 4·94	25 = 4·34	19 = 3·50	15 = 2·81
17 = 3·83	15 = 3·38	9 = 2·25	5 = 1·30
7 = 1·85	5 = 1·30		
Radius of Section V. at 35′ from the ground in inches.	**Radius of Section VI. at 45′ from the ground in inches.**	**Radius of Section VII. at 55′ from the ground in inches.**	**Radius of Section VIII. at 64′ from the ground in inches.**
Total = 6·03	Total = 5·81	Total = 3·54	Total = 2·12
80 = 5·96	72 = 5·75	64 = 3·46	34 = 2·07
70 = 5·71	62 = 5·40	54 = 2·98	24 = 1·66
60 = 5·28	52 = 4·87	44 = 2·40	14 = ·88
50 = 4·80	42 = 4·31	34 = 1·85	4 = ·24
40 = 4·37	32 = 3·74	24 = 1·40	**Radius of Section IX. at 68′ from the ground in inches.**
30 = 3·85	22 = 3·05	14 = 1·03	Total = 1·43
20 = 2·95	12 = 1·90	4 = ·41	26 = 1·39
10 = 1·35	2 = ·35		16 = 1·02
			6 = ·50

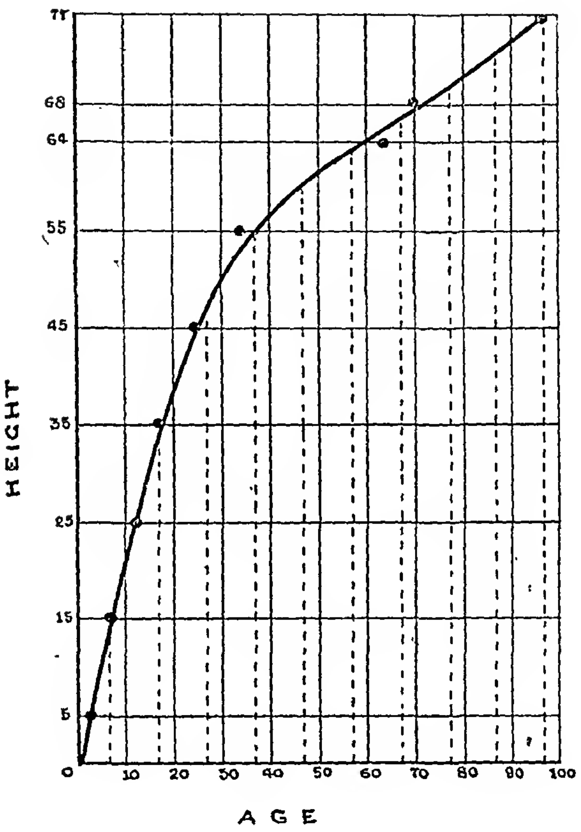


Fig. 29.—Graphic representation of the Height Increment.

Note.—The tree grew in a very favourable locality, but it was overtopped by other trees at the age of about 40 years; hence the abnormal height growth.

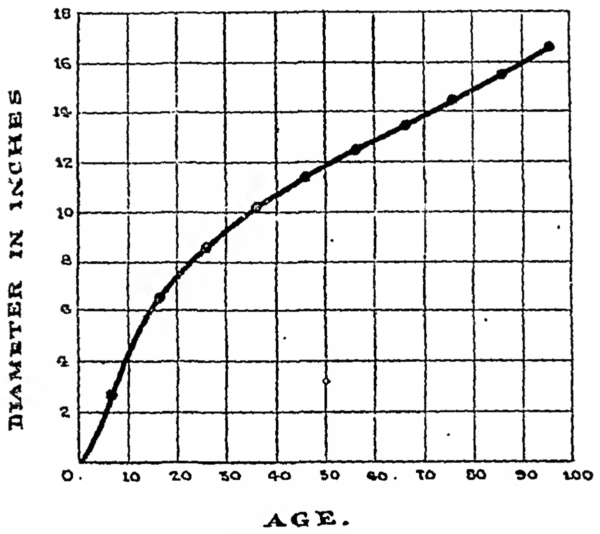


Fig. 30.—Graphic representation of the Diameter Increment.

CALCULATION OF THE VOLUME OF THE TREE AT DIFFERENT AGES.

Number of Section.	Diameter in Inches.	Basal Area in Square Feet.	Length in Feet.	Volume in Cubic Feet.
Whole Tree, including Bark; age = 97 Years.				
1	17·6	1·69	10	16·9
2	13·8	1·04	10	10·4
3	12·8	·89	10	8·9
4	12·1	·80	10	8·0
5	11·6	·73	10	7·3
6	7·1	·27	10	2·7
7	4·2	·10	8	·8
		Total Timber =		55·0
8	2·9	·05	$\frac{8}{3}$*	·15
		Total Timber and Fuel =		55·15

Number of Section.	Diameter in Inches.	Basal Area in Square Feet.	Length in Feet.	Volume in Cubic Feet.
Whole Tree, without Bark; age = 97 Years.				
1	16·6	1·50	10	15·0
2	13·6	1·01	10	10·1
3	12·4	·84	10	8·4
4	11·9	·77	10	7·7
5	11·5	·72	10	7·2
6	6·9	·26	10	2·6
7	4·1	·09	8	·7
		Total Timber =		51·7
8	2·8	·04	$\frac{8}{3}$	·12
		Total Timber and Fuel =		51·82

* The top is considered as representing a cone, the volume of which = basal area × one-third of the height.

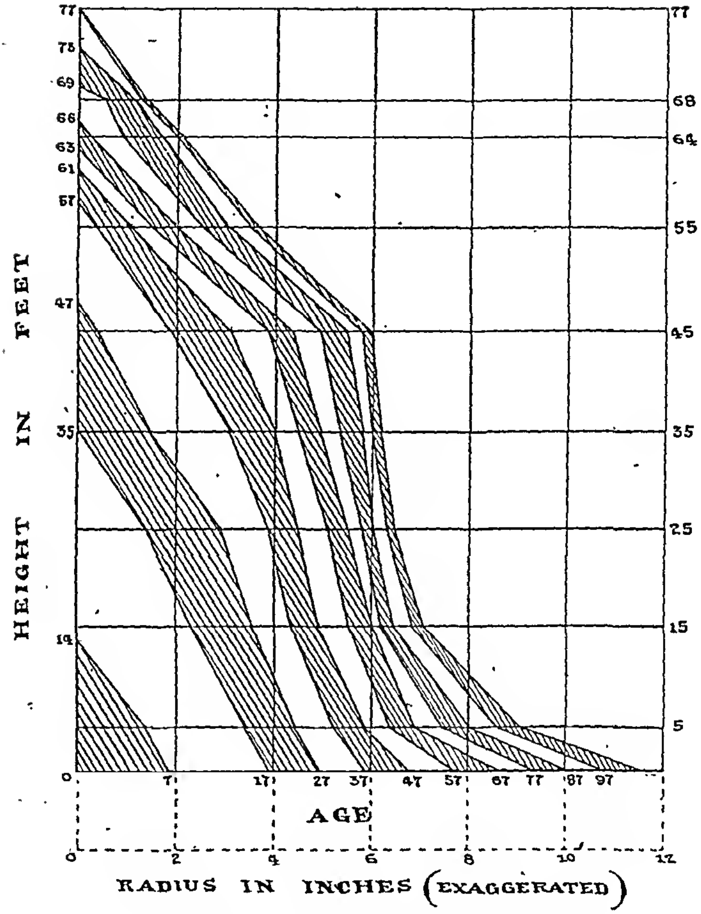


Fig. 31.—Graphic representation of a Tree Analysis.

Number of Section.	Diameter in Inches.	Basal Area in Square Feet.	Length in Feet.	Volume in Cubic Feet.
Tree 87 Years Old.				
1	15·7	1·34	10	13·4
2	12·9	·91	10	9·1
3	11·9	·77	10	7·7
4	11·4	·71	10	7·1
5	10·8	·64	10	6·4
6	6·0	·20	10	2·0
7	3·3	·06	8	·5
		Total Timber =		46·2
8	2·0	·02	$\frac{5}{3}$	·03
		Total Timber and Fuel =		46·23
Tree 77 Years Old.				
1	14·7	1·18	10	11·8
2	12·3	·83	10	8·3
3	11·2	·68	10	6·8
4	10·6	·61	10	6·1
5	9·7	·51	10	5·1
6	4·8	·13	10	1·3
		Total Timber =		39·4
7	1·8	·02	8	·16
8	1·0	·01	$\frac{1}{3}$	—
		Total Timber and Fuel =		39·56
Tree 67 Years Old.				
1	13·5	·99	10	9·9
2	12·1	·80	10	8·0
3	10·6	·61	10	6·1
4	9·6	·50	10	5·0
5	8·6	·40	10	4·0
6	3·7	·07	10	·7
		Total Timber =		33·7
7	2·0	·02	$\frac{6}{3}$	·04
		Total Timber and Fuel =		33·74
Tree 57 Years Old.				
1	12·5	·85	10	8·5
2	10·9	·65	10	6·5
3	10·0	·55	10	5·5
4	8·7	·41	10	4·1
5	7·5	·31	10	3·1
		Total Timber =		27·7
6	2·8	·04	10	·4
7	1·0	·01	$\frac{3}{3}$	·01
		Total Timber and Fuel =		28·11
Tree 47 Years Old.				
1	11·5	·72	10	7·2
2	9·9	·53	10	5·3
3	8·8	·42	10	4·2
4	7·7	·32	10	3·2
5	6·1	·20	10	2·0
		Total Timber =		21·9
6	2·1	·02	10	·20
7	·6	·002	$\frac{1}{3}$	—
		Total Timber and Fuel =		22·10
Tree 37 Years Old.				
1	10·1	·56	10	5·6
2	8·5	·39	10	3·9
3	7·6	·32	10	3·2
4	5·9	·19	10	1·9
5	3·8	·08	10	·8
		Total Timber =		15·4
6	2·4	·03	$\frac{7}{3}$	·07
		Total Timber and Fuel =		15·47
Tree 27 Years Old.				
1	8·7	·41	10	4·1
2	7·0	·27	10	2·7
3	5·6	·17	10	1·7
		Total Timber =		8·5
4	2·7	·04	10	·40
5	1·6	·01	$\frac{7}{3}$	·02
		Total Timber and Fuel =		8·92
Tree 17 Years Old.				
1	6·8	·25	10	2·5
2	4·5	·11	10	1·1
		Total Timber =		3·6
3	2·6	·04	10	·40
4	1·2	·01	$\frac{5}{3}$	·02
		Total Timber and Fuel =		4·02
Tree 7 Years Old.				
		Total Timber =		0·0
1	2·6	·04	10	·40
2	1·2	·01	$\frac{4}{3}$	·01
		Total Timber and Fuel =		0·41

Recapitulation.

The stem of the tree had, at the age of 97 years:

A total volume of = 55·15 cubic feet.

Of this was

Bark $\begin{cases} = 55{\cdot}15 - 51{\cdot}82 = 3{\cdot}33 \text{ cubic feet} \\ = \ 6 \text{ per cent. of total volume.} \end{cases}$

Leaving

Timber =	51·7	cubic feet.
Firewood =	·12	,, ,,

Total Timber and Firewood = 51·82 cubic feet.

By graphically representing the volume of wood at the several ages, figure 32 is obtained, which, with the previous diagrams, gives the following data:—

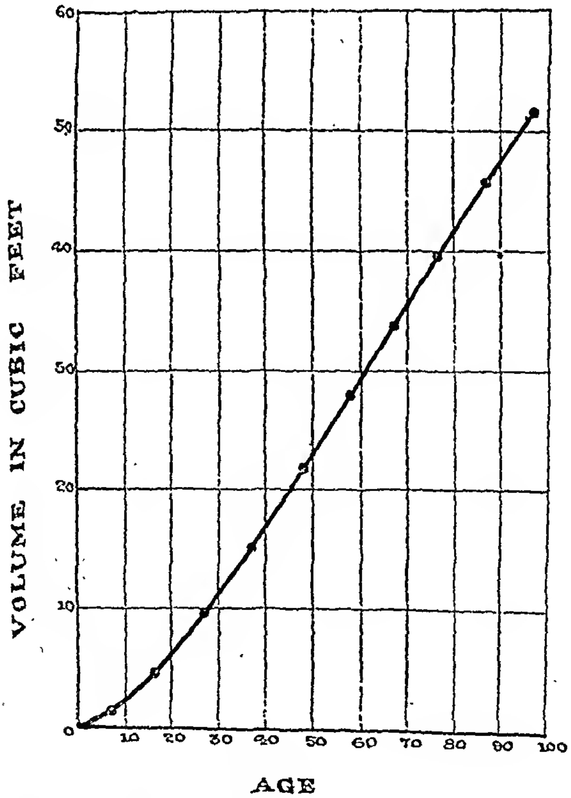


Fig. 32.—Graphic representation of the Volume Increment.

Age of Tree.	Height in Feet.	Diameter without Bark, at 5' above Ground, Inches.	Volume without Bark in Cubic Feet.	Periodic Increment for every Ten Years.
0	—	—	—	
10	20	4·0	1·8	1·8
20	38	7·4	6·0	4·2
30	51	9·1	12·0	6·0
40	58	10·6	18·0	6·0
50	62	11·9	24·0	6·0
60	64	12·8	29·9	5·9
70	67	13·9	35·3	5·4
80	70	15·0	42·0	6·7
90	74	16·0	48·0	6·0
97	77	16·6	51·8	3·8
			Total	51·8

ii. Determination of Increment by the Middle Section.

If somewhat less accurate results suffice, the volumes can be ascertained by multiplying the basal area in the middle by the height. Let V be the volume of the tree at the present time, v that n years ago, H and h the corresponding heights, figure 33, and S and s the corresponding basal areas at $\frac{H}{2}$ and $\frac{h}{2}$, then

$$I = V - v = S \times H - s \times h.$$

The height and basal area at $\frac{H}{2}$ can easily be measured;

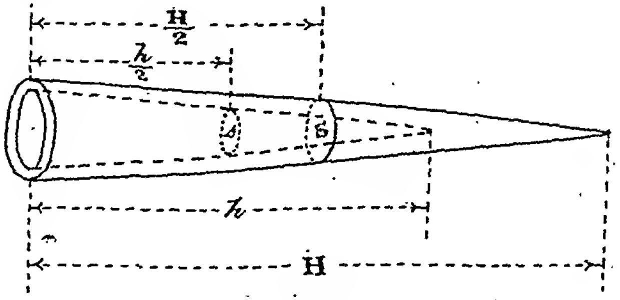


Fig. 33.

h is ascertained by cutting off a piece from the top downwards, and repeating the operation, until the point has been ascertained where the basal area contains n concentric rings. Then the basal area s at $\frac{h}{2}$ is ascertained, either by making a cross section at the spot, or by ascertaining the breadth of the last n rings with the increment borer, thus ascertaining the diameter which the tree had n years ago. In either case the diameter increment must be measured in several places of the circumference, so as to obtain the mean.

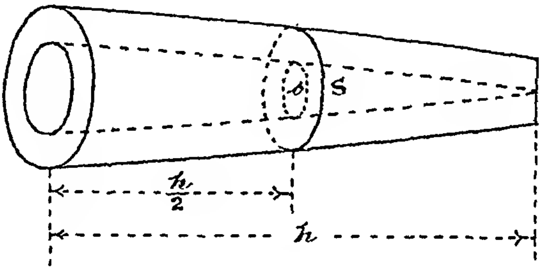


Fig. 34.

In order to simplify the operation, Pressler proposed to cut off a length corresponding to n years height growth in the first place, and then to measure the basal area in both cases at $\frac{h}{2}$. He obtains the increment according to the formula:

$$I = S \times h - s \times h = (S - s)\ h.$$

The error due to omitting the top is said to be compensated for by S having been taken somewhat below $\frac{H}{2}$.

iii. Determination of Increment by Form Factors.

Let S be the basal area of a tree, taken at chest height; s the basal area of the tree n years ago taken at the same height; H and h the corresponding heights, and F and f the corresponding form factors, then the increment:—

$$I = S \times H \times F - s \times h \times f.$$

In the case of a standing tree, H is measured with a height

measurer, h is estimated, S is obtained by measuring the diameter with a calliper, and s with the assistance of an increment borer. F and f must be taken out of form factor tables, or estimated. If F is taken as $= f$, the formula becomes:

$$I = (S \times H - s \times h) \times F.$$

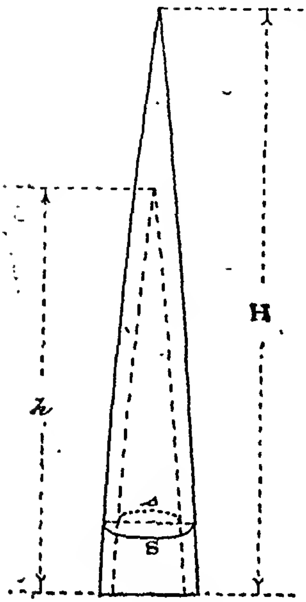


Fig. 35.

The method can only give approximately correct results, because h has to be estimated. It must also not be overlooked, that form factor tables give only averages; hence the method is not adapted to the measurement of a single tree, but only to that of a large number of trees.

b. Volume Increment of the Future.

The increment which a tree may be expected to lay on in the future can be estimated either from its own past increment, especially that of the immediate past, or by comparing it with other older trees.

i. Estimate according to Past Increment.

The increment is represented by the formula:

$$I_n = S \times H \times F - s \times h \times f,$$

where $s \times h \times f$ is the value of the present volume, and $S \times H \times F$ that to be expected after n years. The formula shows that, in order to obtain fairly accurate results, it is necessary to estimate S, H and F from s, h, and f. How this should be done as regards basal area and height, has been explained above. The form factor F may be obtained from tables, if such are available; otherwise it must be estimated, or it may be taken as equal to f.

Instead of estimating the separate factors, the volume increment of the next n years may be estimated direct from that laid on during the last n years, taking into consideration how far the latter should be modified with regard to the age of the tree, locality, future treatment of the wood, especially the proposed degree of thinning, etc.

According to Pressler's method the probable increment can be ascertained by estimating the probable diameter increment, and then proceeding by the formula:

$$I_n = (S - s)\ h,$$

where s represents the present section in the middle, S the expected section in the same spot after n years, and h the present height. The method applies only to felled trees; hence it necessitates the felling of one or more sample trees.

ii. Estimate by Comparison with other Older Trees.

The increment I which a tree A now a years old, and containing a volume v_a is likely to lay on during the next n years, can be ascertained by examining another tree B grown under the same conditions but now $a + n$ years old, which with a present volume $= v_{a+n}$ had, when a years old, a volume equal v_a. The increment which the second tree has laid on during the last n years, may be assumed as equal to that which the first tree is likely to lay on during the next n years. The assumption holds good only if the weather and treatment are the same during both periods.

It is, however, not always easy to find exactly the tree wanted, and as a rule a tree differing somewhat in volume and age must be taken, which involves the following corrections:—

Supposing the age of the second tree is $a + t$ instead of $a + n$, and its volume $= v_{a+t}$, then, if the increment has been steadily laid on, the following equation holds good:—

$$t : n = I_t : I_n = v_{a+t} - v_a : I_n$$

and

$$I_n = (v_{a+t} - v_a) \times \frac{n}{t}.$$

If, on the other hand, $t = n$, but the second tree had only a volume v_a' in the year a, then the increments of the two trees may be assumed to bear the same proportion as the volumes, or:

$$v_a : v_a' = I_n : I_n' \text{ and } I_n = I_n' \times \frac{v_a}{v_a'} = (v'_{a+n} - v'_a) \times \frac{v_a}{v'_a}.$$

Now:

$$v_{a+n} = v_a + I_n = v_a + (v'_{a+n} - v_a') \times \frac{v_a}{v_a'}.$$

$$v_{a+n} = v_a + \frac{v'_{a+n}}{v_a'} \times v_a - v_a = v'_{a+n} \times \frac{v_a}{v_a'},$$

or—

$$v_{a+n} : v_a = v'_{a+n} : v_a';$$

in words: the proportion which exists between the volumes of the second tree n years ago and now, holds good also as regards the volumes of the first tree now and n years hence.

If both, the ages and volumes, differ, it may again be assumed that the increments of the two trees show the same proportion as the volumes. In that case:—

$$v_a : v_a' = I_n \text{ of } v_a : I_n \text{ of } v_a'$$

$$v_a : v_a' = I_n : I_t \times \frac{n}{t}$$

And—

$$I_n = \frac{\frac{I_t}{t} \times n \times v_a}{v_a'} = \frac{\frac{v'_{a+t} - v_a'}{t} \times n}{v_a'} \times v_a,$$

$$I_n = (v'_{a+t} - v_a') \times \frac{v_a}{v_a'} \times \frac{n}{t} = \frac{(v'_{a+t} - v'_a) v_a \times n}{v_a' \times t}.$$

All these calculations assume that the increment of n years has been laid on in annually equal quantities, which is only approximately correct. The degree of accuracy depends on the length of the time n, and the differences between n and t and between v_a and v'_a.

This method of calculating the volume of a single tree is so complicated, that it is not used in practice; it has been

explained here, as it forms the basis of calculating the increment of whole woods, to be explained further on.

Example:—

Find the increment which a tree now 50 years old with a volume of 30 c′ is likely to lay on during the next 10 years.

A tree has been found, now 58 years old, with a volume of 40 c′, and which had 33 cubic feet when it was 50 years old.

$$I_{10} = (40 - 33)\,\frac{30}{33} \times \frac{10}{8} = 7{\cdot}95.$$

Section II.—Determination of the Increment of Whole Woods.

It has been shown that, in the case of single trees, the accumulation of the volume, as well as of the factors which lead up to it, height, diameter, or basal area increment, can be followed backwards with a considerable degree of accuracy. This is not the case as regards whole woods, because trees die or are taken away in thinnings. Investigations made on sample trees selected in a wood show only the successive development of the individuals existing at the time of examination, but they throw no light on that of those trees which have disappeared in course of time, since the wood was created. Height growth alone makes an exception. An analysis of a number of sample trees will indicate the mean height of these trees during previous periods, which may be taken as the upper height of the wood at those periods. These would of course not represent the mean heights at the several ages, because it may safely be assumed that the now existing trees were, as a rule, always the leading trees. Investigations have proved that the mean height of woods can be deduced from the upper height. For instance, in the case of the Scotch pine the difference ranges from about 3 per cent. to 5 per cent. according to the age of the wood. But no such relation has as yet been found as regards the basal area or the volume, and to evolve

the former amounts of these out of the present quantity is more or less speculative.

Under these circumstances one of the following two methods may be followed:—

A. Determination of the Future Increment according to the Mean Annual Increment of the Past.

The present volume of the wood is ascertained and divided by its age, the quotient giving the mean annual increment calculated on the growing stock present at the time of measurement. According to the age of the wood, it may be assumed that the mean annual increment will be laid on for a number of years to come, or a somewhat diminished or increased increment.

The method gives fair results, if the calculation is made for the time when the mean annual increment culminates, and even for older woods; it is less accurate in the case of younger woods. Moreover, it is only applicable for a limited number of years, during which no thinnings are made, say 10 years.

B. Determination of Increment by means of Yield Tables.

I. *OF YIELD TABLES GENERALLY.*

1. *Definition of Yield Table.*

It has already been explained that the progress of height, diameter, basal area, and volume increment can be represented by curves constructed on the principle that the successive ages are marked as abscissæ, and that the corresponding ordinates represent the height, diameter, basal area, or volume. Such curves indicate the appropriate quantities for any age up to a fixed limit, generally the highest rotation likely to be adopted. Instead of employing curves, the data which they represent are read off and arranged in tables, which are called Yield or Increment Tables.

By a yield table is understood a tabular statement which gives the course of the development of a wood from early youth up to a fixed age, either from year to year, or for intervals of a certain number of years.

2. *Object and Contents of Yield Tables.*

Yield tables are used for a great variety of purposes, as:

(*a*) Determination of the volume of woods.
(*b*) " " " increment of woods.
(*c*) " " " quality of localities or of woods.
(*d*) " " " most profitable species, method of treatment and rotation.
(*e*) " " " value of the soil, growing stock, or both.
(*f*) " " " yield of forests.

In order to meet all these requirements, yield tables should show, per unit of area (acre):

(1) The progressive volume which may be found in a fully-stocked wood.
(2) The number of trees.
(3) The basal area of trees.
(4) The height of the wood.
(5) The form factors.
(6) The current annual and mean annual increment.

Separate yield tables must be prepared for

(*a*) Each species.
(*b*) Each method of treatment, as high forest, coppice wood, and combination forest.
(*c*) Each quality of locality.

The volume is given divided into the different classes of wood, as timber, firewood, fagots, etc. The volume of thinnings is entered separately from that of final yields.

Yield tables are only prepared for "normal" woods, that is to say for woods which are fully stocked, taking into considera-

tion the species, quality of locality, and the adopted method of treatment. Such woods are produced if no extraordinary influences have interfered with their progress, such as natural phenomena, faulty treatment, etc.

3. *Local and General Yield Tables.*

If a yield table has been prepared for a particular district of limited extent, it is called a "local" yield table; if for a whole province or county, a "general" yield table.

The question, what limits should be assigned to the applicability of a yield table, is still under discussion, but so much is certain, that in the preparation of such tables a considerable extent of country can be thrown together without incurring any appreciable inaccuracy.

4. *Quality Classes.*

Localities of different quality or yield capacity produce woods, which follow in their development different laws. The law of increment of the one cannot be evolved out of that of the other. The preparation of a yield table should therefore be based on data obtained from localities of precisely the same quality. Practically, however, an immense number of different qualities exist, hence in practical forestry some concession must be made, by being satisfied with a limited number, which rarely exceeds five, and frequently three are quite sufficient. The best quality is generally designated as I. quality (though the reverse would be better).

In proceeding to construct yield tables it is obviously of the first importance to have a ready method by which the quality of a locality may be indicated. It has been explained in "Sylviculture" that the several factors of the locality, such as the chemical and physical conditions of the soil and subsoil, the climate, etc., do not enable the forester to determine the quality of the locality for forest purposes with any degree of

accuracy, and that the only satisfactory indication is given by the wood which has been produced on it. In other words, a locality which produces, in a given time, a large volume, is of good quality; one which produces a small volume, of inferior quality. The volume then, is in the first place the surest indication of the quality of the locality.

As it is, however, a somewhat cumbrous process to ascertain the volume when searching for a certain quality, the question arises, whether one or more of the elements from which the volume is calculated, would not do equally good service. It has been shown above that the volume is

$$V = s \times h \times f.$$

Of these three elements, the form factor moves between comparatively narrow limits, and it is not suited for the present purpose, apart from the fact that the volume would first have to be ascertained in order to determine the form factor. Basal area and height together give a sure indication of the quality, that is to say, two woods which show the same basal area and height, may safely be assumed to have the same volume, hence the localities which have produced them would be of the same quality.

If only one indicating element is used, the height is far preferable to the basal area. While two woods which have the same basal area may have very different heights, experience has shown that two normal woods of the same height have approximately the same basal area. It follows that the height is, next to the volume itself, the best indicator of the quality of the locality. Great height growth means good quality, small height growth inferior quality of locality.

Neither the mean diameter nor the number of trees can be used for the above purpose, as they are not in due proportion to the volume. Nor can the product of number of trees multiplied by the mean diameter be used.

5. *Methods of Constructing Yield Tables.*

The following methods have been proposed :—

a. Annual or Periodical Measurement of the Growing Stock of one and the same Wood; in the second case the Intermediate Values are found by Interpolation.

The method gives absolute certainty that all figures of the yield table are derived from the same quality class, but as the preparation of the table would take a century and more, the method has only theoretical value. Moreover, accidents may happen which would render the wood unfit for further observation.

b. Annual or Periodical Measurement of the Growing Stock of a limited number of Woods of different Ages.

In order to save time, it has been proposed to select several woods differing in age by a certain number of years, say 20, and to obtain from the measurements of each, extending over 20 years, part of the yield table. To make sure that the quality of the several woods is the same, it is necessary that they should have the same volume at the same age; in other words, the wood now 40 years old should have had, when 20 years old, the same volume as the present 20-years-old wood has; again, the 60-years-old wood the same volume when it was 40 years old, as the present 40-years-old wood, etc. Or, to put the matter differently, the 20-years-old wood should have, when it becomes 40 years old, the same volume as the 40-years-old wood has now, etc. In addition, the progress of the increment should be steady throughout.

Although it is difficult to select localities on these lines, which are exactly of the same quality, or woods which will develop in the same manner, there can be no doubt that ultimately satisfactory yield tables can only be obtained by observing and periodically measuring suitable woods for a series of years. Hence the method is actually followed. For each quality class and age gradation several sample plots are

selected, and these are periodically measured, and the mean taken. In this way yield tables will ultimately be obtained. It is necessary to take several plots for each quality and age gradation, so as to obtain average results, and because one or other may become unfit for the purpose, in consequence of unforeseen events.

c. Measurement of a large number of Woods of different ages once, so that Yield Tables are obtained immediately.

Until yield tables, prepared as indicated under *b*, become available, others for immediate use are required. These are obtained by measuring fully-stocked sample plots in a sufficient number of woods, representing all ages with moderate intervals. Out of the data thus obtained, steady curves and tables are prepared. A separate set of woods are required for each quality class, and the great difficulty consists in selecting for each set localities of the same quality. For this purpose various methods have been suggested. Most of these start from an indicating wood, while one, specially elaborated by Baur,* starts from a different principle; it will be dealt with in detail further on.

i. Selection of Woods for each Quality Class by Means of an Indicating Wood.

The method is based upon the fact that the older wood has been evolved out of the younger, in other words that the older wood had at one time the same volume as the younger. Hence it should be possible, by analysing a number of sample trees, to ascertain the volume, or its forming factors, as basal area, height and form factor, which the trees of a mature wood had at the several periods of its life. Guided by the data thus obtained, woods are selected, the dominant trees of which show the same dimensions as those of the mature trees at the same age. Such woods are assumed to give a true representation of what the now mature wood was at the same age. When a

* Professor of Forestry at the University of Munich.

sufficient number of woods of various ages have been selected, normal sample plots are measured in them, and the data worked up into a yield table for the corresponding quality class. The same procedure is followed for all other quality classes.

Various authors have gradually elaborated this system, first Seutter as early as 1799, then Hossfeld in 1823. Huber, in 1847, was the first to give a regular method of working with an indicating wood. He calculated the mean tree of a normal mature wood, analysed it and searched for younger normal woods, the mean tree of which possesses the same dimensions as the mean tree of the mature wood at the same time. His method is, however, wrong, because the mean tree of the mature wood was not the mean tree at all former stages of life.

Theodor Hartig, and afterwards Robert Hartig, analysed only the largest trees of the mature wood, and then searched for younger woods, an equal number of the largest trees of which show the same dimensions as the largest trees of the mature wood had at the same age. Such woods are considered as having been produced on localities of the same quality, so that they can be united into one yield table.

The system presupposes that the largest trees of the mature wood were at all times amongst the largest trees at previous periods of the wood's life. Although this holds good generally, exceptions occur. Besides, the method is very troublesome in execution.

ii. Baur's Method of Preparing Yield Tables.

After a sufficient number of normal sample plots on all sorts of qualities have been carefully measured (at least 30 for each desired quality class), the volumes are marked as ordinates over the corresponding ages as abscissæ (see figure 36).

Next two curves are drawn, so that the lower touches the lowest points, and the upper the highest points indicating these volumes. Then the area thus confined is divided into as many equal strips, as there are quality classes to be distinguished. The woods falling into each strip are considered as belonging to

the same quality class. By drawing a mean curve through each strip, the mean volume curve for the quality is obtained, from which the volume table is prepared for successive years. In a similar way mean curves for the height, basal area and number of trees are constructed for each quality class. The method is of easy application, and it yields good results.

Instead of limiting the quality classes in the manner described above, the average quantity of final yield which corresponds to each quality class can be once for all determined. Thus in Germany it has been decided to consider that normal Scotch pine woods, which contain (in round numbers) at an age of 100 years :—

10,000	cubic feet	per acre,	shall be	considered as	I.	quality
7,900	,,	,,	,,	,,	II.	,,
6,000	,,	,,	,,	,,	III.	,,
4,300	,,	,,	,,	,,	IV.	,,
2,900	,,	,,	,,	,,	V.	,,

EXAMPLE OF PREPARING YIELD TABLES ACCORDING TO BAUR'S METHOD.

Scotch Pine : 3 Quality Classes to be distinguished.
Woods Measured as follows :—

No.	Age. Years.	No. of Trees.	Basal Area, sq. ft.	Mean Height, feet.	Volume in solid cub. ft.	No.	Age. Years.	No. of Trees.	Basal Area, sq. ft.	Mean Height, feet.	Volume in solid cub. ft.
1	15	...	62	16	1800	21	76	295	173	70	5500
2	17	...	60	14	1400	22	79	265	177	72	6300
3	18	...	61	13	1100	23	81	245	192	86	7200
4	21	...	84	20	1700	24	85	290	156	62	5030
5	27	1400	130	33	3300	25	94	190	196	93	9000
6	29	2400	99	25	2050	26	94	240	150	67	6000
7	34	1480	133	35	3250	27	94	218	177	80	7300
8	35	1670	113	32	2800	28	96	248	150	69	5300
9	35	910	156	46	4450	29	97	200	176	82	6950
10	46	620	165	55	4800	30	99	170	194	93	8200
11	47	740	150	47	4230	31	104	160	192	94	9200
12	48	860	132	40	2900	32	106	169	177	86	7150
13	49	680	154	52	4700	33	106	220	160	69	5700
14	50	750	132	44	3500	34	108	173	179	86	8100
15	54	450	182	69	6400	35	109	210	152	72	6700
16	62	450	169	65	4700	36	109	151	196	96	8500
17	62	369	184	73	6200	37	112	148	194	98	9700
18	68	420	148	56	4450	38	115	150	176	88	7500
19	74	270	192	83	7800	39	118	145	194	98	9000
20	74	350	146	61	4000	40	120	186	157	75	6000

WOODS SEPARATED INTO QUALITIES. (See figure 36.)

No.	Age.	No. of Trees.	Basal Area.	Mean Height.	Volume
		I. Quality.			
1	15	...	62	16	1800
5	27	1400	130	33	3300
9	35	910	156	46	4450
10	46	620	165	55	4800
15	54	450	182	69	6400
17	62	369	184	73	6200
19	74	270	192	83	7800
23	81	245	192	86	7200
25	94	190	196	93	9000
30	99	170	194	93	8200
31	104	160	192	94	9200
36	109	151	196	96	8500
37	112	148	194	98	9700
39	118	145	194	98	9000
		II. Quality.			
2	17	...	60	14	1400
4	21	...	84	20	1700
7	34	1480	133	35	3250
8	35	1670	113	32	2800
11	47	740	150	47	4230
13	49	680	154	52	4700
16	62	450	169	65	4700
21	76	295	173	70	5500
22	79	265	177	72	6300
27	94	218	177	80	7300
29	97	200	176	82	6950
32	106	169	177	86	7150
34	108	173	179	86	8100
38	115	150	176	88	7500
		III. Quality.			
3	18	...	61	13	1100
6	29	2400	99	25	2050
12	48	860	132	40	2900
14	50	750	132	44	3500
18	68	420	148	56	4450
20	74	350	146	61	4000
24	85	290	156	62	5030
26	94	240	150	67	6000
28	96	248	150	69	5300
33	106	220	160	69	5700
35	109	210	152	72	6700
40	120	186	157	75	6000

YIELD TABLE FOR THE SCOTCH PINE, I. QUALITY.

Derived from the Curves in figures 36, 37, 38 and 39.

Age.	Number of Trees.	Basal Area. Square Feet.	Mean Height. Feet.	Volume. Cubic Feet, solid.	Increment.	
					Current Annual.	Mean Annual.
10	...	30	10	900	90	90
20	2000	92	23	2100	120	105
30	1200	133	40	3300	120	110
40	770	160	54	4500	120	112
50	520	175	64	5400	90	108
60	380	186	73	6250	85	104
70	300	190	80	6950	70	99
80	250	192	86	7600	65	95
90	200	193	90	8200	60	91
100	160	194	94	8650	45	86
110	150	194	97	9100	45	83
120	140	194	100	9500	40	79

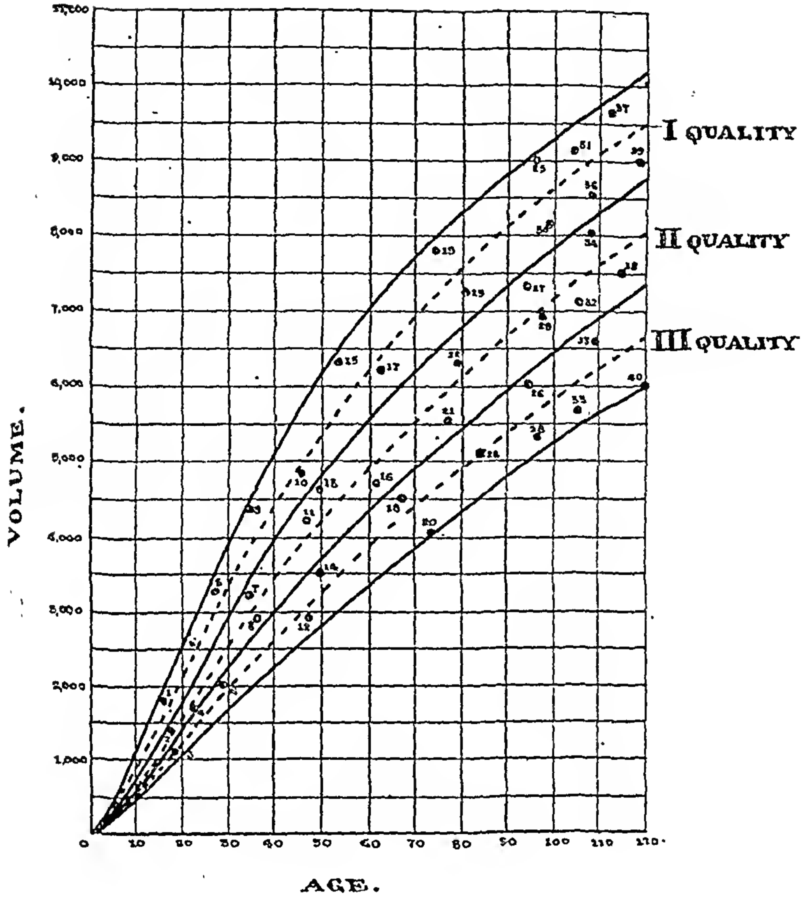


Fig. 36.—Graphic representation of the Volume per acre of 40 different Woods and their allotment to Three Quality Classes, according to Baur's method.

II. *DETERMINATION OF THE INCREMENT OF WOODS BY MEANS OF YIELD TABLES.*

If yield tables are available, and it is desired to estimate the increment of a wood forward or backward, it is necessary to decide in the first place which of the quality classes of the tables corresponds with the given wood; in other words, it must be ascertained to which quality class the wood belongs.

The best way of doing this is to measure the volume of a normal sample plot in the wood and compare it with the

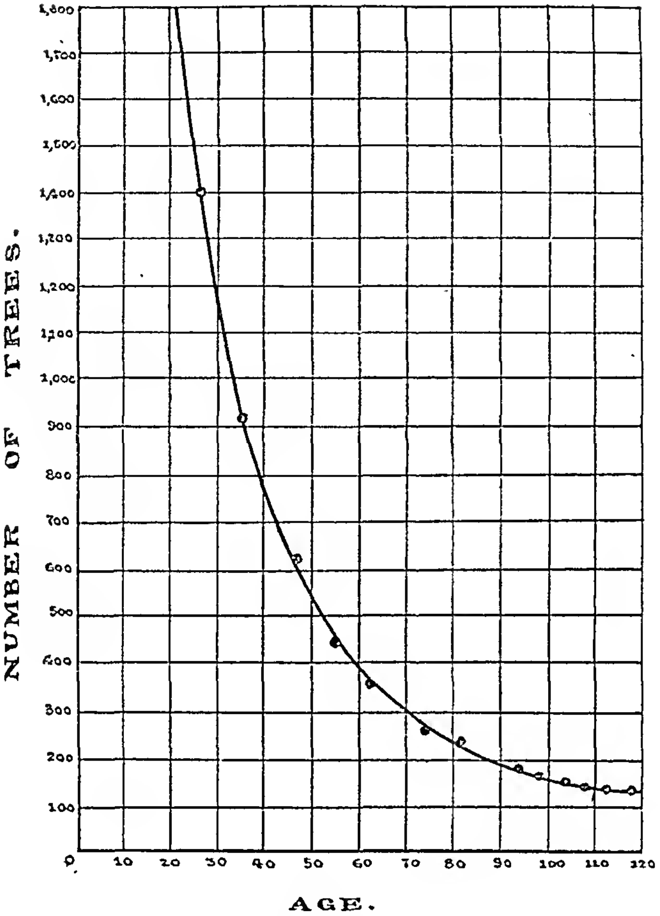


Fig. 37.—Graphic representation of the Number of Trees per Acre.

volumes given in the tables for the same age and the different quality classes. If it agrees with one of these volumes, the two are of the same quality class, and the increment shown in the table applies also to the wood in question.

If the volume of the wood does not agree with any of the volumes in the tables, then that quality class is selected which comes nearest to it, and the increment is ascertained in proportion to the two volumes. Let v_a be the present volume of the wood, V_a the nearest volume given in the table; V_{a+n} the volume given in the same table for the year $a + n$; and v_{a+n} the desired volume of the wood in the year $a + n$, then

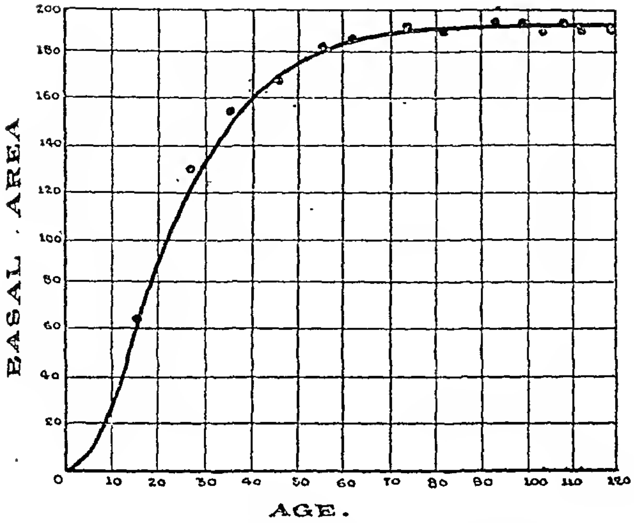


Fig. 38.—Graphic representation of the Basal Areas per Acre.

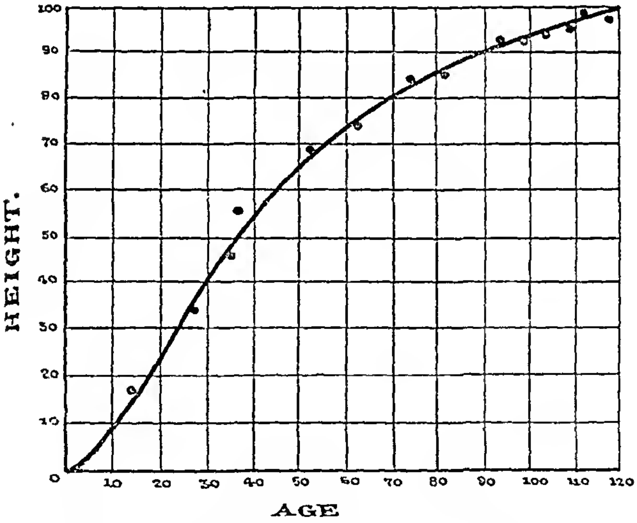


Fig. 39.—Graphic representation of the Height Growth.

the following equation may be assumed to hold good (see figure 40):—

$$V_a : v_a = V_{a+n} : v_{a+n}$$

and

$$v_{a+n} = \frac{v_a \times V_{a+n}}{V_a}$$

and

$$\text{Increment in } n \text{ years} = v_{a+n} - v_a = \frac{V_{a+n} \times v_a}{V_a} - v_a = v_a\left(\frac{V_{a+n}}{V_a} - 1\right)$$

This method rests upon the assumption that the selected yield table represents correctly the progressive increment of the wood, of which the increment is to be ascertained. As

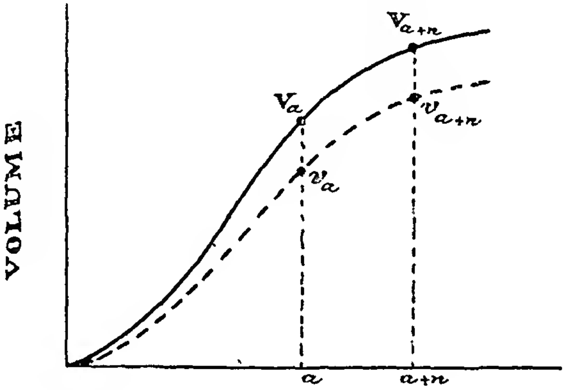


Fig. 40.

this is only approximately the case, the degree of accuracy of the method depends—

(*a*) On the degree to which v_a approaches V_a, and

(*b*) On the difference of ages, that is to say, the difference between a and $a+n$; the smaller this is, the more accurate will be the result.

In following the above method, it is essential to measure the volume on a "normal" sample plot, because only then can the true quality class be ascertained by means of the volume. If no fully stocked sample plot is available, that nearest to it should be selected, and the proportion between the actual and normal stocking ascertained. The actual volume must then be augmented in the same proportion, before it is used for the determination of the quality class. At the same time this procedure is subject to errors, as it is not always easy to determine correctly the proportion between the actual and normal stocking.

Generally, the method is better adapted to woods which have passed middle age than to younger woods, as in the latter the factors of the locality have not in all cases found full expression. In the case of very young woods it is altogether useless to measure the volume for the purpose of selecting the proper yield table. For such woods the quality class must be selected by means of an older wood growing in the vicinity on a locality of similar quality. The same procedure is followed in the case of blanks; if no such older wood is available, the soil and climate must be examined and the best possible estimate of the quality made accordingly.

As the measurement of the volume takes much time, and as it is difficult to estimate the exact proportion between the actual and normal stocking, it has been proposed to select the proper yield table for a wood by means of one factor only of the volume. It has already been explained that of all, such as number of trees, diameter, form factors, basal area and height, the last is the most suitable. Indeed, actual investigation has proved that in the case of all woods of middle age and upwards the volume of two woods, other conditions being the same, is fairly proportionate to their mean heights. The mean height is, therefore, an excellent indication of the quality class; it, as well as the age, are comparatively easy to ascertain. In selecting the appropriate yield table the mean height is used in the same way as has been described for the volume. If the height agrees with one of the heights given in the yield table for the same age, the increment can be read off directly. If it differs, the nearest is selected and the increment of the table modified in proportion to the difference between the actual height and that given in the table. If, moreover, the wood is not fully stocked, the increment given in the table must be further modified in the manner indicated above.

The height is no true indicator of the quality for young woods; for such, as well as for blanks, other woods growing in the vicinity must be utilized, or the soil and climate examined.

Example :—

A Scotch pine wood has a height of 53 feet when 60 years old, and a volume of 3,800 cubic feet; find the probable increment for the next ten years?

YIELD TABLES.

Quality.	Height.	Volume in the Year 60.	Volume in the Year 70.
I.	72	6750	—
II.	60	5420	—
III.	51	4060	4530
IV.	42	3360	—
V.	35	2670	—

The wood belongs to the III. Quality.

According to the formula—

$$I_n = (v_{a+n} - v_a) \times \frac{h_a'}{h_a} \times \frac{v_a'}{v_a}$$

$$I_{60-70} = (4530 - 4060) \times \frac{53}{51} \times \frac{3800}{4060} = 457 \text{ cubic feet.}$$

PART II.

FOREST VALUATION.

FOREST VALUATION.

Forest Valuation deals with the determination of the value of forest soil, the growing stock, the forest as a whole, and the rental derivable from the soil, growing stock, or forest.

These values must be determined in all cases of sale, or for the purpose of assessing forest property, or where it is proposed to divide a property. Soil and growing stock also form the capital invested in forestry, and their value must be ascertained for the purpose of gauging the financial success of the forest industry. The latter subject is generally dealt with in a separate part called, by continental foresters, "Forest Statics." In the present instance it is proposed to compress that part into one chapter and to add it to "Forest Valuation."

In order to deal with the matter here contemplated, it is necessary to explain the various methods according to which the value of property may be ascertained, to determine the rate of interest applicable to the forest industry, to give certain formulæ for calculating with compound interest, and to explain the methods of estimating receipts and expenses. All these matters will be dealt with in a preliminary chapter, and the subjects here under consideration will be arranged as follows:—

Chapter I. Preliminary Matters.
,, II. Valuation of Forest Soil.
,, III. ,, ,, the Growing-Stock.
,, IV. ,, ,, Whole Woods or Forests.
,, V. ,, ,, the Rental.
,, VI. Methods of Calculating the Financial Results of the Forest Industry and of determining the most Profitable Treatment of Forests.

CHAPTER I.

PRELIMINARY MATTERS.

SECTION I.—VALUE OF PROPERTY GENERALLY.

PROPERTY means an object which serves for the satisfaction of a requirement. The degree of utility of a property indicates its value.

The value of property may present itself in various ways:—A piece of property may possess value, because it can be used for a certain purpose (as articles of food), or for the production of another kind or class of property (as a set of carpenters' tools, or raw materials).

Again, the value of a property may be general or special; the former is that which a property has in the open market; the latter is that which it has for a particular person (as a piece of land situated in the middle of another estate), or under special circumstances (as a loaf of bread in a famine-stricken district).

By the price of a property is understood the amount of another class of property which is offered for it in exchange; the ordinary means of exchange is money.

The value of property can be ascertained in various ways, of which four must be mentioned:—

(1) *The expectation value*, by which is understood the present net value of all yields which a property may be able to give; it is determined by discounting all net incomes derivable from the property to the present time.

(2) *The cost value*, or the total outlay on the acquisition or production of a property.

(3) *The sale value*, or the price which can be realized by the sale of a property; if the sale is open to competition, the sale value becomes the *market price*, which depends on supply and demand.

(4) *The rental value*, by which is understood the capital which corresponds to the rental which the property is capable of yielding. This value is ascertained by capitalizing the rental according to the formula—

$$\text{Capital} = \frac{\text{Rental} \times 100}{\text{rate per cent.}} = \frac{R \times 100}{p}.$$

In determining the value of a forest property, or indeed any other property, all calculations must be made with compound interest, as all money, whether principal or interest, is capable of again yielding interest.

Section II.—Choice of Rate of Interest.

By rate of interest is understood the proportion between the yearly interest (I) and the capital (C) which has yielded it, as represented by the formula—

$$\text{Rate of interest} = \frac{I}{C}.$$

By rate per cent. or, shortly, per cent., is understood the yearly interest yielded by a capital of 100; hence the per cent. :—

$$\mathrm{p} = \frac{\mathrm{I}}{\mathrm{C}} \times 100.$$

The rate of interest which is applicable to an industry depends on many things, of which the most important are :—

(*a*) The degree of security of the investment and the safety with which it yields a return. In a general way it may be said that the rate of interest is inversely proportional to the safety of the investment.

(*b*) The supply and demand of capital, which changes from time to time, and with the locality.

(*c*) The general credit of the country in which the industry

is carried on, in other words the interest yielded by Government securities (called Consols in England).

It follows that the general rate of interest applicable to the forest industry is not a fixed quantity, but that it changes with the locality, time, and a variety of other circumstances.

The question then arises, what rate of interest is applicable to the forest industry under a given set of conditions? In attempting to answer this question, the following points must be taken into consideration:—

(1) The safety of capital invested in forests. The soil offers (apart from changes in prices) almost absolute security. The growing stock is subject to damage by men, insects, fungi, wind, snow, rime, and above all, by fire. The degree of danger differs much according to species, method of treatment, length of rotation, climate, etc.; in temperate climates the damage keeps within narrow limits.

(2) The price of forest produce is, on the whole, subject to less sudden fluctuations than the value of money.

(3) Investment in forest property possesses the great convenience of yielding a steady income.

(4) Forests cannot easily be let on lease, as inroads on the growing stock are difficult to control; for the same reason forests, beyond the value of the land, are not well suited as security for loans.

(5) Compared with the cultivation of field crops it must be noted that:—

 (*a*) A forest once placed under systematic management yields annually equal returns, or nearly so, while those of fields differ much according to the seasons.

 (*b*) Forests require much less labour, and the management is comparatively simple.

 (*c*) Temporary high prices can be fully utilized by cutting more than the normal yield for a time, or *vice versâ*.

(*d*) As a rule successful forest management requires larger estates than the cultivation of field crops.

Bearing these matters in mind, attempts may be made to determine the rate of interest for the forest industry in one of the following ways :—

(1) Determination based upon the rental and value of the soil.

It has been shown above that—

$$p = \frac{I}{C} \times 100.$$

By substituting the value S of the soil for C, and the rental R of the soil for I, the above equation becomes—

$$p = \frac{R}{S} \times 100.$$

The drawback of this method lies in the difficulty of ascertaining correctly the value of the soil and of the rental without having previously determined the rate of interest; in other words such a determination would turn in a circle.

(2) Determination based upon the rental and value of the forest.

If the value of a forest (F), which is managed so as to yield an annual net return R', is known from a sale which has taken place, the per cent. would be :

$$p = \frac{R'}{F} \times 100.$$

The conditions for the applicability of the method are :—

(*a*) That the annual rental of the forest is accurately known.

(*b*) That the forest is, at any rate approximately, in such a condition that it can yield a steady annual return.

(*c*) That the price realized for the forest was the result of genuine competition.

There is great difficulty in complying with all these conditions, so that, on the whole, neither this nor the first methods are of much practical value.

(3) Determination based upon the rate of interest obtainable in agriculture.

As indicated above, differences exist between the two methods of using the soil, which are difficult to estimate. At the same time, forestry and agriculture resemble each other in many respects, so that in many cases the agricultural rate of interest can be applied to the forest industry.

(4) Determination according to the rate of interest yielded by Government securities.

The rate of interest of Consols may be too high in some cases, and too low in others, according to the general credit of the State, and the nature of the forests. On the whole, however, this is a safe way of determining the rate of interest for the forest industry in all well regulated States.

The rate of interest generally adopted in the calculations contained in the following chapters is that of British Consols, that is to say, $2\frac{1}{2}$ per cent.

Section III.—Formulæ of Compound Interest.

The following formulæ, based upon compound interest, are required in forest valuation:—

1. *Amount or the Future Value to which a Capital accumulates.*

A capital C_o put out at p per cent. compound interest accumulates in the course of n years to the value:—

$$C_n = C_o \times 1{\cdot}op^n \quad . \quad . \quad . \quad . \quad \text{(I.)}$$

or

$$\log.\ C_n = \log.\ C_o + n \times \log.\ 1{\cdot}op.$$

where

$$1{\cdot}op = \frac{100+p}{100} \text{ and } 1{\cdot}op^n = (1{\cdot}op)^n = \left(\frac{100+p}{100}\right)^n.$$

2. *Discount, or Determination of Present Value.*

The present value C_o of a capital C_n to be realized n years hence, is:—

$$C_o = \frac{C_n}{1{\cdot}op^n} \quad . \quad . \quad . \quad . \quad \text{(II.)}$$

$$\log.\ C_o = \log.\ C_n - n \times \log.\ 1{\cdot}op.$$

3. *Summation of Rentals.*

a. Future Value.

A rental R becomes due for the first time after m years, and is repeated altogether n times at intervals of m years; its value after $m \times n$ years is:—

$$C_{mn} = \frac{R\,(1{\cdot}op^{mn} - 1)}{1{\cdot}op^{m} - 1} \quad . \quad . \quad . \quad \text{(III.)}$$

A rental r is due at the end of each year, altogether n times; its value at the end of n years is:—

$$C_{n} = \frac{r\,(1{\cdot}op^{n} - 1)}{{\cdot}op} \quad . \quad . \quad . \quad \text{(IV.)}$$

b. Present Value.

A rental R becomes due for the first time after m years, and is repeated n times after intervals of m years; its present value is:—

$$C_{o} = \frac{R\,(1{\cdot}op^{mn} - 1)}{1{\cdot}op^{mn}\,(1{\cdot}op^{m} - 1)} \quad . \quad . \quad . \quad \text{(V.)}$$

A rental r is due at the end of each year, altogether n times; its present value is:—

$$C_{o} = \frac{r\,(1{\cdot}op^{n} - 1)}{1{\cdot}op^{n} \times {\cdot}op} \quad . \quad . \quad . \quad \text{(VI.)}$$

A rental r is due at the end of each year, for ever; its present value is:—

$$C_{o} = \frac{r}{{\cdot}op} \quad . \quad . \quad . \quad . \quad \text{(VII.)}$$

A rental R is due after n years, and again every n years, for ever; its present value is:—

$$C_{o} = \frac{R}{1{\cdot}op^{n} - 1} \quad . \quad . \quad . \quad \text{(VIII.)}$$

A rental R is due after m years, and again every n years, for ever; its present value is:—

$$C_{o} = \frac{R \times 1{\cdot}op^{n-m}}{1{\cdot}op^{n} - 1} \quad . \quad . \quad . \quad \text{(IX.)}$$

A rental R is due now, and then after every n years, for ever; its present value is:—

$$C_o = \frac{R \times 1 \cdot op^n}{1 \cdot op^n - 1} \qquad . \qquad . \qquad . \qquad . \qquad (X.)$$

4. *Conversion of an Intermittent into an Annual Rental.*

A rental R is due every n years, commencing from the present time; it is equal to an annual rental of—

$$r = \frac{R}{1 \cdot op^n - 1} \times \cdot op \qquad . \qquad . \qquad . \qquad (XI.)$$

A rental R is due after m years, and then every n years for ever; its value is equal to an annual rental of

$$r = \frac{R \times 1 \cdot op^{n-m}}{1 \cdot op^n - 1} \times \cdot op \qquad . \qquad . \qquad . \qquad (XII.)$$

A rental R is due now, and again every n years for ever; its value is equal to an annual rental of—

$$r = \frac{R \times 1 \cdot op^n}{1 \cdot op^n - 1} \times \cdot op \qquad . \qquad . \qquad . \qquad (XIII.)$$

All the above calculations can be made with logarithms, or by means of tables which give future or present values for various rates of interest. The Tables at page 394, which give the values for Formulæ I., II., VIII., and VI., suffice for all ordinary forest calculations.

Section IV.—Estimate of Receipts and Expenses.

1. *Receipts.*

The receipts of forests are derived from a variety of produce, which are generally divided into two classes: Major or principal produce, and minor produce.

a. Major Produce.

This comprises all yields of wood, that is to say, timber and firewood. Bark is generally included; if it is severed from

the timber before disposal, it is sometimes classed as minor produce.

Major produce is subdivided into that obtained from final cuttings (final yield), and that from thinnings, etc. (intermediate yields).

The value of major produce is ascertained by means of *money yield tables*, which are calculated from *volume yield tables*.

The preparation of volume yield tables and the selection of the proper table have been explained in Forest Mensuration. If tables are available, the age, volume and height of each wood are ascertained, and these data are used in the selection of the proper yield table. If the data do not agree with any table, the nearest is selected and its quantities are modified in proportion to the volumes, or heights, as the case may be.

When the produce yield table has been determined, it is converted into a money yield table, for which purpose the local prices of the several classes of timber and firewood must be ascertained. In doing this it must be remembered that the average value of material per unit rises with the rotation.

These tables refer to fully stocked or normal woods; hence the quantities must be modified before use in the same degree as a particular wood differs from the normal condition. In some cases a further reduction is made as a kind of insurance against future events. Some authors give 10 per cent. as a proper reduction.

b. *Minor Produce.*

This comprises all yields which do not consist of timber and firewood; their amounts and values must be locally ascertained.

2. *Expenses.*

The expenses comprise the cost of administration, protection, formation, harvesting, construction of roads, slides, houses, taxes, etc. The amounts must be locally ascertained.

3. *Examples of Money Yield Tables.*

Subjoined follow two money yield tables, the first being based upon Weise's yield table for Scotch pine, III. quality, and the second upon Schwappach's yield table for the beech, III. quality.

MONEY YIELD TABLE FOR ONE ACRE OF SCOTCH PINE WOOD.

According to Weise, III. or middling quality; calculated with English prices; brushwood under 3 inches diameter omitted.

Age.	Yield in Cubic Feet.		Net Value of Returns.					Remarks.
			Per Cubic Foot, Pence.		Total, Shillings.			
	Final.	Intermediate	Final.	Intermediate	Final.	Intermediate	Total Final and Intermediate	
a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	
20	30							For simplicity's sake, it has been assumed that the thinnings are made at the end of every 10 years; hence column *h* gives the values of the wood before the thinnings are made, and column *f* the values which remain after the thinnings. *Example.*—In the year 60 the volume amounts to 3300 + 413=3713*c'*, of which 413 fall in the thinning, while 3300 remain. The corresponding values are 1100+86=1186 shillings.
30	830	48	2·	1·	138	4	142	
40	1970	289	2·5	1·5	406	36	442	
50	2700	403	3·	2·	675	67	742	
60	3300	413	4·	2·5	1100	86	1186	
70	3820	363	5·	3·	1592	91	1683	
80	4260	327	6·	3·5	2130	95	2225	
90	4620	282	7·	4·	2695	94	2789	
100	4910	248	8·	5·	3273	103	3376	
110	5150	213	9·	6·	3862	106	3968	
120	5340	164	10·	7·	4450	96	4546	

Money Yield Table for One Acre of Beech Wood.

According to Schwappach, III. or middling quality, calculated with English prices; brushwood under 3 inches diameter omitted.

Age.	Yield in Cubic Feet.		Net Value of Returns.					Remarks.
			Per Cubic Foot, Pence.		Total, Shillings.			
	Final.	Inter-mediate	Final.	Inter-mediate	Final.	Inter-mediate	Total Final and Inter-mediate	
a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	
30	86	—	2·	—	14	—	14	See remarks on money yield table for a Scotch pine wood.
40	943	—	3·	—	236	—	236	
50	2001	86	4·5	3·	750	21	771	
60	2915	272	6·	4·	1457	91	1548	
70	3716	329	7·	4·5	2167	123	2290	
80	4430	357	8·	5.	2953	149	3102	
90	5045	400	9·	6·	3784	200	3984	
100	5573	414	10·	7·	4644	241	4885	
110	6045	400	11·	8·	5541	267	5808	
120	6460	400	12·	9·	6460	300	6760	
130	6817	386	13·	10·	7385	322	7707	
140	7117	372	14·	11·	8303	341	8644	

CHAPTER II.

VALUATION OF FOREST SOIL.

THE soil can be utilized in two ways:—

Either by being used direct, as for mining, quarrying, construction of dwellings, etc.;

Or by letting it produce other goods, as field- or forest-crops.

In each of these cases the soil may have a different value. Forest valuation ascertains the value of the soil under the supposition that it is used for the production of forest crops; for this purpose it determines the expectation, cost, and sale value.

SECTION I.—THE EXPECTATION VALUE OF FOREST SOIL.

1. *Method of Calculation.*

In conformity with the definition given at page 114, by the expectation value of forest soil is understood the sum of the present values of all returns expected from the soil in the course of time, less the present value of all expenses which must be incurred to obtain those returns.

a. Present Value of Returns.

(1) *Final Yields.*—Let the rotation contain r years, and let the value of each final yield, less cost of harvesting, $= Y_r$, to be realized every r years, then the present value of all final yields for ever amounts, according to formula VIII. (page 119), to:—

$$\text{Present value of all final yields} = \frac{Y_r}{1{\cdot}op^r - 1}.$$

(2) *Intermediate Cuttings, or Thinnings.*—Let T_a, T_b, T_c . . . T_q, represent the value of thinnings, less cost of harvesting, occurring in the years a, b, c, . . . q. Each of these thinnings occurs during every rotation, that is to say, every r years. Thus the first thinning occurs after a years, again after $a + r$ years, again after $a + 2 \times r$ years, and so on; hence the present value of all these thinnings, according to formula IX.:—

$$= \frac{T_a \times 1{\cdot}op^{r-a}}{1{\cdot}op^r - 1} + \frac{T_b \times 1{\cdot}op^{r-b}}{1{\cdot}op^r - 1} + \ldots + \frac{T_q \times 1{\cdot}op^{r-q}}{1{\cdot}op^r - 1}$$

$$= \frac{T_a \times 1{\cdot}op^{r-a} + T_b \times 1{\cdot}op^{r-b} + \ldots + T_q \times 1{\cdot}op^{r-q}}{1{\cdot}op^r - 1}.$$

(3) *Minor Produce.*—These are dealt with in the same way as thinnings, if they occur at regular intervals. Let M_s, M_t, . . . M_z, be the values of minor produce occurring in the years s, t . . . z, and again every r years, then their present value is—

$$\frac{M_s \times 1{\cdot}op^{r-s} + M_t \times 1{\cdot}op^{r-t} + \ldots + M_z \times 1{\cdot}op^{r-z}}{1{\cdot}op^r - 1}.$$

If items of minor produce occur regularly every year, as grazing fees, rent from shooting, etc., and of equal value year after year, their present value amounts, according to formula VII., to $\frac{M}{{\cdot}op}$.

b. Present Value of Expenses.

(1) *Cost of Formation.*—Assuming that c represents the cost of formation, whether artificial or natural, which has to be incurred at the commencement of each rotation, then the present value of all such expenses comes, according to formula X., to:—

$$\text{Total present value of cost of formation} = \frac{c \times 1{\cdot}op^r}{1{\cdot}op^r - 1}.$$

If the first cost differs from that on future occasions, and if the latter is represented by c', then the total present value is—

$$c + \frac{c'}{1{\cdot}op^r - 1}.$$

(2) *Annually Recurring Expenses.*—Let e represent the annual value of all annually recurring expenses, as cost of administration, taxes, etc., then the total present value of all such expenses comes to—

$$\frac{e}{\cdot op} = E,$$

or the capitalised value of the annual expenses.

(3) *Periodically Occurring Expenses.*—These are dealt with as has been shown in the case of thinnings or items of minor produce.

(4) *Expenses of Harvesting and of Collection of Revenue.*—These are always deducted from the receipts, and they do not appear in the account.

c. *Formula for the Expectation Value of Forest Soil.*

This formula would be represented by an addition of all the items enumerated under *a* less those under *b*. In order to shorten the formula, without destroying its accuracy, it may be assumed—

(1) That T_a, T_b, . . . T_q, represent thinnings as well as items of minor produce;

(2) That the expenses are reduced to the cost of formation and the annually recurring expenses; and

(3) That the cost of formation at the commencement of each rotation comes to the same amount.

The expectation value S_e of the soil is then represented by the formula—

$$S_e = \frac{Y_r + T_a \times 1{\cdot}op^{r-a} + \ldots + T_q \times 1{\cdot}op^{r-q} - c \times 1{\cdot}op^r}{1{\cdot}op^r - 1} - E$$

or, as

$$\frac{c \times 1{\cdot}op^r}{1{\cdot}op^r - 1} = c + \frac{c}{1{\cdot}op^r - 1}$$

$$S_e = \frac{Y_r + T_a \times 1{\cdot}op^{r-a} + \ldots + T_q \times 1{\cdot}op^{r-q} - c}{1{\cdot}op^r - 1} - (c + E).$$

Example :—An acre of land is to be cultivated at once with Scotch pine, and to be worked under a rotation of 80 years. It is expected to yield the returns given in the money yield

table at page 122. The expenses are expected to be as follows :—

Cost of formation every 80 years = 60 shillings.

Annual expenses for administration, taxes, etc. = 3 shillings.

Interest = $2\frac{1}{2}$ per cent.

The expectation value of the soil will, in that case, amount to :—

$$S_e = \frac{2225 + 4 \times 1{\cdot}025^{50} + 36 \times 1{\cdot}025^{40} + 67 \times 1{\cdot}025^{30} + 86 \times 1{\cdot}025^{20} + 91 \times 1{\cdot}025^{10} - 60 \times 1{\cdot}025^{80}}{1{\cdot}025^{80} - 1} - \frac{3}{\cdot 025}.$$

Calculating with the tables at page 394, the above becomes

$$S_e = (2225 + 4 \times 3{\cdot}4371 + 36 \times 2{\cdot}6851 + 67 \times 2{\cdot}0976 + 86 \times 1{\cdot}6386 + 91 \times 1{\cdot}2801 - 60 \times 7{\cdot}2096) \times {\cdot}1610 - 120.$$

$$S_e = (2225 + 13{\cdot}7484 + 96{\cdot}6636 + 140{\cdot}5392 + 140{\cdot}9196 + 116{\cdot}4891 - 432{\cdot}5760) \times {\cdot}1610 - 120.$$

$$S_e = 2300{\cdot}7839 \times {\cdot}1610 - 120 = 250 \text{ shillings} = £12\ \ 10\ \ 0.$$

This shows that, under the above conditions, and calculating with $2\frac{1}{2}$ per cent. compound interest, it pays to plant Scotch pine on land of the III. quality, if it can be purchased for £12 10*s*. an acre or under.

Assuming now that an acre of land is planted with beech, that it yields the returns given in the money yield table at page 123, and that all other conditions remain the same, the expectation value of the soil will be—

$$S_e = \frac{3102 + 21 \times 1{\cdot}025^{30} + 91 \times 1{\cdot}025^{20} + 123 \times 1{\cdot}025^{10} - 60 \times 1{\cdot}025^{80}}{1{\cdot}025^{80} - 1} - 120.$$

$$S_e = 366 \text{ shillings} = £18\ \ 6\ \ 0.$$

It follows that under the above conditions it pays to plant beech on lands of the III. quality, if it can be purchased for £18 6*s*. an acre or under.

Note.—It must not be overlooked that a beech soil of the III. or middling quality, would at least represent a soil of the II. quality, if planted with Scotch pine.

2. *Matters which affect the Expectation Value of the Soil.*

As indicated by the formula, the expectation value of the soil depends on—

(*a*) The absolute amount of receipts and expenses.
(*b*) The length of the rotation.
(*c*) The time when the intermediate returns are realized.
(*d*) The time when the costs of production have to be incurred; and
(*e*) The rate of interest with which the calculation is made.

Each of these matters must be further considered.

a. The absolute amount of Receipts and Expenses.

These are influenced, not only by careful management, but also by the choice of species. If, for one and the same piece of land, the calculation is made for different species, the expectation values thus obtained are likely to differ considerably; hence the importance of selecting the most suitable species for each piece of land.

b. The Length of Rotation.

As the growing stock has, in the majority of cases, very little value during the first part of the life of a wood, so that the yield would not even cover the cost of harvesting, it follows that the expectation value of the soil may under a very short rotation be negative. With advancing age the value of the growing stock increases, so that the expectation value of the soil becomes positive; it goes on augmenting until it reaches a maximum, after which period it falls again. A second maximum may occur owing to a sudden heavy increase in the value of the growing stock. Under an excessively high rotation the expectation value of the soil would again become negative.

Example.—Making the calculation for successive rotations for a Scotch pine wood, with the same conditions as before, the following values are obtained for the expectation value :—

S_e for a rotation of	30 years	=	− 105 shillings.
S_e ,, ,, ,,	40 ,,	=	+ 50 ,,
S_e ,, ,, ,,	50 ,,	=	+ 121 ,,
S_e ,, ,, ,,	60 ,,	=	+ 196 ,,
S_e ,, ,, ,,	70 ,,	=	+ 236 ,,
S_e ,, ,, ,,	80 ,,	=	+ 250 ,,
S_e ,, ,, ,,	90 ,,	=	+ 245 ,,
S_e ,, ,, ,,	100 ,,	=	+ 229 ,,
S_e ,, ,, ,,	110 ,,	=	+ 207 ,,
S_e ,, ,, ,,	120 ,,	=	+ 180 ,,

The above data show that, financially, the most favourable rotation must be that under which the expectation value reaches its maximum; in the above example a rotation of about 80 years.

c. The Time when the Intermediate Returns are Realized.

The earlier the intermediate returns occur, other matters being the same, the higher will be the expectation value.

Example.—The present value of a thinning, worth 86 shillings, made in the year 60, and again every 100 years, is =

$$\frac{86 \times 1{\cdot}025^{40}}{1{\cdot}025^{100} - 1} = 21 \text{ shillings.}$$

If the same thinning were made in the year 30, and again every 100 years, its present value would be =

$$\frac{86 \times 1{\cdot}025^{70}}{1{\cdot}025^{100} - 1} = 45 \text{ shillings.}$$

It follows that strong thinnings made at an early age can considerably increase the expectation value. At the same time it must not be overlooked that the small material obtained from early thinnings is not always saleable at remunerative rates; moreover, such thinnings may disastrously affect the later returns, especially the final yield, so that the advantage gained in the first instance may be more than counterbalanced by loss later on.

Early receipts from minor produce affect the expectation value in the same way as those from thinnings; hence they are of great financial importance, as long as they do not unduly affect later returns.

d. The Time when the Costs of Production have to be Incurred.

This affects the expectation value in a manner the reverse of that produced by early thinnings and incomes from minor produce. Expenses incurred during the early part of the rotation affect the expectation value unfavourably; hence it is important to keep the cost of formation as low as possible.

Example.—The present value of the cost of formation amounting to 60 shillings to be incurred at once and again every 100 years is=

$$\frac{60 \times 1{\cdot}025^{100}}{1{\cdot}025^{100} - 1} = 65{\cdot}5 \text{ shillings.}$$

If the cost amounted to 120 shillings each time, the present value would be=

$$\frac{120 \times 1{\cdot}025^{100}}{1{\cdot}025^{100} - 1} = 131 \text{ shillings.}$$

Thus it may happen that the expectation value is higher under the system of natural regeneration, than under the clear cutting system with planting.

e. The Rate of Interest with which the Calculation is made.

A high rate of interest gives a low expectation value of the soil, and *vice versâ*. The value is, however, not in inverse proportion to the rate of interest; the former rises more rapidly than the latter falls.

Again, under a low rate of interest the expectation value culminates later than under a high rate of interest.

Example.—Taking the same data as above, and calculating

the expectation value of the soil with 2½ and again with 3 and 4 %, the following values are obtained :—

Rotation. Years.	Calculation made with		
	2½%	3%.	4%.
	s.	s.	s.
60	+ 196	+ 104	– 15
70	+ 236	+ 123	+ 5
80	+ 250	+ 124	– 3
90	+ 245	+ 113	– 15

This example shows :—

(1) By raising the per cent. from 2½ to 3, the expectation value for a rotation of 80 years falls from 250 to 124 shillings, and calculated with 4% to – 3 shillings.

(2) By making the calculation with 4 % the expectation value becomes practically nil, and under a higher per cent. it becomes negative.

(3) The expectation value of the soil culminates :—

Calculated with 2½% at about 82 years.
" " 3 " " 78 "
" " 4 " " 72 "

3. *Merits of the Method of the Expectation Value.*

The expectation value indicates the true economic value of the soil for forest culture, because it is based upon the productive power of the land when used for the production of forest crops. It gives the value which corresponds to the net returns calculated with the adopted rate of interest. On the other hand, the method gives correct results only under the following conditions :—

a matter which introduces much uncertainty into the calculation.

(b) That the calculation is made with a suitable rate of interest. It has been shown above that this is a matter beset by considerable difficulty.

(c) That the rotation corresponding to the maximum expectation value of the soil can be adopted and retained without thereby lowering the price of forest produce; in other words, that the market can readily absorb any extra cuttings which may be necessary in order to introduce a rotation lower than that hitherto followed.

Generally speaking, the expectation value of forest soil is not a fixed quantity; it changes, not only in the ways indicated above, but also with alterations in the price of forest produce consequent on changes in the areas set aside for the production of forest crops.

Section II.—The Cost Value of Forest Soil.

By the cost value of the soil is understood the sum of all expenses incurred in acquiring the land and rendering it fit for forest culture. These expenses consist of:—

(1) The price paid for the land.

(2) The sum expended in rendering it fit for cultivation, such as drainage or irrigation, levelling, fixation, etc.

(3) The interest accumulated on the outlay mentioned under (1) and (2) up to the date when the first forest crop is started.

Example.—An acre of land has been purchased for £10; a sum of £5 has been expended at once in breaking through an impermeable substratum in strips 6 feet apart; a further sum of £2 has been spent after the lapse of 3 years in breaking up the intermediate strips; the land was allowed to lie fallow for another 2 years. The cost value of the land at the end of 5 years, when it is planted, amounts, calculating with $2\frac{1}{2}\%$, to

$$S_c = (10+5)\times 1{\cdot}025^5 + 2\times 1{\cdot}025^2 = £19\ 1\ 5$$

The cost value of the land may be accepted as the true value:—

(1) If the owner agrees to let the land go at the price which represents his own outlay on it :

(2) If the expectation value of the soil cannot be ascertained with any degree of accuracy :

The cost value of the soil may be equal to, smaller, or larger than the expectation value.

Section III.—The Sale Value of Forest Soil.

By the sale value of forest soil is understood the value which it realizes in the open market. It represents the true economic value only, if it agrees with the expectation value.

In most localities a sale price has established itself, but this represents in the majority of cases the value which the land has for other purposes, such as agriculture. It may differ considerably from the value which the land has if used for the production of forest crops, the sale value being generally higher in the case of good lands, and lower in the case of inferior lands, because the former yield a higher rental under field crops, and the latter under forest crops.

The sale value of forest soil may be taken as expressing the true value :—

(1) If forest soil is to be disposed of voluntarily for other uses.

(2) In the case of forced sales, when the local value has to be ascertained, rather than the forest value.

CHAPTER III.

VALUATION OF THE GROWING STOCK.

THE value of the growing stock can, as in the case of the soil, be determined as the expectation, cost, or sale value.

The valuation may refer to :—

(1) A whole wood or a series of woods.

(2) A part of a wood, such as one or more trees, one or more units of measurement, or one or several years increment.

SECTION I.—VALUE OF THE GROWING STOCK OF A WHOLE WOOD.

1. *Expectation Value of a Whole Wood.*

The expectation value of the growing stock of a wood now m years old is equal to the present value of all incomes, which may be expected from the wood, less the present value of all expenses which must be incurred between the year m and the time when the wood is finally cut over.

a. *Method of Calculation.*

Starting from the same data as those given in the case of the valuation of forest soil, the *receipts* consist of :—

(1) Final yield in the year $r = Y_r$; present value =

$$\frac{Y_r}{1{\cdot}op^{r-m}}.$$

(2) Intermediate yields, as thinnings, to be realized in the years n, o, p, and q ; their present value amounts to :—

Thinning in the year $n = \frac{T_n}{1{\cdot}op^{n-m}} = \frac{T_n \times 1{\cdot}op^{r-n}}{1{\cdot}op^{r-m}}$.

" " " o . . $= \frac{T_o \times 1{\cdot}op^{r-o}}{1{\cdot}op^{r-m}}$.

" " " p . . $= \frac{T_p \times 1{\cdot}op^{r-p}}{1{\cdot}op^{r-m}}$.

" " " q . . $= \frac{T_q \times 1{\cdot}op^{r-q}}{1{\cdot}op^{r-m}}$.

Expenses :

(1) Annual expenses to be incurred from the year m to the year r; their present value amounts to :—

$$\frac{e}{1{\cdot}op} + \frac{e}{1{\cdot}op^2} + \ldots + \frac{e}{1{\cdot}op^{r-m}} = \text{(according to Formula VI.)}$$

$$= \frac{e\,(1{\cdot}op^{r-m}-1)}{1{\cdot}op^{r-m} \times {\cdot}op} = \frac{\frac{e}{{\cdot}op}\,(1{\cdot}op^{r-m}-1)}{1{\cdot}op^{r-m}} = \frac{E\,(1{\cdot}op^{r-m}-1)}{1{\cdot}op^{r-m}}.$$

(2) Rent of soil to be paid from the year m to the year r; its annual amount may be denoted by $S \times {\cdot}op$; the total present value of the rent of the soil during $r - m$ years amounts to :—

$$\frac{S \times {\cdot}op}{1{\cdot}op} + \frac{S \times {\cdot}op}{1{\cdot}op^2} + \ldots + \frac{S \times {\cdot}op}{1{\cdot}op^{r-m}} = \frac{S \times {\cdot}op\,(1{\cdot}op^{r-m}-1)}{1{\cdot}op^{r-m} \times {\cdot}op}$$

$$= \frac{S\,(1{\cdot}op^{r-m}-1)}{1{\cdot}op^{r-m}}.$$

The *formula* for the expectation value of a wood stands therefore as follows :—

$$^{m}G_{e} = \frac{Y_r + T_n \times 1{\cdot}op^{r-n} + \ldots + T_q \times 1{\cdot}op^{r-q} - (S+E)\,(1{\cdot}op^{r-m}-1)}{1{\cdot}op^{r-m}}$$

Example.—A fully-stocked Scotch pine wood, worked under a rotation of 80 years, is feloniously burned when 45 years old; what compensation per acre should be paid to the owner, if the expected returns are those indicated in the table at page 122.

Rate of interest = $2\frac{1}{2}$ per cent.
Value of soil = 250 shillings.
Annual expenses = 3 ,, to be incurred at the end of each year :—

$$ {}^{45}G_e = \frac{2225 + 67 \times 1{\cdot}025^{30} + 86 \times 1{\cdot}025^{20} + 91 \times 1{\cdot}025^{10} - (250 + 120)\,(1{\cdot}025^{35} - 1)}{1{\cdot}025^{35}}. $$

$$ {}^{45}G_e = 891 \text{ shillings} = £44\ 11\ 0 $$

b. Notes on the Expectation Value of the Growing Stock.

The expectation value of the growing stock depends on the following matters :—

i. The Absolute Amount of Receipts and Expenses.

Regarding the value of the soil to be introduced into the calculation it should be noted, that the maximum expectation value should be chosen if the soil is again to be used for forest purposes; if the soil can be more profitably used for agriculture or other purposes, the correspondingly increased value must be introduced into the account.

ii. The Length of the Rotation.

In the case of fully-stocked or normal woods, the highest expectation value is obtained for that rotation for which the expectation value of the soil culminates, provided that value is introduced into the account. If a larger value of soil is introduced, then the maximum expectation value of the growing stock is obtained for a lower rotation than that for which the expectation value of the soil culminates, and *vice versâ.*

In the case of insufficiently stocked or abnormal woods, the rotation for which the highest expectation value of the growing stock is obtained, can only be determined by experimental calculations based upon the data of each special case.

Example.—Taking the same data as before.

For a rotation of 70 years :—

$$ {}^{45}G_e = \frac{1683 + 67 \times 1{\cdot}025^{20} + 86 \times 1{\cdot}025^{10} - (250 + 120)\,(1{\cdot}025^{25} - 1)}{1{\cdot}025^{25}} = 856 \text{ shillings}. $$

For a rotation of 80 years:—

$$ {}^{45}G_e = 891 \text{ shillings.} $$

For a rotation of 90 years:—

$$ {}^{45}G_e = \frac{2789 + 67 \times 1{\cdot}025^{40} + 86 \times 1{\cdot}025^{30} + 91 \times 1{\cdot}025^{20} + 95 \times 1{\cdot}025^{10} - (250 + 120)\,(1{\cdot}025^{45} - 1)}{1{\cdot}025^{45}} $$

$$ = 878 \text{ shillings.} $$

The maximum value is obtained for a rotation of 80 years, that is to say, the rotation under which the expectation value of the soil culminates (see page 129).

Assuming now that the wood has been injured by wind at a previous period, so that no thinnings can be made before the year 90, and that the growing stock would realize—

At an age of 70 years	. . . =	1200 shillings.
" " 80 "	. . . . . =	2000 "
" " 90 "	. . . . =	2789 "

then the expectation value of the wood under these rotations would be as follows:—

For a rotation of 70 years:—

$$ {}^{45}G_e = \frac{1200 - 370\ (1{\cdot}025^{25} - 1)}{1{\cdot}025^{25}} = 477 \text{ shillings.} $$

For a rotation of 80 years:—

$$ {}^{45}G_e = \frac{2000 - 370\ (1{\cdot}025^{35} - 1)}{1{\cdot}025^{35}} = 628 \text{ shillings.} $$

For a rotation of 90 years:—

$$ {}^{45}G_e = \frac{2789 - 370\ (1{\cdot}025^{45} - 1)}{1{\cdot}025^{45}} = 673 \text{ shillings.} $$

For a rotation of 100 years:—

$$ {}^{45}G_e = \frac{3376 + 94 \times 1{\cdot}025^{10} - 370\ (1{\cdot}025^{55} - 1)}{1{\cdot}025^{55}} = 625 \text{ shillings.} $$

It will be observed that the maximum expectation value of the growing stock is, in this case, obtained under a rotation of 90 years.

iii. The Age of the Wood.

The expectation value of the growing stock rises (given a fixed rotation) with the age of the wood, but not in exact proportion. If the thinnings are made after regular intervals, say every 10 years, it generally happens that immediately before making thinnings the expectation value is slightly higher, than immediately after the thinning has been made. For instance, if a thinning is made in the year 60, the expectation value of the growing stock for the year 59 will probably be higher than for the year 61.

Immediately after the area has been stocked in the beginning of the rotation the expectation value of the growing stock is equal to the cost of formation, and at the end of the rotation it is equal to the value of the final yield.

iv. The Rate of Interest with which the Calculation is made.

A high rate of interest gives a low expectation value of the growing stock, and *vice versâ*, similar to what has been shown for the expectation value of the soil.

2. *Cost Value of the Growing Stock of a Whole Wood.*

The cost value of the growing stock of a wood now m years old, is equal to the present value of all costs of production less the present value of all returns which the wood has yielded before the year m.

a. Method of Calculation.

Costs of Production.—(1) The present value of the rent of the soil during m years comes to :—

$$S \times 1{\cdot}op^m - S = S\ (1{\cdot}op^m - 1).$$

(2) The present value of the annual expenses during m years (see Formula IV.) :—

$$= e \times 1{\cdot}op^{m-1} + e \times 1{\cdot}op^{m-2} + \ldots + e,$$

$$= \frac{e}{{\cdot}op}\ (1{\cdot}op^m - 1) = E\ (1{\cdot}op^m - 1).$$

(3) Present value of cost of formation :—

$$= c \times 1{\cdot}op^{m}.$$

Receipts.—These consist of all previous thinnings and items of other incomes ; they may be represented by T_a, T_b, . . . T_l. Their present value is—

$$= T_a \times 1{\cdot}op^{m-a} + T_b \times 1{\cdot}op^{m-b} + \ldots + T_l \times 1{\cdot}op^{m-l}.$$

Should any items of income have occurred in annually equal amounts, such as shooting, grazing, etc., they can be summed up according to the Formula IV.

The general formula for the cost value is therefore :—

$$^{m}G_c = (S+E)\ (1{\cdot}op^m - 1) + c \times 1{\cdot}op^m - (T_a \times 1{\cdot}op^{m-a} + T_b \times 1{\cdot}op^{m-b} + \ldots + T_l \times 1{\cdot}op^{m-l}).$$

Here $^{m}G_c$ still includes the thinning of the year m. If the calculation is made for what remains as final yield to go on with, then the thinning in the year $m = T_m$ must also be deducted.

Example.—Taking the same data as before, the cost value comes to :—

$$^{45}G_c = (250+120)\ (1{\cdot}025^{45} - 1) + 60 \times 1{\cdot}025^{45} - (4 \times 1{\cdot}025^{15} + 36 \times 1{\cdot}025^{5}).$$

$^{45}G_c$ = 891 shillings, as in the case of the expectation value.

b. Notes on the Cost Value.

The cost value of the growing stock depends on :—

(1) *The absolute amount of the receipts and expenses up to the year* m.

(2) *The Age of the Wood.*—The value rises with the age, but temporary exceptions occur, as immediately after a thinning the cost value may be smaller, than immediately before it.

At the commencement of the rotation the cost value is equal to the cost of formation.

At the end of the rotation the cost value is equal to the value of the final yield, provided that :—

(*a*) the calculation has been made with the expectation value of the soil;

(*b*) the receipts and expenses were of the normal amounts, and

(*c*) the wood is fully stocked, or normal, at the end of the rotation.

Proof.—Let $m = r$, then,

$$^{r}G_c = (S+E)\ (1{\cdot}op^r - 1) + c \times 1{\cdot}op^r - (T_a \times 1{\cdot}op^{r-a} + \ldots + T_q \times 1{\cdot}op^{r-q}).$$

By introducing the expectation value of the soil, the above equation becomes :—

$$^{r}G_c = \left(\frac{Y_r + T_a \times 1{\cdot}op^{r-a} + \ldots + T_q \times 1{\cdot}op^{r-q} - c \times 1{\cdot}op^r}{1{\cdot}op^r - 1} - E + E\right) \times (1{\cdot}op^r - 1) + c \times 1{\cdot}op^r - (T_a \times 1{\cdot}op^{r-a} + \ldots + T_q \times 1{\cdot}op^{r-q}),$$

which, after reduction, leads to :—

$$^{r}G_c = Y_r.$$

(3) *The Rate of Interest.*—If the calculation is made with the maximum expectation value of the soil, and the receipts and costs corresponding to it, then a higher rate of interest yields a lower cost value, and *vice versâ*.

If the above assumptions do not hold good, it depends on the value of the soil and the amounts of receipts and costs which are introduced into the calculation, whether a higher rate of interest gives a greater or smaller cost value of the growing stock.

3. *Sale Value, or Utilization Value, of the Growing Stock of a Whole Wood.*

Under the sale value of the growing stock of a wood is understood that price which it would realize in the open market.

A wood may be sold under one of two conditions :—

(*a*) The wood is allowed to grow on for a number of years. In this case the purchaser would have to rent the soil for a number of years, and he would have to meet certain other expenses. Hence the sale value should be equal to the expectation value.

(*b*) The wood is to be cut down at once. In this case the sale value would be the price which the cut material realizes in the open market. It is ascertained by determining the volume of the growing stock and multiplying it by the net mean rate per unit of measurement.

The sale value of very young woods under condition *b* is generally negative, until the receipts obtained by the sale of the produce cover the cost of harvesting; from that period it becomes positive, rising at first slowly, then more rapidly, reaching its maximum value far beyond the period at which the mean annual increment culminates, in fact not until the annual increase in the value per unit of measurement is no longer sufficient to cover the falling off caused by thinning or decay. This period occurs, generally speaking, earlier in the case of light-demanding species, than in the case of shade-bearing species which maintain a full stocking for a longer space of time.

4. *Relation existing between the Expectation and Cost Values of the Growing Stock of a Normal Wood.*

Looking at the formulæ for the expectation and cost values, it will be observed that the soil rental and annual expenses appear in the negative form in the one, and in the positive form in the other; again, the thinnings appear positive in the first and negative in the second. It follows that any change in these items affects the two values in opposite directions; what raises the one value, reduces the other, and *vice versâ*. Nevertheless, the one value can become equal to the other. This is the case, other items remaining the same in both instances, if the calculation is made in either case with the expectation value of the soil.

Proof.—Let ${}^{m}G_e = {}^{m}G_c$, immediately after the thinning in the year m has been made, then,

$$\frac{Y_r + T_n \times 1{\cdot}op^{r-n} + \ldots + T_q \times 1{\cdot}op^{r-q} - (S+E)\ (1{\cdot}op^{r-m} - 1)}{1{\cdot}op^{r-m}}$$
$$= (S+E)\ (1{\cdot}op^{m} - 1) + c \times 1{\cdot}op^{m} - (T_a \times 1{\cdot}op^{m-a} + \ldots + T_l \times 1{\cdot}op^{m-l} + T_m).$$

After making the necessary reduction, it will be seen that this equation can only hold good, if—

$$S = \frac{Y_r + T_a \times 1{\cdot}op^{r-a} + \ldots + T_q \times 1{\cdot}op^{r-q} - c \times 1{\cdot}op^{r}}{1{\cdot}op^{r} - 1} - E,$$

in other words, if the expectation value of the soil is introduced for S.

It must, however, not be overlooked that the above holds good only in the case of normally stocked woods. If a wood has been too thinly stocked from early youth, so that both thinnings and final yield are below the normal amounts, the cost value will be found to be greater than the expectation value.

5. *Relation existing between the Expectation and Cost Values of the Growing Stock of a Normal Wood on the one hand, and the Utilization Value on the other hand.*

The utilization value of the growing stock is equal to the expectation, or cost value at the end of the rotation, provided the maximum expectation value of the soil is introduced into the account, and the rotation is that for which the expectation value of the soil culminates. An equality can also occur at a previous stage, according to the values introduced into the account.

Generally speaking, the utilization value of young woods is smaller than the expectation, or cost value. On approaching the end of the rotation the difference is small, and it is then the safest plan to value the woods according to their utilization value, as the calculation of the expectation and cost values is based upon more or less uncertain data.

SECTION II.—VALUE OF A PART OF A WOOD.

1. *Value of Single Trees.*

The average value of a single tree is obtained by dividing the value of the whole wood, whether it be the expectation, cost, or utilization value by the number of trees.

The special value of a certain tree can be ascertained by estimating its utilization value, or according to the formula of the expectation or cost value, by introducing the data rèferring to the special tree in question, an operation which is beset by considerable difficulties.

2. *Value per Unit of Volume.*

This is obtained by dividing the value of a wood (or tree) by the number of units of volume contained in the wood (or tree).

3. *Value of the Increment of One or More Years.*

The value of x years increment is equal to the value of the wood x years hence less the present value of the wood. The calculation can be made for x years forward or backward. In either case the expectation, or cost value may be calculated.

a. The Expectation Value.

(1) Calculated for the year $m + x$:—

$$\text{Value} = \frac{Y_r + T_n \times 1{\cdot}op^{r-n} + \ldots - (S+E)\,(1{\cdot}op^{r-(m+x)} - 1)}{1{\cdot}op^{r-(m+x)}}$$

$$- \frac{Y_r + T_n \times 1{\cdot}op^{r-n} + \ldots - (S+E)\,(1{\cdot}op^{r-m} - 1)}{1{\cdot}op^{r-m}}.$$

$$\text{Value} = \frac{(Y_r + T_n \times 1{\cdot}op^{r-n} + \ldots + S + E)\,(1{\cdot}op^{x} - 1)}{1{\cdot}op^{r-m}}.$$

(2) Calculated for the year m:—

$$\text{Value} = \frac{(Y_r + T_n \times 1{\cdot}op^{r-n} + \ldots + S + E)\,(1{\cdot}op^{x} - 1)}{1{\cdot}op^{r+x-m}}.$$

b. The Cost Value.

(1) Calculated for the year $m + x$:

$$\text{Value} = (S+E)\,(1{\cdot}op^{m+x}-1)+c\times 1{\cdot}op^{m+x}-(T_a\times 1{\cdot}op^{m+x-a}\\ +\ldots)-[(S+E)\,(1{\cdot}op^{m}-1)+c\times 1{\cdot}op^{m}\\ -(T_a\times 1{\cdot}op^{m-a}+\ldots)].$$

$$\text{Value} = 1{\cdot}op^{m}\left[S+E+c-\left(\frac{T_a}{1{\cdot}op^{a}}+\ldots\right)\right](1{\cdot}op^{x}-1).$$

(2) Calculated for the year m:—

$$\text{Value} = 1{\cdot}op^{m-x}\left[S+E+c-\left(\frac{T_a}{1{\cdot}op^{a}}+\ldots\right)\right](1{\cdot}op^{x}-1.)$$

The cost value becomes equal to the expectation value, if the expectation value of the soil is substituted for S, a matter which is easy to show. In either case it becomes, calculated for the year $m + x$:—

Value

$$= 1{\cdot}op^{m}\left(Y_r+T_n\times 1{\cdot}op^{r-n}+\ldots+\frac{T_a}{1{\cdot}op^{a}}+\ldots-c\right)\left(\frac{1{\cdot}op^{x}-1}{1{\cdot}op^{r}-1}\right).$$

For the year m this value has to be divided by $1{\cdot}op^{x}$.

SECTION III.—VALUE OF THE GROWING STOCK OF A NORMAL SERIES OF AGE GRADATIONS. (NORMAL GROWING STOCK.)

If a forest is so managed that it yields annually an equal return, it must contain a regular series of woods of equal yield capacity ranging in age from 1 year up to r years, with one year's difference between every two successive gradations. Whether these age gradations are found on separate areas, or are mixed with each other, makes no difference. In either case, every year the oldest age gradation will be cut over, giving a yield $= Y_r$, and every year there will be thinnings in the gradations which have reached the ages of $a, b \;\ldots\; q$ years; at the same time every year formation would cost c shillings, while supervision would cost $r \times e$ shillings. It is of interest to the forester to ascertain the value of the growing stock in such a forest, the same being known as "the growing stock of a normal series of age gradations."

1. *Time of Year for which the Calculation should be made.*

The annual net return of a normal series of age gradations (or a working section) forms the rental of the soil and normal growing stock of that working section. Like the interest yielded by any ordinary capital that rental is produced within the year, so that the growing stock at the end of the year represents the capital plus one year's rental. Hence, the capital alone is present immediately after the year's rental has been removed. At that moment the oldest age gradation is $(r-1)$ years old, the next $(r-2)$, etc., and the youngest (which has just been cleared), is 0 year old.

Where cuttings are made in winter the normal growing stock is present in spring, before the trees have commenced to lay on the new annual increment.

2. *Expectation Value of the Normal Growing Stock.*

For simplicity's sake, it shall, in the first place, be assumed, that only one intermediate return is obtained, in the year q; then the values of the various age gradations will be as follows:—

$$^{r-1}G_e = \frac{Y_r - (S+E)\,(1{\cdot}op^1 - 1)}{1{\cdot}op^1}$$

$$^{r-2}G_e = \frac{Y_r - (S+E)\,(1{\cdot}op^2 - 1)}{1{\cdot}op^2}$$

etc.

$$^{q}G_e = \frac{Y_r - (S+E)\,(1{\cdot}op^{r-q} - 1)}{1{\cdot}op^{r-q}}$$

$$^{q-1}G_e = \frac{Y_r + T_q \times 1{\cdot}op^{r-q} - (S+E)\,(1{\cdot}op^{r-(q-1)} - 1)}{1{\cdot}op^{r-(q-1)}}$$

etc.

$$^{o}G_e = \frac{Y_r + T_q \times 1{\cdot}op^{r-q} - (S+E)\,(1{\cdot}op - 1)}{1{\cdot}op^{r}}.$$

By adding up all these quantities the following sum is obtained:—

$$Y_r\left(\frac{1}{1{\cdot}op}+\frac{1}{1{\cdot}op^2}+\ldots+\frac{1}{1{\cdot}op^r}\right)-(S+E)\left(\frac{1{\cdot}op}{1{\cdot}op}+\frac{1{\cdot}op^2}{1{\cdot}op^2}+\ldots+\frac{1{\cdot}op^r}{1{\cdot}op^r}\right)$$

$$+(S+E)\left(\frac{1}{1{\cdot}op}+\frac{1}{1{\cdot}op^2}+\ldots+\frac{1}{1{\cdot}op^r}\right)$$

$$+T_q\times 1{\cdot}op^{r-q}\left(\frac{1}{1{\cdot}op^{r-(q-1)}}+\frac{1}{1{\cdot}op^{r-(q-2)}}+\ldots+\frac{1}{1{\cdot}op^{r-(q-q)}}\right)$$

$$=\frac{Y_r\,(1{\cdot}op^r-1)}{1{\cdot}op^r\times{\cdot}op}-r\,(S+E)+\frac{(S+E)\,(1{\cdot}op^r-1)}{1{\cdot}op^r\times{\cdot}op}$$

$$+\frac{T_q\times 1{\cdot}op^{r-q}(1{\cdot}op^q-1)}{1{\cdot}op^r\times{\cdot}op}.$$

Introducing now the other intermediate returns which occur in the age gradations aged a, b, ... their values will appear in the same manner as the return which occurred in the age gradation q, namely as :—

$$\frac{T_a\times 1{\cdot}op^{r-a}\,(1{\cdot}op^a-1)}{1{\cdot}op^r\times{\cdot}op};\quad \frac{T_b\times 1{\cdot}op^{r-b}(1{\cdot}op^b-1)}{1{\cdot}op^r\times{\cdot}op},\ \text{etc.}$$

Hence, the general formula for the expectation value of the normal growing stock runs thus :—

$$\left.\begin{array}{r}\text{Norm. Gr.}\\ \text{Stock } G_e\end{array}\right\}=\frac{\begin{array}{c}(Y_r+S+E)\,(1{\cdot}op^r-1)+T_a\times 1{\cdot}op^{r-a}\,(1{\cdot}op^a-1)\\ +\ldots+T_q\times 1{\cdot}op^{r-q}\,(1{\cdot}op^q-1)\end{array}}{1{\cdot}op^r\times{\cdot}op}$$

$$-r\,(S+E).$$

Assuming that in the above formula Y_r, T_a . . . T_q, S and E are given for the unit of area, say 1 acre, then the formula represents the value of the normal growing stock for r acres. Consequently the normal growing stock for the unit of area is :—

$$\frac{\begin{array}{c}(Y_r+S+E)\,(1{\cdot}op^r-1)+T_a\times 1{\cdot}op^{r-a}\,(1{\cdot}op^a-1)+\ldots\\ +T_q\times 1{\cdot}op^{r-q}\,(1{\cdot}op^q-1)\end{array}}{r\times 1{\cdot}op^r\times{\cdot}op}-(S+E).$$

By introducing the soil expectation value into the formulas given above, and substituting its value

$$S_e=\frac{Y_r+T_a\times 1{\cdot}op^{r-a}+\ldots+T_q\times 1{\cdot}op^{r-q}-c\times 1{\cdot}op^r}{1{\cdot}op^r-1}-E$$

in the first part of the formula, it reduces to:

$$\frac{Y_r+T_a+\ldots+T_q-c}{\cdot op}-r\,(S_e+E),$$

or, as $E=\frac{e}{\cdot op}$,

$$\frac{Y_r+T_a+\ldots+T_q-(c+r\times e)}{\cdot op}-r\times S_e.$$

In words: The expectation value of the growing stock of a normal working section is equal to the capitalized annual net income minus the value of the soil.

For the unit of area the formula is:

$$\frac{Y_r+T_a+\ldots+T_q-(c+r\times e)}{r\times\cdot op}-S_e.$$

3. *Cost Value of the Normal Growing Stock.*

Assuming the same conditions as before, and also that only the a year old gradation has given an intermediate yield, the values of the successive age gradations come to:—

$$^{o}G_c=(S+E)\,(1{\cdot}op^{o}-1)+c\times 1{\cdot}op^{o}$$

$$^{1}G_c=(S+E)\,(1{\cdot}op^{1}-1)+c\times 1{\cdot}op$$

etc.

$$^{a}G_c=(S+E)\,(1{\cdot}op^{a}-1)+c\times 1{\cdot}op^{a}-T_a$$

$$^{a+1}G_c=(S+E)\,(1{\cdot}op^{a+1}-1)+c\times 1{\cdot}op^{a+1}-T_a\times 1{\cdot}op$$

etc.

$$^{r-1}G_c=(S+E)\,(1{\cdot}op^{r-1}-1)+c\times 1{\cdot}op^{r-1}-T_a\times 1{\cdot}op^{r-a-1}.$$

The sum of these expressions comes to:—

$$(S+E)\,(1{\cdot}op^{o}+1{\cdot}op^{1}+\ldots+1{\cdot}op^{r-1})-r\,(S+E)+c\,(1{\cdot}op^{o}+1{\cdot}op^{1}+\ldots+1{\cdot}op^{r-1})-T_a\,(1+1{\cdot}op^{1}+\ldots+1{\cdot}op^{r-a-1})$$

$$=\frac{(S+E)\,(1{\cdot}op^{r}-1)}{\cdot op}-r\,(S+E)+\frac{c\,(1{\cdot}op^{r}-1)}{\cdot op}-\frac{T_a\,(1{\cdot}op^{r-a}-1)}{\cdot op}.$$

By introducing now the further intermediate returns

$$\frac{T_b\,(1{\cdot}op^{r-b}-1)}{\cdot op}\ldots\frac{T_q\,(1{\cdot}op^{r-q}-1)}{\cdot op},$$

the general formula for the cost value becomes :—

$$\left.\begin{array}{r}\text{Norm. Gr.}\\ \text{Stock } G_c\end{array}\right\} = \frac{(S+E+c)(1{\cdot}op^r-1)-[T_a(1{\cdot}op^{r-a}-1)+\ldots+T_q(1{\cdot}op^{r-q}-1)]}{{\cdot}op} - r(S+E).$$

For the unit of area :—

$$\left.\begin{array}{r}\text{Norm. Gr.}\\ \text{Stock } G_c\end{array}\right\} = \frac{(S+E+c)(1{\cdot}op^r-1)-[T_a(1{\cdot}op^{r-a}-1)+\ldots+T_q(1{\cdot}op^{r-q}-1)]}{r\times{\cdot}op} -(S+E).$$

If again the calculation is made with the soil expectation value, as before—

$$\left.\begin{array}{r}\text{Norm. Gr. Stock of}\\ \text{Work. Section } G_c\end{array}\right\} = \frac{Y_r+T_a+\ldots+T_q-(c+r\times e)}{{\cdot}op} - r\times S_e,$$

$$\left.\begin{array}{r}\text{Norm. Gr. Stock}\\ \text{of Unit of Area}\end{array}\right\} = \frac{Y_r+T_a+\ldots+T_q-(c+r\times e)}{r\times{\cdot}op} - S_e,$$

or the same as was obtained for the expectation value.

4. *Capitalized Rental of Normal Growing Stock.*

The value of the normal growing stock may also be obtained by capitalizing the annual net rental and deducting therefrom the value of the soil.

The annual net income is represented by the expression :—

$$Y_r+T_a+\ldots+T_q-(c+r\times e);$$

hence the value of the growing stock is equal to :—

$$\frac{Y_r+T_a+\ldots+T_q-(c+r\times e)}{{\cdot}op} - r\times S_e,$$

or for the unit of area

$$\frac{Y_r+T_a+\ldots+T_q-(c+r\times e)}{r\times{\cdot}op} - S_e;$$

that is to say, the same as the expectation and cost values calculated with the soil expectation value.

CHAPTER IV.

VALUATION OF WHOLE WOODS OR FORESTS.

THE value of a whole wood, or a forest, is equal to the value of the soil plus the value of the growing stock; hence it can be ascertained by adding together these two values. At the same time, that value can also be determined direct out of the returns and expenses, or from sales of forests, or it can be calculated out of the rental which the forest yields. The present chapter shows how the forester proceeds in each of these cases:—

1. *Expectation Value of a Forest.*

As this is based upon future returns and costs, which differ under different methods of treatment and species, two cases must be distinguished:—

a. Calculation under the supposition that the same Species and System of Management are retained, after the present Crop has been Harvested.

(1) The value of the forest is equal to the value of the soil and the expectation value of the growing stock. The greatest expectation value of the forest is obtained for that rotation, under which the expectation value of the soil culminates.

Let the age of the existing growing stock $= m$, then the expectation value of the forest is:—

$$^{m}F_e = \frac{Y_r + T_n \times 1{\cdot}op^{r-n} + \ldots + T_q \times 1{\cdot}op^{r-q} - ({}^{r}S_e + E)\ (1{\cdot}op^{r-m} - 1)}{1{\cdot}op^{r-m}} + {}^{r}S_e$$

$$= \frac{Y_r + T_n \times 1{\cdot}op^{r-n} + \ldots + T_q \times 1{\cdot}op^{r-q} - E\ (1{\cdot}op^{r-m} - 1) + {}^{r}S_e}{1{\cdot}op^{r-m}}.$$

By introducing the value of ${}^rS_e$—

$$ {}^rS_e = \frac{Y_r + T_a \times 1{\cdot}op^{r-a} + \ldots + T_q \times 1{\cdot}op^{r-q} - c \times 1{\cdot}op^r}{1{\cdot}op^r - 1} - E, $$

the above formula becomes

$$ F_e = \frac{1{\cdot}op^m \left(Y_r + T_n \times 1{\cdot}op^{r-n} + \ldots + T_q \times 1{\cdot}op^{r-q} + \frac{T_a}{1{\cdot}op^a} + \ldots + \frac{T_m}{1{\cdot}op^m} - \right.}{1{\cdot}op^r - 1} $$

$$ - E. $$

If m is smaller than a, that is to say, if the age of the wood is less than the year when the first thinning occurs, then the above two formulæ become respectively:

$$ {}^mF_e = \frac{Y_r + T_a \times 1{\cdot}op^{r-a} + \ldots + T_q \times 1{\cdot}op^{r-q} - E\,(1{\cdot}op^{r-m} - 1) + {}^rS_e}{1{\cdot}op^{r-m}} $$

and

$$ {}^mF_e = \frac{1{\cdot}op^m\ (Y_r + T_a \times 1{\cdot}op^{r-a} + \ldots + T_q \times 1{\cdot}op^{r-q} - c)}{1{\cdot}op^r - 1} - E. $$

If $m = 0$, and cultivation has not yet taken place—

$$ {}^oF_e = \frac{\begin{array}{c} Y_r + T_a \times 1{\cdot}op^{r-a} + \ldots + T_q \times 1{\cdot}op^{r-q} - c \times 1{\cdot}op^r \\ - E\ (1{\cdot}op^{r-m} - 1) + {}^rS_e \end{array}}{1{\cdot}op^r} $$

$$ = \frac{{}^rS_e\,(1{\cdot}op^r - 1) + {}^rS_e}{1{\cdot}op^r} = {}^rS_e = $$ to the soil expectation value.

(2) The forest expectation value can also be calculated direct out of the expected receipts and expenses, in which case—

$$ {}^mF_e = \frac{Y_r}{1{\cdot}op^{r-m}} + \frac{Y_r}{1{\cdot}op^{2r-m}} + \ldots + \frac{T_n}{1{\cdot}op^{n-m}} + \frac{T_n}{1{\cdot}op^{r+n-m}} $$

$$ + \frac{T_n}{1{\cdot}op^{2r+n-m}} + \ldots + \frac{T_a}{1{\cdot}op^{r-(m-a)}} + \frac{T_a}{1{\cdot}op^{2r-(m-a)}} $$

$$ + \ldots - \left(\frac{c}{1{\cdot}op^{r-m}} + \frac{c}{1{\cdot}op^{2r-m}} + \ldots\right) - E; $$

$$^mF_e = \frac{Y_r \times 1{\cdot}op^m}{1{\cdot}op^r - 1} + \frac{T_n \times 1{\cdot}op^{r+m-n}}{1{\cdot}op^r - 1} + \ldots + \frac{T_a \times 1{\cdot}op^{m-a}}{1{\cdot}op^r - 1}$$
$$+ \ldots - \frac{c \times 1{\cdot}op^m}{1{\cdot}op^r - 1} - E\,;$$

$$^mF_e = \frac{1{\cdot}op^m \left(Y_r + T_n \times 1{\cdot}op^{r-n} + \ldots + \frac{T_a}{1{\cdot}op^a} + \ldots - c \right)}{1{\cdot}op^r - 1} - E,$$

as before.

Example.—Determine the expectation value of a forest now 45 years old, if that forest yields the returns given in the money yield table for the Scotch pine at page 122; if rotation $=80$ years; cost of formation $=60$ shillings; per cent. $=2\frac{1}{2}$; and annual costs per acre $=3$ shillings.

$$^{45}F_e = \frac{1{\cdot}025^{45}\left(2225 + 67 \times 1{\cdot}025^{30} + 86 \times 1{\cdot}025^{20} + 91 \times 1{\cdot}025^{10} + \frac{4}{1{\cdot}025^{30}} + \frac{36}{1{\cdot}025^{40}} - 60\right)}{1{\cdot}025^{80} - 1} - 120.$$

$$^{45}F_e = 1141 \text{ shillings} = £57\ 1\ 0.$$

It was found before:—

At page 127 .	$^{80}S_e$	= 250 shillings.		
,, 136 .	$^{45}G_e$	= 891	,,	
	Total .	. 1141	,,	= £57 1 0

as above.

In the case of the present growing stock being abnormal, the corresponding values must be introduced into the account (see page 137).

b. Calculation under the supposition that, after the cutting over of the present crop, another species or another method of using the soil is introduced.

In this case, the value S' of the soil corresponding to the new conditions must be introduced into the account; then the rotation r' must be determined, under which the

expectation value of the present growing stock reaches its maximum.

The value of the forest is then represented by the formula—

$$ {}^{m}F_e' = \frac{Y_{r'} + T_n \times 1{\cdot}op^{r'-n} + \ldots - E\ (1{\cdot}op^{r'-m} - 1) + S'}{1{\cdot}op^{r'-m}}. $$

If the present growing stock is abnormal, a further modification is required, by substituting the abnormal for the normal returns.

2. *Cost Value of a Forest.*

a. The Cost Value of a Forest is equal to the Cost Value of the Soil, plus that of the Growing Stock.

(1) For any soil value:—

$$ \begin{aligned} {}^{m}F_c &= S + (S+E)\ (1{\cdot}op^m - 1) + c \times 1{\cdot}op^m - [T_a \times 1{\cdot}op^{m-a} + \ldots] \\ &= (S+E+c) 1{\cdot}op^m - [T_a \times 1{\cdot}op^{m-a} + \ldots + E]. \end{aligned} $$

(2) By introducing the expectation value of the soil, the above becomes, for normal woods:—

$$ \begin{aligned} {}^{m}F_c &= \left(\frac{Y_r + T_a \times 1{\cdot}op^{r-a} + \ldots - c \times 1{\cdot}op^r}{1{\cdot}op^r - 1} - E + E + c \right) 1{\cdot}op^m \\ &\quad - [T_a \times 1{\cdot}op^{m-a} + \ldots + E] \\ &= \frac{1{\cdot}op^m \left(Y_r + T_n \times 1{\cdot}op^{r-n} + \ldots + \dfrac{T_a}{1{\cdot}op^a} + \ldots - c \right)}{1{\cdot}op^r - 1} - E. \end{aligned} $$

This, it will be observed, is equal to the expectation value of the forest.

b. The Cost Value can be calculated direct out of the Expenses incurred.

The method is similar to that followed in calculating the cost value of the growing stock, but the value of the soil is added instead of the rental only; hence—

$$ \begin{aligned} {}^{m}F_c &= S \times 1{\cdot}op^m + E\ (1{\cdot}op^m - 1) + c \times 1{\cdot}op^m - (T_a \times 1{\cdot}op^{m-a} + \ldots) \\ &= (S+E+c)\ 1{\cdot}op^m - [T_a \times 1{\cdot}op^{m-a} + \ldots + E], \end{aligned} $$

as before.

3. *Sale Value of a Forest.*

By the sale value of a forest is understood the value estimated according to prices realized for forests of a similar description. As it is difficult to estimate existing differences, and hence to estimate the sale value, the latter is of subordinate importance.

4. *Rental Value of a Forest.*

Under the rental value of a forest is understood the capitalized rental which it is capable of yielding. If the annually equal rental is $=R$, the rental value would be :—

$$\text{Rental value} = \frac{R}{\cdot op}.$$

This method is only applicable in the case of a forest which can be so managed that it yields an annually (or periodic) equal rental. The rental is represented by—

$$Y_r + T_a + T_b + \ldots + T_q - (c + r \times e),$$

and the rental value of the forest is—

$$= \frac{Y_r + T_a + T_b + \ldots + T_q - (c + r \times e)}{\cdot op};$$

Or, if $\frac{e}{\cdot op}$ is denoted by $=E$,

$$\text{Rental value of a whole series} = \frac{Y_r + T_a + \ldots + T_q - c}{\cdot op} - r \times E.$$

The mean value of one age gradation would amount to—

$$\text{Rental value per unit of area} = \frac{Y_r + T_a + \ldots + T_q - c}{r \times \cdot op} - E.$$

CHAPTER V.

DETERMINATION OF THE RENTAL OF FORESTS.

In order to convert any item of income, whether it occurs once or after stated intervals, into an annual rental, it is necessary to ascertain the capital value of the income and then to multiply it by $\cdot op$. For instance, the rental which corresponds to the thinning in the year a, and its recurrence every r years, is equal to :—

$$\frac{T_a \times 1\cdot op^{r-a}}{1\cdot op^r - 1} \times \cdot op.$$

The annual payment corresponding to the cost of cultivation, is expressed by—

$$\frac{c \times 1\cdot op^r}{1\cdot op^r - 1} \times \cdot op.$$

1. *Rental of the Soil.*

Under the rental of the soil is understood the annual net return of the soil. It is represented by the difference between the rentals of incomes and the annual payment of expenses of a wood, hence :—

Soil rental R_s

$$= \frac{Y}{1\cdot op^r - 1} \times \cdot op + \frac{T_a \times 1\cdot op^{r-a}}{1\cdot op^r - 1} \times \cdot op + \ldots + \frac{T_q \times 1\cdot op^{r-q}}{1\cdot op^r - 1} \times \cdot op.$$

$$- \left[\frac{c \times 1\cdot op^r}{1\cdot op^r - 1} \times \cdot op + e\right]$$

$$= \left[\frac{Y_r + T_a \times 1\cdot op^{r-a} + \ldots + T_q \times 1\cdot op^{r-q} - c \times 1\cdot op^r}{1\cdot op^r - 1} - \frac{e}{\cdot op}\right] \times \cdot op.$$

This rental, it will be observed, is the rental of the soil expectation value $= {}^rS_e \times \cdot op$.

2. *Rental of the Growing Stock.*

This is calculated from the value of the growing stock in the same way as in the case of the soil rental.

3. *Rental of the Forest.*

The rental of the forest is equal to the net return yielded by the forest (soil plus growing stock). In the case of the annual working, the forest rental is equal to :—

$$R_f = Y_r + T_a + \ldots + T_q - (c + r \times e).$$

If Y_r, T_a . . . . c and e refer to the unit of area, the rental for the unit of area is :—

$$\text{Unit of area } R_f = \frac{Y_r + T_a + \ldots + T_q - c}{r} - e.$$

CHAPTER VI.

THE FINANCIAL RESULTS OF FORESTRY.

The subjects which will be treated in this chapter belong to that part of scientific forestry which is called on the continent "Forest Statics"; that is to say, the science which weighs and considers the comparative merits of the different methods of treatment to which forests may be subjected. The financial results, or the rent-yielding power of an undertaking, are expressed by the proportion which exists between the yield and the capital which produces it. Hence, when several methods of treatment lead to the realization of the otherwise desired object, it should always be ascertained which of them does so in the most profitable manner; in other words, which of them gives the highest rate of interest on the invested capital.

In the present instance only the most necessary matters will be given, namely the methods of calculating the financial results of forestry, and their application to a few of the more important questions.

Section I.—The Methods of Calculating the Financial Results of Forestry.

The financial results of forestry can be determined in one of two ways:—

(1) by ascertaining the "profit," that is to say, the surplus of receipts over costs of production;

(2) by ascertaining the rate of interest yielded by the capital invested in forestry, here called the "forest per cent."

Each of these two methods must be explained in detail.

1. *Determination of the Profit of Forestry.*

a. Calculation for the Intermittent Working.

In the case of a single wood of approximately uniform age, the returns and costs do not occur at the same time, but at intervals of various duration; hence they must be calculated for one and the same time. In the first place, that time shall be the commencement of the rotation when the area is about to be planted or sown for the production of a forest crop.

i. Calculation for a Blank Area.

Let as before:—

Y_r be the final yield occurring in the year r and again in $2 \times r$, $3 \times r$, and so on for ever;

T_a, T_b . . . T_q, the thinnings occurring in the years a . . . q, and again in $a + r$ . . . $q + r$ years, again in $a + 2 \times r$ . . . $q + 2 \times r$ years and so on;

S_c the cost value of soil;

E the capitalized annual expenses $= \frac{e}{\cdot op}$;

c the cost of formation expended now, and again every r years;

C the capitalized cost of formation $= \frac{c \times 1 \cdot op^r}{1 \cdot op^r - 1}$;

p the per cent. at which money can be made available for investment in forestry, and at which money taken out of the forest can be invested with equal security.

Then the present value of all returns is represented by

$$= \frac{Y_r + T \times 1 \cdot op^{r-a} + \ldots + T_q \times 1 \cdot op^{r-q}}{1 \cdot op^r - 1}$$

while the present value of all costs comes to:—

$$S_c + E + C.$$

Hence the profit of forestry is expressed by the formula:—

$$\text{Profit} = P = \frac{Y_r + T_a \times 1 \cdot op^{r-a} + \ldots + T_q \times 1 \cdot op^{r-q}}{1 \cdot op^r - 1} - (S_c + E + C).$$

This formula can also be written as follows :—

$$P=\left(\frac{Y_r+T_a\times 1{\cdot}op^{r-a}+\ldots+T_q\times 1{\cdot}op^{r-q}-c\times 1{\cdot}op^r}{1{\cdot}op^r-1}-E\right)-S_c.$$

It will be observed that the part in brackets represents the expectation value of the soil; hence the above formula reduces to :—

$$P=S_e-S_c.$$

In words: the profit in the case of a blank area is equal to the difference between the expectation and cost values of the soil. From this fact the following conclusions can be drawn :—

(1) A profit is realized if the soil has been acquired at a lower rate than that indicated by the expectation value of forest soil.

(2) A profit is also realized if, although $S_e = S_c$ at the outset, the soil expectation value is afterwards increased, either by higher returns or by smaller costs, or by both; in other words, by improved and more economic management.

(3) The greatest profit is obtained by the adoption of the rotation, species and method of treatment which give the highest expectation value of the soil.

(4) If the cost value of the soil is equal to the expectation value, the profit is nil and the capital invested in the forest yields exactly the per cent. p. If the cost value is greater than the expectation value, the forest industry involves a financial loss; in that case it is more profitable to take the capital out of the forest and invest it otherwise, as long as in this way p per cent. can be obtained with equal security.

ii. Calculation of the Profit for a Wood now m years old.

It is assumed that the returns and costs are of the normal amount.

The receipts consist of:—

(*a*) All items of income realized between the formation of the wood and the year m, with accumulated interest to the year m:—

$$T_a \times 1{\cdot}op^{m-a} + \ldots T_m.$$

(*b*) All items of income to be realized between the year m and the end of the present rotation r, discounted to the year m:—

$$\frac{T_n \times 1{\cdot}op^{r-n} + \ldots + T_q \times 1{\cdot}op^{r-q} + Y_r}{1{\cdot}op^{r-m}}.$$

(*c*) All items of income to be realized during subsequent rotations, amounting to:—

$$\frac{T_a \times 1{\cdot}op^{r-a} + \ldots + T_q \times 1{\cdot}op^{r-q} + Y_r}{(1{\cdot}op^r - 1) \times 1{\cdot}op^{r-m}}.$$

The costs are:—

(*a*) Past costs with compound interest to the year m:—

$$S_c \times 1{\cdot}op^m + E\ (1{\cdot}op^m - 1) + c \times 1{\cdot}op^m.$$

(*b*) Costs to be incurred from the year m to the end of the rotation:—

$$\frac{E\ (1{\cdot}op^{r-m} - 1)}{1{\cdot}op^{r-m}}.$$

(*c*) Costs to be incurred during future rotations:—

$$\frac{E}{1{\cdot}op^{r-m}} + \frac{c \times 1{\cdot}op^r}{(1{\cdot}op^r - 1) \times 1{\cdot}op^{r-m}}.$$

By deducting all costs from the receipts, the profit is obtained and represented by the following formula:—

Profit $P =$

$$\frac{Y_r + T_a \times 1{\cdot}op^{r-n} + \ldots + T_q \times 1{\cdot}op^{r-q} - E\ (1{\cdot}op^{r-m} - 1)}{1{\cdot}op^{r-m}} \qquad \text{(I.)}$$

$$+ \frac{1}{1{\cdot}op^{r-m}} \left(\frac{Y_r + T_a \times 1{\cdot}op^{r-a} + \ldots - c \times 1{\cdot}op^r}{1{\cdot}op^r - 1} - E \right) \quad . \qquad \text{(II.)}$$

$$-[(S_c+E)(1{\cdot}op^m-1)+c\times 1{\cdot}op^m-T_a\times 1{\cdot}op^{m-a}-\ldots-T_m]$$
$$-S_c \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(III.)}$$

In this formula expression (II.) represents the expectation value of the soil discounted for $r-m$ years, and the bracketed part of expression (III.) represents the cost value of the m years old growing stock; hence the above formula can be written as follows:—

Profit $P=$
$$\frac{Y_r+T_a\times 1{\cdot}op^{r-a}+\ldots+T_q\times 1{\cdot}op^{r-q}+{}^rS_e-E\,(1{\cdot}op^{r-m}-1)}{1{\cdot}op^{r-m}}$$
$$-(S_c+{}^mG_c).$$

Here again the positive part represents the expectation value of the forest (see page 149), and the negative part the cost value of the forest; hence:—

$$\text{Profit } P={}^mF_e-{}^mF_c.$$

It will be seen that this formula agrees with that given for a blank because for the year o the expectation value of the forest is equal to the expectation value of the soil, and the same holds good for the cost values.

The conclusions drawn on page 158, headings (1) to (4), with regard to the formula $P=S_e-S_c$ also hold good with regard to the formula $P=F_e-F_c$.

b. *Calculation for the Annual Working.*

In this case the returns and costs occur regularly every year. If they are of annually equal amounts, the returns are $=Y_r+T_a+\ldots\ldots+T_q$.

The costs consist every year of—

(1) Interest on the value of the soil . . $=r\times S_c\times{\cdot}op$;

(2) " " " " normal growing stock . . . . . $=r\times{}^nG_c\times{\cdot}op$;

(3) The annually recurring cost of administration, protection, taxes, etc. . . $=r\times E\times{\cdot}op$;

(4) The cost of formation . . . . $=c$;

hence total annual costs =

$$(r \times S_c + r \times {}^nG_c + r \times E) \times {\cdot}op + c\,;$$

and the annual profit :—

$$\text{Annual } P = Y_r + T_a + \ldots + T_q - [(r \times S_c + r \times {}^nG_c + r \times E) \times {\cdot}op + c].$$

This formula can be written thus :—

$$\text{Annual } P = Y_r + T_a + \ldots + T_q - c - r \times e - [r \times S_c + r \times {}^nG_c] \times {\cdot}op\,;$$

or,

$$\frac{\text{Annual } P}{{\cdot}op} = \frac{Y_r + T_a + \ldots + T_q - c - r \times e}{{\cdot}op} - (r \times S_c + r \times {}^nG_c).$$

Now :—

$\dfrac{\text{Annual } P}{{\cdot}op}$ = capitalized value of annual profit = total profit;

$\dfrac{Y_r + T_a + \ldots + T_q - c - r \times e}{{\cdot}op}$ = to the expectation value of the forest under the annual working $= F_e$;

$$r \times S_c + r \times {}^nG_c = \text{cost value of forest} = F_c\,;$$

hence total, or capital, value of profit of the whole forest :—

$$\text{Total } P = F_e - F_c.$$

In words, the profit is equal to the difference between the expectation and cost values of a forest. For the rest, what has been said as regards the intermittent working holds also good in respect of the annual working, and *vice versâ*, because a series of age gradations may be considered as so many separate woods of various ages; the profit calculated for the series as a whole must be equal to that obtained by adding together the profits derived from the several age gradations, each calculated for itself.

2. *Determination of the Rate of Interest yielded by the Capital invested in Forestry, called the Forest Per cent.*

a. Calculation for the Intermittent Working.

Under the intermittent working the rate of interest changes from year to year, because the capital, as well as the increment, change with the advancing age of the wood; hence it is necessary to distinguish between the current annual and the mean (or average) annual rate of interest.

i. Current Annual Rate of Interest.

The current annual rate of interest is equal to the net annual value increment of a wood divided by the cost value of the wood at the commencement of the year in question. The current annual per cent. is equal to that quotient multiplied by 100.

Let:—

Y_m be the utilization value of a wood at the end of the year m,
Y_{m+1} " " " " " " $m+1$,

then—

$$Y_{m+1} - Y_m$$

represents the increase in value which has been laid on during the year $m+1$. Deducting from this e, the annual costs, the net increase is equal to—

$$Y_{m+1} - Y_m - e.$$

The cost value of the wood at the end of the year m (or beginning of the year $m+1$) is represented by:—

(*a*) the value of the soil with compound interest to the year $m =$

$$S_e \times 1{\cdot}op^m.$$

(*b*) the cost of formation with compound interest to the year $m =$

$$c \times 1{\cdot}op^m.$$

(*c*) The value of the annual costs during m years with

compound interest calculated for the end of the year $m=$

$$\frac{e\ (1{\cdot}op^m - 1)}{{\cdot}op} = E\ (1{\cdot}op^m - 1).$$

(*d*) From these amounts must be deducted all returns realized between the formation of the wood and the end of the year m—

$$T_a \times 1{\cdot}op^{m-a} + T_b \times 1{\cdot}op^{m-b} + \ldots + T_l \times 1{\cdot}op^{m-l} + T_m.$$

The forest per cent. $= {}^{c}p_f$ with which the invested capital is working during the year $m+1$ is, therefore, expressed by the formula :—

$${}^{curr.}p_f = \frac{(Y_{m+1} - Y_m - e)\ 100}{(S_c + E + c) \times 1{\cdot}op^m - E - (T_a \times 1{\cdot}op^{m-a} + T_b \times 1{\cdot}op^{m-b} + \ldots)}.$$

The denominator in the above formula can also be written thus :—

$$(S_c + E)\ (1{\cdot}op^m - 1) + c \times 1{\cdot}op^m - (T_a \times 1{\cdot}op^{m-a} + \ldots) + S_c.$$

On reference to page 139, it will be seen that this expression represents the cost value of the growing stock immediately after the thinning in the year m has been made plus the cost value of the soil; hence the formula for the forest per cent. reduces to—

$${}^{curr.}p_f = \frac{(Y_{m+1} - Y_m - e) \times 100^*}{S_c + {}^{m}G_c} = \frac{(Y_{m+1} - Y_m - e) \times 100}{{}^{m}F_c}.$$

By substituting the utilization value of the growing stock for its cost value, the formula becomes—

$${}^{curr.}\mathrm{p_f} = \frac{(\mathrm{Y_{m+1}} - \mathrm{Y_m} - \mathrm{e}) \times 100}{\mathrm{S_c} + \mathrm{Y_m}}.$$

* This formula differs from that usually given in continental works, which is as follows:

$${}^{curr.}p_f = \frac{(Y_{m+1} - Y_m)\ 100}{S_c + {}^{m}G_c + E}.$$

It is easy to show that this formula is only correct for the year in which ${}^{curr.}p$ = general per cent. p.

Similar to the progress of the current annual increment of a wood, the current annual forest per cent. is at first small, then rises, reaches a maximum, and then falls again. The maximum is greater than that attained by the mean annual increment, and also it occurs earlier (see fig. 41).

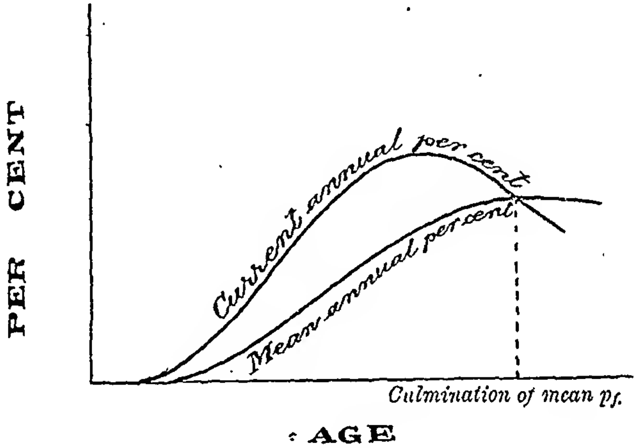


Fig. 41.

Of special interest to the forester is the relation which exists between the current annual forest per cent. $= {}^cp_f$ and the

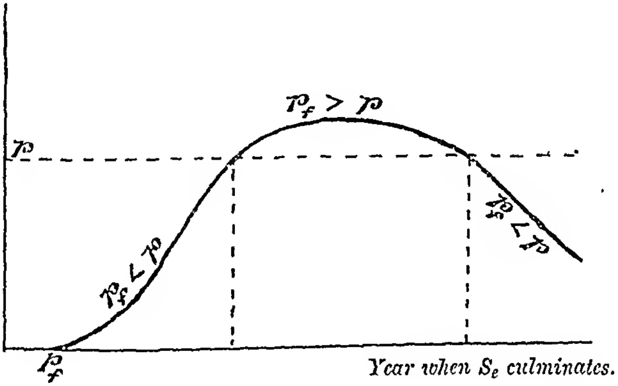


Fig. 42.

general per cent. p. At first ${}^cp_f$ is $< p$, then comes a time when the two are equal; after that ${}^cp_f > p$, and after a further lapse of time ${}^cp_f$ becomes $= p$ a second time (fig. 42), but beyond that period the forest per cent. is permanently smaller

than p, unless exceptional conditions produce once more a sudden rise in the current annual forest per cent.

The second occasion occurs in the year when the expectation value of the soil reaches its maximum. Hence the formula for the current annual forest per cent. can be used to gauge the financial ripeness of a wood, as will be shown further on.

ii. Mean Annual Rate of Interest.

The mean (or average) rate of interest is ascertained by converting all net returns into an equal annual rental and dividing it by the producing capital. That quotient multiplied by 100 gives the mean annual forest per cent.

The best time for making the calculation is the commencement of the rotation. At that time the annual net rental is represented by the expression :—

$$\left(\frac{Y_r + T_a \times 1{\cdot}op^{r-a} + \ldots + T_q \times 1{\cdot}op^{r-q} - c \times 1{\cdot}op^r}{1{\cdot}op^r - 1} - \frac{c}{{\cdot}op}\right) \times {\cdot}op.$$

The producing capital at the commencement of the rotation is equal to the cost value of the soil $= S_c$. Hence the mean annual forest per cent. under the intermittent working is :—

$$^{mean}p_f = \frac{\left(\frac{Y_r + T_a \times 1{\cdot}op^{r-a} + \ldots + T_q \times 1{\cdot}op^{r-q} - c \times 1{\cdot}op^r}{1{\cdot}op^r - 1} - E\right) \times {\cdot}op}{S_c} \times 100$$

$$^{mean}p_f = \frac{S_e}{S_c} \times p.$$

$$\text{If } S_e \gtrless S_c, \text{ then } {}^{mean}p_f \gtrless p.$$

If the expectation value of the soil is equal to the cost value, then the mean annual forest per cent. is equal to the general per cent. p, which proves the correctness of the above formula.

The highest mean annual forest per cent. is obtained under that rotation for which the expectation value of the soil culminates; it is then equal to the current annual forest per cent. (see fig. 41).

b. Calculation for the Annual Working.

Under the annual working, with equal annual increment yield and costs, the current annual forest per cent. is equal to the mean annual forest per cent. In this case the annual net return amounts to:—

$$Y_r + T_a + \ldots + T_q - c - r \times e,$$

and the producing capital to—

$$r \times S_c + r \times {}^nG_c;$$

hence

$$^{mean}p_f = \frac{Y_r + T_a + \ldots + T_q - c - r \times e}{r \times S_c + r \times {}^nG_c} \times 100.$$

$$^{mean}p_f = \frac{Y_r + T_a + \ldots + T_q - c - r \times e}{F_c} \times 100.$$

By multiplying and dividing the enumerator in this formula by $\cdot op$, the following is obtained:—

$$^{mean}p_f = \frac{\left(\dfrac{Y_r + T_a + \ldots + T_q - c - r \times e}{\cdot op}\right) \times 100 \times \cdot op;}{F_c}$$

or, as the part in brackets is equal to the expectation value of the forest $= F_e$:—

$$^{mean}p_f = \frac{F_e}{F_c} \times p.$$

This formula is identical with that obtained above for the profit $= F_e - F_c$:

$$\text{If } F_e \gtrless F_c, \text{ then } {}^{mean}p_f \gtrless p.$$

$$\text{If } F_e = F_c, \text{ then } {}^{mean}p_f = p.$$

The highest mean annual forest per cent. under the annual working is obtained for that rotation under which the expectation value of the forest reaches its maximum.

Section II.—The Financial Test applied to the Method of Treatment.

If the profitableness of the method of treatment is to be tested, or if several methods are to be compared, it must be assumed that in each case those conditions exist, which render the method in itself as profitable as possible. In this case, it may be said that: The most advantageous method of treatment, from a financial point of view, is that which yields the highest profit, or the highest mean annual forest per cent., provided, in the latter case, the capital is the same under each method. If the capitals differ, the following cases must be distinguished:

(1) The method employing the greater capital is the more profitable, if it gives the higher forest per cent.

(2) The one with the smaller capital is the more profitable, if it yields an equal or a greater amount of interest. If it yields less interest and yet a higher forest per cent., it cannot be decided off-hand whether it is the more profitable or not, as the total profit depends on two factors, namely, the rate of interest and the amount of the invested capital; hence it is necessary to calculate the actual amounts of profit for each case and to compare them.

The above-mentioned tests may be applied to all questions connected with forest management. Of these the following are the most prominent:—

1. *Choice between Forestry and Agriculture as regards a Piece of Land.*

Of these two methods of using the land that is the more profitable which yields the highest net rental of the soil.

Rental of soil under forest =

$$\left[\frac{Y_r + T_a \times 1{\cdot}op^{r-a} + \ldots + T_q \times 1{\cdot}op^{r-q} - c \times 1{\cdot}op^r}{1{\cdot}op^r - 1} - E\right] \times {\cdot}op.$$

Rental of soil under agriculture=

Annual gross rental − annual costs = net letting value.

The decision can also be based upon the profit in each case, when the formula stands as follows :—

$$\left[\frac{Y_r + T_a \times 1{\cdot}op^{r-a} + \ldots + T_q \times 1{\cdot}op^{r-q}}{1{\cdot}op^r - 1} - (S + E + {}^rC)\right] \times {\cdot}op$$

$$\gtreqless Y' - (e' + S \times {\cdot}op).$$

where Y' represents the annual gross income from agriculture, e' the annual expenses under agriculture, and S the cost value of the soil.

2. *Choice of Species and Sylvicultural System.*

In both cases the choice is determined by the expectation value of the soil. It depends on :—

(*a*) the value of the returns and the time when they occur.

(*b*) the cost of formation, and the amount of the annual expenses if they differ for different species or different sylvicultural systems.

(*c*) the rate of interest.

3. *Choice of Method of Formation.*

The choice depends on the differences in the expectation value of the soil under the various methods of formation, such as planting, sowing, or natural regeneration.

4. *Choice of Method of Tending, especially in respect of the Time and Strength of Thinnings.*

This is best effected by calculating the expectation value of the growing stock for the different methods of thinning, and comparing them.

5. *Determination of the Financial Rotation.*

The financial rotation, or that which gives the best financial results, may be determined in various ways, as :—

(*a*) The rotation which yields the highest expectation value of the soil, or the highest soil rental.

(*b*) The rotation which gives the highest net return for the forest.

(*c*) The rotation which, taking a certain value of the soil, gives the largest profit or the highest mean annual forest per cent.

6. *Determination of the Financial Ripeness of a Wood.*

The financial ripeness of a wood, say *m* years old, is gauged by means of the current annual forest per cent. As long as the forest per cent. :—

$$^{curr.}p_f = \frac{(Y_{m+1} - Y_m - e) \times 100}{S_c + Y_m}$$

is greater than the general per cent. p, the wood is not ripe. If current $p_f = p$, then the wood is just ripe. If current $p_f < p$, then the wood is past ripeness and, if left standing, a financial loss is incurred.

The above short notes will suffice for the present. The application of these principles will be found in the next part of this volume.

PART III.

PRINCIPLES OF FOREST WORKING PLANS,

OR,

DESCRIPTION OF THE NORMAL FOREST.

PRINCIPLES OF FOREST WORKING PLANS.

INTRODUCTORY.

Forest working plans regulate, according to time and locality, the management of forests in such a manner, that the objects of the industry are as fully as possible realized. As the latter differ widely, it follows that working plans cannot be drawn up according to any special pattern. The working plan for a protection forest, or a park-like forest, is altogether different from that of a forest which is managed on economic principles. In this volume only forests of the latter class will be considered, that is to say, it will be explained how forests should be managed so as to produce the best financial results, or the greatest volume, or the most suitable class of produce.

The yield (or the return) of a forest consists of major or principal and minor produce. Under the former, timber, firewood and bark are understood. It is in the nature of things that forests should chiefly yield such articles; at the same time, articles of minor produce (such as turpentine, fodder, grazing, fruits, caoutchouc, etc.) are frequently of considerable importance, and demand modifications of that management, which would be indicated by considering only the realization of major produce.

Major produce is again divided into the final and intermediate yields. The latter comprise the thinnings which are made from time to time during the course of the life of a wood, while the former is the return yielded by the final cutting of the wood, to be followed by a new crop.

The major produce of forests, wood, is one of the indispen-

sable articles of life, but it is bulky, and not adapted for a long transport by land. To this must be added that long periods of time elapse between the planting and harvesting of trees. Both these matters make it desirable that the yield of forests should be continuous and brought into the market in annually equal or approximately equal quantities, necessitating a management based upon the principle of a sustained yield.

Generally speaking, a "sustained yield" is secured, if all areas which have been cleared are restocked within a reasonable time, and the young woods which spring up properly tended, so that the soil continues to produce crops of wood. At the same time a distinction must be made between:—

(1) The intermittent working, if the successive returns are separated by a varying number of years.

(2) The annual working, if final cuttings occur in each year. If the latter are approximately equal in quantity year by year, the method is called the "strict or equalized annual working."

The regulation of the *yield* of forests worked intermittently is very simple. It is *only* necessary to ascertain the most profitable rotation, taking into consideration the objects of management, and to make the thinnings whenever they become necessary.

Although the method of annual working, and especially of the equalized annual working, is not an absolute necessity, still it is in the majority of cases highly desirable, more especially where extensive areas are under treatment, or where a steady market has to be regularly satisfied. Moreover, it has considerable advantages, of which the following may be mentioned:

(1) It favours the development of a regular and steady market, with a sustained competition of purchasers.

(2) It affords equal employment year after year, and enables the administration to maintain a regular number of workmen, and to instruct them thoroughly in their work.

(3) It secures to the owner equal, or approximately equal, annual incomes, and facilitates budget arrangements.

On the other hand the method has disadvantages, such as :—

(*a*) It cannot as a rule be introduced without cutting certain woods at an age differing from that which is most profitable.

(*b*) Owing to the necessity of bringing annually the same quantity into the market, it interferes with the complete utilization of special demands for forest produce, or the omission of cuttings when the demand is slack.

These remarks show that the intermittent as well as the annual working possess peculiar advantages, and that the choice depends on circumstances. In the majority of cases the annual working will be found more suitable, without, however, strictly adhering to it when it would involve sacrifices out of proportion to the general advantages of the method.

Correctly speaking, in order to have equal annual returns, it would be necessary to regulate the intermediate cuttings or thinnings, as well as the final returns. Against such an arrangement the following reasons may be given :—

(1) Areas which yield equal final returns, do not always yield equal intermediate returns.

(2) Thinnings depend much more than final cuttings on the method of formation and tending.

(3) The yield of thinnings depends frequently on events which do not occur regularly, or which cannot be foreseen, so that it is almost impossible to estimate it correctly beforehand.

Hence, it is desirable to confine the regulation of the annual yield to the final cuttings, and to be satisfied with an approximate equalization of the intermediate returns, such as will naturally happen, if the final cuttings are systematically equalized; provided always that the thinnings are not made so heavy as to affect the subsequent final returns.

If a forest is to yield a return, either annually or periodically, it must be in a certain state. In order to determine what this state should be under a given set of conditions, it is useful to construct an ideal pattern in a simple form, which is uninfluenced by external interfering circumstances. Such an ideal state differs, of course, for every different method of treatment, in accordance with the objects at which the management aims. In all these cases a forest which corresponds in every way to the objects of management is called a *normal forest.* It enables the forester to study the laws which must govern the management, and it serves as an ideal to be aimed at, though it may never be altogether reached, and at any rate not permanently maintained.

The normal state of a forest, under a given set of conditions, depends chiefly on the presence in it of:—

(1) A normal increment.

(2) A normal distribution of the age classes.

(3) A normal growing stock.

By *normal increment* is understood that which is *possible,* given a certain locality, species and rotation. An abnormal increment may be caused by faulty formation, faulty treatment, injurious external influences, and also by a preponderance of certain age classes.

By a *normal distribution of age classes* is understood a series of age gradations, so arranged that at all times when cuttings are to be made, woods of the normal age are available in such a position that no obstacles to their cutting exist.

The *normal growing stock* is that which is present in a forest in which the age gradations are arranged normally, and show the normal increment. It can, however, also be present (in quantity) in an abnormal forest, if the deficiency of some woods is made good by a surplus in others.

For the strictly annual working and the clear cutting system a forest is, therefore, normal, if it consists of a series of fully stocked woods equal in number to the number of years in the rotation, so that each year a wood of the normal age

can be cut, and the returns are equal, at any rate in quantity if not in value.

From a financial point of view, the further condition must be added, that there should be no woods in the forest, the forest per cent. of which has sunk below the general per cent. p (see p. 169).

In accordance with these definitions, the following matters demand special attention :—

1. The increment.
2. The rotation, or the normal age at which woods should be cut over.
3. The normal age classes.
4. The normal growing stock.
5. The normal yield.
6. The relations which exist between increment, growing stock, and yield.
7. The real forest compared with the normal forest.

CHAPTER I.

THE INCREMENT.

Every tree or wood may lay on three different kinds of increment, namely:—

1. Quantity or volume increment.
2. Quality increment.
3. Price increment.

Section I.—Quantity Increment.

By quantity increment is understood the increase in the volume caused by the growth of a tree or a wood. It is measured by the cubic foot solid, or the cubic foot stacked. The different kinds of quantity increment and the modes of measuring them have been explained in Forest Mensuration, p. 82. For the purpose of working plans it must be added that for short periods, say 5–10 years, the periodic mean annual increment can be put equal to the current annual increment, without any appreciable error.

The calculations of increment may refer to the final yield only, or to the intermediate yields, or to both together.

In the tables at pages 188 to 191, the various classes of increment have been calculated separately for final, intermediate, and both yields together.

1. *Progress of Volume Increment.*

a. Of Single Trees.

The volume increment is produced by an annual elongation of the crown and roots, and by the laying on of a new layer

between wood and bark all over the stem, branches and roots. As a general rule the stem or trunk is the most important part of the tree; hence the forester is specially interested in the height and diameter growth.

It has been explained in Volume I. of this Manual, p. 161, that the energy of height growth differs not only according to species, but also, in the case of one and the same species, according to the locality and method of treatment; besides, there is in this respect a great difference between seedlings and coppice shoots.

Generally speaking, in the case of *seedlings* the height growth during earliest youth is, in temperate climates, comparatively slow; it then increases rapidly, remains steady for a time, then decreases, and ceases altogether, or nearly so.

The periods when the current annual and mean annual height increment show their maxima are of special interest to the forester, but the data at present available give wide limits for those periods. Taking the various quality classes together the following table gives the limits of the periods and the mean year of the maximum height growth:—

Species.	Current Annual Height Increment.		Mean Annual Height Increment.	
	Limits of Period when the Maximum occurs.	Average Year when Maximum occurs.	Limits of Period when Maximum occurs.	Average Year when Maximum occurs.
Scotch pine	15—40	28	30— 55	43
Spruce	20—55	38	38— 80	59
Beech	25—55	40	41— 92	67
Silver Fir	20—90	55	50—140	95

On the whole, the culmination occurs earlier:—

(1) In the case of light-demanding species, and

(2) In the better localities.

In the case of teak, the current annual height increment

generally reaches its maximum during the first five years of the tree's life, frequently in the second or third year. Deodar shows a height growth similar to that of spruce, though somewhat quicker during early life. Sál shows, as far as is known at present, a remarkably even rate of height growth up to an age of 80 or 100 years.

Coppice shoots show, generally, the greatest height growth during the first few years of their existence; the rate of increment begins to fall off early, nor do such shoots, rare cases excepted, reach the same ultimate height as seedling trees.

The comparative height growth of different species has been dealt with at p. 163 of Volume I. of this Manual.

The lateral increment of the trunk of a tree, *i.e.*, diameter or sectional area increment, depends on the surface of the leaf canopy and on its activity. Hence, free growing trees increase more rapidly in diameter than those grown in dense or crowded woods. At the same time the position of the leaf surface is of importance. Trees with a crown coming close to the ground are comparatively more tapering, while those with the crown reduced to the upper part of the stem show a more cylindrical shape. The form or shape of the stem depends therefore on the distribution of the crown. If, with advancing age, the crown of trees in crowded woods moves higher up the stem, the difference in diameter increment between the lower and upper part of the stem decreases, and this is accompanied by what may be called the "form increment"; in other words, the tree becomes less tapering. The forester expresses this, as explained at p. 36, by the "form factor," or the coefficient by which the volume of a cylinder of the same base and height as the tree must be multiplied, in order to obtain the volume of the stem of the tree.

It has been stated, at p. 38, that in practice only the form factors based on a measurement of the base at height of chest, or 4¼ feet above the ground, are used, and at p. 39 the form factors for the following trees were given:—

Scotch pine,	according to	T.	Kunze.
Spruce,	,,	,,	Baur.
Silver Fir,	,,	,,	Lorey.
Beech	,,	,,	Baur.

These factors refer to trees grown in fairly crowded woods.

Similar figures for oak, based on the measurements of a sufficiently large number of woods, are not yet available, but until they have been obtained the form factors for beech may, within reasonable limits, be used for oak grown in fully stocked woods. In the case of oak trees grown in coppice with standards, form factors are out of the question.

b. Increment of whole Woods.

The increment of a wood consists, during the first period of life, of the full increment of the individual trees. As soon as the trees close overhead, the extension of the crowns is interfered with, followed by a decrease in the diameter increment. As long as the degree of crowdedness is not too great, the height growth is not reduced; on the contrary, a moderate degree of density of the leaf canopy encourages height growth. Although, during this period, the individual tree has less increment than it would have in a free position, the increment of a fairly crowded wood can have, and generally has, a larger increment per unit of area than an open wood, because the total increment is equal to the mean increment per tree multiplied by the number of trees. What degree of density of a wood gives the maximum increment is a question which awaits solution. In the meantime it must not be forgotten that a fairly crowded condition encourages height growth, decreases the tapering of the stems, and kills off the lower branches, thus producing more valuable trunks.

While the loss of material is very small in trees grown in the open, it becomes considerable in the case of fully stocked woods. Not only do all the lower branches die off, but the greater number of the trees, of which the wood originally consisted, must be removed by degrees, because they are gradually

overtopped and suppressed; these form ordinarily the material of which the thinnings consist.

In fully stocked woods, especially in those treated as high forest, a distinction must be made between the dominant and suppressed trees; the former may be called the major or primary part of the growing stock, and the latter the minor or secondary part. Not only the latter, but also a considerable portion of the former, will be removed in the thinnings, in the same degree as, with the advancing age of the wood, they lose their dominant character and join the secondary part of the growing stock.

The progress of the increment in whole woods has by no means been determined for all important species, though much material bearing on this question has been collected of late years on the continent of Europe. In India matters are still more backward. So much, however, has been determined, that both the current and mean annual increment culminate much earlier than had been supposed.

The yield tables for some of the more important European species justify the following conclusions:—

(1) The current annual increment rises rapidly after the first youth is passed, and reaches its maximum about the time when the height growth culminates; it then falls, and reaches zero at the death of the wood.

(2) The mean annual increment keeps below the current annual increment, until the two become equal; after that period the mean annual increment is greater than the current annual increment.

(3) The mean annual increment reaches its maximum at the precise moment when it is equal to the current annual increment. Gustav Heyer has proved this in the following manner:—

Let c_1, c_2, c_3, . . . c_n, c_{n+1}, be the current annual increments of successive years; m_1, m_2, m_3, . . . m_n, m_{n+1}, the mean annual increments for the same years; then the current annual increment of the year $(n+1)$ is represented by—

$$c_{n+1} = (n+1)\, m_{n+1} - n \times m_n,$$

or

$$c_{n+1} = n \times m_{n+1} + m_{n+1} - n \times m_n,$$

and

$$c_{n+1} - m_{n+1} = n\,(m_{n+1} - m_n).$$

It follows that if $m_{n+1} \gtrless m_n$, then also $c_{n+1} \gtrless m_{n+1}$; and if $m_{n+1} = m_n$, then also $c_{n+1} = m_{n+1}$, as it was proposed to prove.

(4) When the mean annual increment culminates, the current annual increment must, naturally, already be past its maximum, and be falling; hence the former culminates later than the latter. During the intermediate period between the two culminations, the mean annual increment is still rising, whereas the current annual increment is already falling.

Example:—

Year.	Current Annual Increment. Cubic feet.	Total Increment. Cubic feet.	Mean Annual Increment. Cubic feet.	Year.	Current Annual Increment. Cubic feet.	Total Increment. Cubic feet.	Mean Annual Increment. Cubic feet.
1	30	30	30	7	120	716	102
2	65	95	47	8	108	824	103
3	105	200	67	9	94	918	102
4	126	326	81	10	80	998	100
5	138	464	93	11	68	1066	96
6	132	596	99	12	62	1128	94

Or graphically represented:—

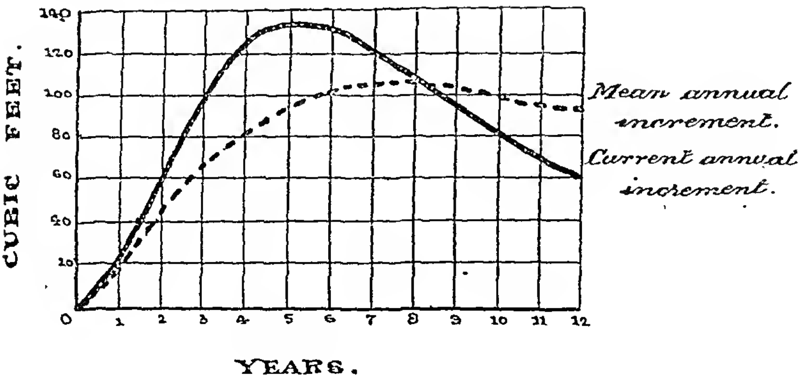


Fig. 43.

(5) Whenever the object of management consists in the realization of the greatest return of volume, the rotation must coincide with the year in which the mean annual increment culminates. The time when the maximum for final yield only occurs, differs from that for final plus intermediate yields; the difference may amount to 10 and even 20 years, especially if heavy thinnings are made at an early period.

2. *Quantity Increment Per cent.*

So far the increment has been expressed in actual volume. In addition, it is useful to ascertain the proportion which exists between the total volume of a tree or wood at a certain age, and the increment laid on during the year before or the following year. In order to express this proportion independently of the actual volume, it is usual to give it in per cents., and to call the proportion the "increment per cent."; by this is, therefore, understood the current annual increment which is laid on by every 100 units of volume.

The increment per cent. is used, sometimes to calculate from the present volume the increment which is likely to be laid on in the immediate future, but is chiefly employed for the purpose of testing the activity of the capital invested in forestry.

Let the volume of a tree or wood at a certain age $= v$,
,, ,, the same tree or wood one year later $= V$;
then the increment of one year $i = V - v$.

Let further the increment per cent. of the volume $= p_v$, then

$$v : V - v = 100 : p_v$$

and

$$p_v = \frac{V - v}{v} \times 100 = \frac{i}{v} \times 100.$$

The same expression is obtained by considering V as the accumulated value of v, produced by v working with interest for one year, in other words:

$$V = v \times 1{\cdot}0p_v.$$

This gives :—

$$p_v=\left(\frac{V}{v}-1\right)100=\frac{V-v}{v}100=\frac{i}{v}\times 100,$$

as before.

The increment per cent. p is naturally very large during the early youth of a tree or wood; but as the volume increases year by year, that is to say the denominator in the above equation, while the annual increment does not increase in anything like the same proportion, and in fact begins to decrease comparatively early, it follows that the increment per cent. becomes smaller year by year. Heavy thinnings can temporarily produce an exception to the above rule, as they may retard the sinking of the increment per cent.

Instead of comparing i with v, it can be brought into relation with V; in that case the increment per cent. becomes:—

$$p_v=\frac{V-v}{V}\times 100=\frac{i}{V}\times 100.$$

As the determination of the increment of a single year is a difficult and inaccurate operation, it is usual to determine it for a number of years, 5, 10, or, generally, n years, and to consider V as the value produced by placing v for n years at compound interest, working with p_v per cent., as :—

$$V=v\times 1{\cdot}op_v^n.$$

From this—

$$1{\cdot}op=\sqrt[n]{\frac{V}{v}},$$

and

$$p_v=100\left(\sqrt[n]{\frac{V}{v}}-1\right),$$

or—

$$\log(100+p)=2+\frac{\log V-\log v}{n}.$$

In order to avoid the use of logarithms, several formulæ have been evolved, which give approximately accurate results. Pressler obtained such a formula by assuming that the increment during the n years is laid on in annually equal quantities, and by comparing the increment with the volume which is present in the middle of the period of n years. He thus obtains the proportion—

$$\text{Capital} : \text{annual increment} = 100 : p_v$$

$$\frac{V+v}{2} : \frac{V-v}{n} = 100 : p_v$$

and

$$p_v = \frac{V-v}{V+v} \times \frac{200}{n}.$$

This formula gives p_v somewhat too small; but the difference is so slight, that it can be neglected for all practical purposes.

Example :—

Let v (in the year 70) = 3820 cubic feet;
,, V (in the year 80) = 4260 ,,

then—

$$p_v = 100\left(\sqrt[n]{\frac{V}{v}} - 1\right) = 100\left(\sqrt[10]{\frac{4260}{3820}} - 1\right) = 1{\cdot}096 \text{ per cent.};$$

or

$$p_v = \frac{V-v}{V+v} \times \frac{200}{n} = \frac{4260-3820}{4260+3820} \times \frac{200}{10} = 1{\cdot}089 \quad ,, \quad ,,$$

If any thinnings have been made during the n years, their amount must be added to V, before the increment per cent. is calculated. Supposing that in the above case 327 cubic feet were cut between the years 60 and 70, then—

$$p_v = 100\left(\sqrt[n]{\frac{4260+327}{3820}} - 1\right) = 1{\cdot}847 \text{ per cent.};$$

or

$$p_v = \frac{4260+327-3820}{4260+327+3820} \times \frac{200}{10} = 1{\cdot}825 \quad ,, \quad ,,$$

A law of considerable importance in the preparation of working plans, which was discovered by Pressler, runs thus:—

"The increment per cent., in its gradual fall, is expressed for the year r, in which the mean annual increment culminates, by the formula—

For final yield only: . . . . $p_v = \frac{100}{r}$.

For final and intermediate yields: $p'_v = \frac{100+t}{r'}$,

where t represents the sum of all thinnings expressed in per cent. of the final yield.

The proof is easy:—

The increment per cent. is,

$$p_v = \frac{i}{v} \times 100.$$

In the year r, when the mean annual increment culminates, the current annual increment is equal to the mean annual increment, that is to say, $i = \frac{v}{r}$; introducing this value in the above formula, it becomes

$$p_v = \frac{v}{r} \times \frac{100}{v} = \frac{100}{r}.$$

For final and intermediate yields let:—

T = total of intermediate yields to the year r',
v' = final yield in the year r',

then the maximum mean annual increment—

$$= \frac{v'+T}{r'}.$$

If p_v' be the corresponding increment per cent., then

$$p_v' = \frac{v'+T}{r'} \times \frac{100}{v'} = \frac{100}{r'}\left(1 + \frac{T}{v'}\right).$$

If now t represents the per centage of T in v', then—

$$t = \frac{T}{v'} \times 100,$$

and

$$T=\frac{t \times v'}{100}.$$

Introducing this value into the above formula of p'_v, the latter becomes:—

$$p_v'=\frac{100}{r'}\left(1+\frac{t \times v'}{100 \times v'}\right)=\frac{100+t}{r'}.$$

The formulæ $p_v=\frac{100}{r}$ and $p_v'=\frac{100+t}{r'}$ are used to gauge the ripeness of growing woods which are worked for volume of production only. If it is found that the increment per cent.

YIELD TABLE FOR ONE ACRE OF A SCOTCH PINE WOOD,
Timber down to

Age of Wood.	Mean Height of Dominant Trees.	Final Yield. Volume in Cubic Feet solid.	Final Yield. Increment in Cubic Feet solid. Periodic.	Current Annual. $\frac{d}{10}$	Mean Annual.	Per Cent.	Intermediate. Volume in Cubic Feet solid.	Incre- Periodic	Current Annual $=\frac{i}{10}$
a	b	c	d	e	f	g	h	i	k
10	5	—							
			—						
20	15	30			1·5				
			800	80		39·88		48	5
30	26	830			28·		48		
			1140	114		9·03		289	29
40	35	1970			49·		289		
			730	73		3·20		403	40
50	43	2700			54·		403		
			600	60		2·03		413	41
60	51	3300			55·		413		
			520	52		1·47		363	36
70	57	3820			55·		363		
			440	44		1·10		327	33
80	63	4260			53·		327		
			360	36		·81		282	28
90	67	4620			51·		282		
			290	29		·61		248	25
100	71	4910			49·		248		
			240	24		·48		213	21
110	73	5150			47·		213		
			190	19		·36		164	16
120	75	5340			44·		164		

in a certain year r is still greater than $\frac{100}{r}$, it shows that the moment when the mean annual increment culminates has not yet been reached—the wood is not yet ripe. If $p_v < \frac{100}{r}$ then ripeness is past; and if $p_v = \frac{100}{r}$, the wood is just ripe.

It remains to add, that the formulæ for the increment per cent. can be applied to height, diameter, or basal area increment, as well as to volume increment.

Example.—The accompanying two tables are yield tables for the Scotch pine III. quality, according to Weise, the first for

III. or Middling Quality, according to Weise.
3 inches diameter only.

Yields.		Total Yield.						Age of Wood.
ment.		Volume in Cubic Feet solid.		Increment.				
Mean Annual $= \frac{m}{a}$	Total to present Age.	Columns $c+h$.	Columns $c+m$.	Periodic $d+i$.	Current Annual $e+k$.	Mean Annual $\frac{o}{a}$	Per Cent.	
l	m	n	o	p	q	r	s	
								10
		30	30			1·5		20
				843	85		40·16	
1·6	48	878	878			29·		30
				1429	143		10·53	
8·0	337	2259	2307			58·		40
				1133	113		4·65	
15·	740	3103	3440			69·		50
				1013	101		3·24	
19·	1153	3713	4453			74·		60
				883	88		2·40	
21·	1516	4183	5336			76·		70
				767	77		1·85	
23·	1843	4587	6103			76·		80
				642	64		1·41	
24·	2125	4902	6745			75·		90
				538	54		1·11	
24·	2373	5158	7283			73·		100
				453	45		·89	
23·	2586	5363	7736			70·		110
				354	35		·67	
23·	2750	5504	8090			67·		120

YIELD TABLE FOR ONE ACRE OF A SCOTCH PINE WOOD,

Timber and

Age of Wood.	Mean Height of Dominant Trees. Feet.	FINAL YIELD. Volume, Cubic Feet solid.	FINAL YIELD. Increment, in cubic feet solid. Periodic.	FINAL YIELD. Increment, in cubic feet solid. Current Annual. $\frac{d}{10}$	FINAL YIELD. Increment, in cubic feet solid. Mean Annual.	FINAL YIELD. Increment, in cubic feet solid. Per Cent.	INTERMEDIATE Volume in Cubic Feet solid.	INTERMEDIATE Incre- Periodic.	INTERMEDIATE Incre- Current Annual. $\frac{i}{10}$
a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*k*
			510	51					
10	5	510			51				
			780	78		9·72			
20	15	1290			64				
			850	85		5·19		343	43
30	26	2140			71		343		
			760	76		3·10		614	61
40	35	2900			72		614		
			630	63		1·98		572	57
50	43	3530			71		572		
			530	53		1·41		529	53
60	51	4060			68		529		
			470	47		1·10		443	44
70	57	4530			64		443		
			420	42		·89		386	39
80	63	4950			62		386		
			350	35		·68		328	33
90	67	5300			59		328		
			280	28		·51		286	29
100	71	5580			56		286		
			240	24		·42		243	24
110	73	5820			53		243		
			190	19		·32		186	19
120	75	6010			50		186		

timber down to 3 inches diameter, and the second for timber and firewood, in each case for one acre; the data refer to the volumes above ground only.

The second table shows that—

(1) The current annual increment, for final returns only, culminates about the year 25.

(2) The mean annual increment, for final returns only, culminates about the year 40, that is to say, when it is equal to the current annual increment.

(3) The current annual increment, for final and intermediate returns, culminates about the year 35.

III. or Middling Quality, according to Weise.

Fagots.

Yield.		Total Yield.						Age of Wood.
ment.		Volume in Cubic Feet solid.		Increment.				
Mean Annual. $\frac{m}{a}$	Total to present Age.	Columns $c+h$.	Columns $c+m$.	Periodic $d+i$.	Current Annual. $e+k$.	Mean Annual. $\frac{o}{a}$	Per Cent.	
l	*m*	*n*	*o*	*p*	*q*	*r*	*s*	
				510	51			
		510	510			51		10
				780	78		9·72	
		1290	1290			64		20
				1193	119		6·77	
12	343	2483	2483			83		30
				1374	137		5·08	
24	957	3514	3857			96		40
				1202	120		3·53	
30	1529	4102	5059			101		50
				1059	106		2·66	
34	2058	4589	6118			102		60
				913	91		2·05	
36	2501	4973	7031			100		70
				806	81		1·65	
36	2887	5336	7837			98		80
				678	68		1·29	
36	3215	5628	8515			95		90
				566	57		1·02	
35	3501	5866	9081			91		100
				483	48		·83	
34	3744	6063	9564			87		110
				376	38		·63	
33	3930	6196	9940			83		120

(4) The mean annual increment, for final and intermediate returns, culminates about the year 60, when it is equal to the current annual increment.

(5) Both the current and mean annual increment of final and intermediate returns culminate later than in the case of final returns only.

(6) The increment per cent. forms a falling series in each case.

(7) The increment per cent. of final yield for the year 40, when the mean annual increment culminates, should be—

$$p_v = \frac{100}{40} = 2{\cdot}5.$$

This agrees with the table, as it shows—

For the period 30-40: $p = 3{\cdot}10$
" " 40-50: $p = 1{\cdot}98$
or about 2·5 for the year 40.

For final and intermediate returns:—

$$p_v' = \frac{100 + \frac{2058}{4060} \times 100}{60} = \frac{100 + 51}{60} = 2{\cdot}52.$$

The table shows—

For the period 50-60: $p' = 2{\cdot}66$
" " 60-70: $p' = 2{\cdot}05$
between which 2·52 lies.

(8) The maximum quantity return will be obtained under a rotation of 60 years.

Section II.—Quality Increment.

By quality increment is understood the increase in the value per unit of volume. It is produced, in the first place, by larger pieces of timber fetching higher prices per unit of measurement, and secondly by a reduction of the cost of harvesting per unit of measurement. Quality increment is independent of any alteration in the general price of forest produce.

If in the course of n years the net value of the unit of volume rises from q to Q, then the quality increment is $= Q - q$, and the corresponding per cent. is obtained by the formula:—

$$Q = q \times 1{\cdot}0p_q^{\,n};$$

and

$$p_q = 100\left(\sqrt[n]{\frac{Q}{q}} - 1\right);$$

or

$$\log(100 + p_q) = 2 + \frac{\log Q - \log q}{n}.$$

An approximately correct value for p_q is obtained by the formula:—

$$p_q = \frac{Q-q}{Q+q} \times \frac{200}{n}.$$

The quality increment may be rising, falling, or its movements may be more or less irregular; hence it is impossible to indicate these movements in a mathematical form.

Woods grown for *firewood* only show little or no quality increment after middle-age; except, perhaps, in so far as the per-centage of stem- to branch wood increases. The latest investigations seem even to indicate that wood taken from middle-aged trees has a higher heating power than wood taken from older trees although they may be perfectly sound.

Matters are different in the case of *timber forests;* here the quality increment rises, in the majority of cases, to an advanced age, because:

(1) Trees of large dimensions are, on the whole, more valuable per unit of volume, than those of small dimensions.

(2) The per centage of timber to firewood increases, at any rate up to a certain age.

The quality increment per cent. sinks, on the whole, with advancing age, though more or less irregularly; it can become nil and even negative if the timber commences to decay, while the quantity increment is still above nil.

Example.—A Scotch pine wood 60 years old contains:—

Timber = 3,300 cubic feet, worth 4*d.* per cubic foot.
Firewood = 760 „ „ „ 1*d.* „ „ „

Hence, mean quality:

$$q = \frac{3300 \times 4 + 760 \times 1}{4060} = 3{\cdot}44 \text{ pence.}$$

The same wood in the year 70 has:—

Timber = 3,820 cubic feet, worth 5*d.* a cubic foot.
Firewood = 710 „ „ „ 1*d.* „ „

Hence:

$$Q=\frac{3820\times 5+710\times 1}{4530}=4\cdot 37 \text{ pence.}$$

And:

$$4\cdot 37=3\cdot 44\times 1\cdot op_q^{10}$$

$$p_q=100\left(\sqrt[n]{\frac{4\cdot 37}{3\cdot 44}}-1\right)$$

$$\log(100+p_q)=2+\frac{\log 4\cdot 37-\log 3\cdot 44}{10}.$$

And

$$p_q=2\cdot 42 \text{ per cent.}$$

Approximate value:

$$p_q=\frac{4\cdot 37-3\cdot 44}{4\cdot 37+3\cdot 44}\times\frac{200}{10}=2\cdot 38 \text{ per cent.}$$

Calculating the quality increment per cent. for timber only with the data given in the table at page 122, the following values are obtained for p_q according to the formula :—

$$Q=q\times 1\cdot op_q^n.$$

Period	30— 40 years . . .	p_q	=	2·26
,,	40— 50 ,, ...	,,	=	1·84
,,	50— 60 ,, . . .	,,	=	2·92
,,	60— 70 ,, ...	,,	=	2·26
,,	70— 80 ,, . . .	,,	=	1·84
,,	80— 90 ,, ...	,,	=	1·55
,,	90—100 ,, . . .	,,	=	1·34
,,	100—110 ,, ...	,,	=	1·18
,,	110—120 ,, . . .	,,	=	1·06

What has been said above can also be applied to the intermediate returns. Indeed, the quality increment of that part of a wood which yields the thinnings can be very considerable, especially while the wood is still young. Here a few years extra growth may cause a great rise in the quality per unit of measurement. On the other hand, if thinnings are kept over too long, they interfere with the proper development of the

major part of the wood, hence extremes in this respect must be avoided.

Section III.—Price Increment.

Under price increment is understood the increment caused by a change in the price of forest produce generally, independent of the accompanying quality increment. It can be positive, nil, or negative.

Example.—A hitherto inaccessible forest is brought into communication with a large town by the construction of a railway; the increase in the prices of the produce of the forest represents the price increment, which in this case is positive.

Or, Owing to an increased import of forest produce the price of the home production falls generally; this represents a fall of prices, in other words a negative price increment.

Price increment depends partly on the forester and partly on external causes, over which he has little or no control. Of the former class of causes are, for instance, the construction of good roads, development of industries which consume forest produce, improvement in the general management leading to a higher net value per unit of measurement.

It is out of the question to construct a law showing the changes in price. In some cases such changes affect all classes of produce, in others only certain kinds. Under any circumstances it is almost impossible to foresee them, except in special definite cases. At the same time the price increment is of considerable importance, as it affects the financial ripeness of woods, and in this way influences the lines upon which the management of the forest should proceed.

The price increment is calculated in the same way as the quality increment. If s represents the value of the unit of measurement at the present time, and S the corresponding value after n years, the price increment is $= S - s$, and

$$S = s \times 1{\cdot}op_s^n$$

$$p_s = 100 \left(\sqrt[n]{\frac{S}{s}} - 1 \right),$$

$$\log(100 + p_s) = 2 + \frac{\log S - \log s}{10}.$$

Again, the approximate value:—

$$p_s = \frac{S - s}{S + s} \times \frac{200}{n}.$$

Section IV.—Addition of the several Increment Per cents. leading to the Forest Per cent.

On reference to page 163 it will be seen that the current annual per cent. with which the capital invested in a wood works, is expressed by the formula:—

$$p_f = \frac{(Y_{m+1} - Y_m - e)\,100}{S_c + {}^mG_c} = \frac{(Y_{m+1} - Y_m - e)\,100}{F_c};$$

or, if ${}^mG_c$ is taken as $= Y_m$,

$$p_f = \frac{(Y_{m+1} - Y_m - e) \times 100}{S_c + Y_m}.$$

This formula could be used to determine the financial activity of a wood at any time, if it were possible to determine accurately the value increment of the wood for a single year. This, however, is a very uncertain operation; hence the difference in the value of the crop produced during a series of years must be ascertained, a difference due to the combined effect of volume-, quantity-, and price increment.

The determination of the combined increment per cent. is done in the following manner: A property which has at present a value $= w$ increases during the next n years

in volume by p_v per cent. annually.
,, quality ,, p_q ,, ,,
,, price ,, p_s ,, ,,

Its value W at the end of the n years may be expressed by the formula:—

$$W = w \times 1{\cdot}op_v^n \times 1{\cdot}op_q^n \times 1{\cdot}op_s^n;$$

or

$$W = w\left(1+\frac{p_v}{100}\right)^n\left(1+\frac{p_q}{100}\right)^n\left(1+\frac{p_s}{100}\right)^n;$$

or

$$\sqrt[n]{\frac{W}{w}} = \left(1+\frac{p_v}{100}\right)\left(1+\frac{p_q}{100}\right)\left(1+\frac{p_s}{100}\right).$$

$$\sqrt[n]{\frac{W}{w}} = 1+\frac{p_v+p_q+p_s}{100}+\frac{p_v\times p_q+p_v\times p_s+p_q\times p_s}{100^2}+\frac{p_v\times p_q\times p_s}{100^3};$$

or

$$100\left(\sqrt[n]{\frac{W}{w}}-1\right) = p_v+p_q+p_s+\frac{p_v\times p_q+p_v\times p_s+p_q\times p_s}{100}+\frac{p_v\times p_q\times p_s}{100^2}.$$

In this equation the right side expresses nothing else than the current per cent. with which the forest capital w works during the period of n years, in other words what has been called at page 163 the current annual forest per cent. The proof is easy: If $n=1$, the original formula becomes:—

$$W = w\ (1{\cdot}op_v)\ (1{\cdot}op_q)\ (1{\cdot}op_s)$$

and also:

$$W = w\ (1{\cdot}op_f).$$

Hence

$$1{\cdot}op_f = (1{\cdot}op_v)\ (1{\cdot}op_q)\ (1{\cdot}op_s)$$

$$1+\frac{p_f}{100} = \left(1+\frac{p_v}{100}\right)\left(1+\frac{p_q}{100}\right)\left(1+\frac{p_s}{100}\right)$$

and

$$p_f = p_v+p_q+p_s+\frac{p_v\times p_q+p_v\times p_s+p_q\times p_s}{100}+\frac{p_v\times p_q\times p_s}{100^2}.$$

Hence the above formula can be written thus:—

$$^{\text{cur.}}\mathrm{p_f} = 100\left(\sqrt[n]{\frac{\mathrm{W}}{\mathrm{w}}}-1\right).$$

This formula was introduced by Pressler, who called the $^{cur}p_f$ thus obtained the indicating per cent. (Weiserpro-

cent.). Pressler went out of his way to call the above expression the approximate value of ${}^{c}p_f$, whereas it represents the absolutely current value of it, as has just been proved.

The indicating per cent. (or current forest per cent.) indicates the per cent. with which the capital represented by a wood works at the various periods of the wood's life; in other words, it indicates at any time, whether a wood is financially ripe or not. (See page 169.) As long as the indicating per cent. is larger than the general per cent. p, at which money can be invested otherwise with equal security, or at which money can be obtained for investment in forestry, the wood is financially not ripe; when the indicating per cent. has become smaller than p the financial ripeness of the wood is past; the wood is financially ripe at the time when the indicating per cent. is equal to p.

It remains to substitute the proper values for w and W. The capital value w of the forest at the present time is represented by the value of the soil and growing stock, correctly $= S + {}^{m}G_c$. As the formula is only used in the case of woods which are at or near maturity, the utilization value may be substituted for the cost value of the growing stock, so that

$$w = Y_m + S.$$

This is the capital which it is proposed to let work for another n years. During that period it increases to the value of the forest in the year $m + n$, from which amount must be deducted the annual costs during n years with compound interest, so that:—

$$W = Y_{m+n} + S - E\ (1{\cdot}op^n - 1)$$

and

$$^{\text{cur.}}p_f = 100\left(\sqrt[n]{\frac{Y_{m+n} + S - E\ (1{\cdot}op^n - 1)}{Y_m + S}} - 1\right).^*$$

If between the years m and $m + n$ a thinning has been

* This formula differs from that given by Pressler and Judeich for the reasons indicated in the footnote at page 163.

made, say in the year x, its value with compound interest to the year $m + n$ must be added to W, so that the formula becomes :—

$$ {}^{cur.}p_f = 100 \left(\sqrt[n]{\frac{Y_{m+n} + T_x \times 1{\cdot}op^{m+n-x} + S - E\,(1{\cdot}op^n - 1)}{Y_m + S}} - 1 \right).* $$

In either case the value S of the soil can be taken as the cost value or as the expectation value.

If n is placed = 1, the above formula reduces to :—

$$ {}^{cur.}p_f = \frac{(Y_{m+1} - Y_m - e) \times 100}{Y_m + S.}, $$

agreeing with that given at page 163 for the current annual forest per cent.

Example.—Taking the data in the table at page 122, and putting $p = 2\frac{1}{2}$ per cent., $S = 250$ shillings, $e = 3$ shillings, the following values of p_f are obtained :—

For the period 70—80 years :—

$$ \log\ (100 + p_f) = 2 + \frac{\log\ (2130 + 95 + 250 - 34) - \log\ (1592 + 250)}{10} $$

$$ {}^{70-80}p_f = 2{\cdot}86. $$

For the period 80 − 90 years :—

$$ \log\ (100 + p_f) = 2 + \frac{\log\ (2695 + 94 + 250 - 34) - \log\ (2130 + 250)}{10} $$

$$ {}^{80-90}p_f = 2{\cdot}36. $$

The current annual forest per cents. given in the table at page 202 have been calculated in this way, and they show that the financial ripeness occurred during the period 80 to 90, or more precisely in the year 82.

* This formula differs from that given by Pressler and Judeich for the reasons indicated in the footnote at page 163.

CHAPTER II.

THE ROTATION.

By rotation is understood that period of years which elapses between the formation of a wood and the time when it is finally cut over and regenerated.

The end of this period, that is to say the age of the wood when cut over, is called the "final age." If it coincides with that which is considered the one best suited to the system of management, it is called the "normal" final age; if a wood has, for one reason or another, to be cut-over at a different age, the latter is called an "abnormal" final age.

The determination of the rotation is one of the most important measures in forest management. At the same time the rotation depends entirely on the various objects of management; hence it differs with every change of conditions. In economic forestry the following deserve to be distinguished:—

1. The financial rotation.
2. The rotation of the highest income.
3. The rotation of the greatest volume production.
4. The technical rotation.
5. The physical rotation.

Each of these may be indicated by the objects of management, and it is necessary to explain them in some detail.

1. *The Financial Rotation.*

a. Calculation of the Financial Rotation.

By the financial rotation is understood that under which a forest yields, if calculated with a given per cent. and

compound interest, the highest net return. The financial rotation is, therefore, identical with that which—

(*a*) Gives the maximum soil rental as expressed by the formula:—

$$\text{Soil rental} = S_e \times {\cdot}op$$
$$= \frac{Y_r + T_a \times 1{\cdot}op^{r-a} + \ldots + T_q \times 1{\cdot}op^{r-q} - c \times 1{\cdot}op^r}{\dfrac{1{\cdot}op^r - 1}{{\cdot}op}} - e;$$

(See page 154.)

(*b*) Or yields the highest profit:—

$$P = S_e - S_c.$$

(See page 158.)

(*c*) Or yields the maximum mean annual forest per cent.:

$$^{mean}p_f = \frac{S_e}{S_c} \times p.$$

(See page 165.)

Of these, the first formula is the most convenient, and the procedure is as follows:—In the forest for which the financial rotation shall be determined, a number of typical woods are examined and as many data as possible collected. These can be augmented by data taken from suitable yield tables if such are available. Then the soil rental is calculated for various rotations, and that, for which the rental becomes a maximum, is the financial rotation.

In order to explain the method the appended table has been calculated from the money yield table for the Scotch pine given at page 122. In calculating that table it has been assumed that the cost of formation comes to 60 shillings, the annually recurring costs to 3 shillings, and that the general per cent. p is $= 2\frac{1}{2}$ per cent. It has also been assumed that the thinnings during the several periods of ten years have been made at the end of each period; for instance, the thinnings during the period of 40—50 are assumed to have been made in the year 50.

FINANCIAL YIELD TABLE FOR

Value of S,

(According to Weise's Volume Yield Table for the III. Quality, calculated

(See Table

a	*b*	*c*	*d*	*e*
Year.	NET VALUE OF YIELDS, IN SHILLINGS.		Sum of Intermediate Yields with Compound Interest to Date, Shillings $p = 2\cdot5$.	Total Yield to Date $(b + d)$ Shillings.
	Final.	Intermediate.		
30	138	4	4	142
40	406	36	41	447
50	675	67	120	795
60	1100	86	239	1339
70	1592	91	397	1989
80	2130	95	603	2733
90	2695	94	866	3561
100	3273	103	1212	4485
110	3862	106	1657	5519
120	4450	96	2218	6668

This table shows that the financial rotation falls between the years 75 and 85. In order to ascertain the exact year, the rentals given in column *k* of the table have been plotted (see figure 44). It will be seen that the financial rotation falls into the year 82, when the rental reaches its maximum.

In column *l* the current annual forest per cent. is given, calculated according to the formula on page 199. These data show that the forest- or indicating per cent. passes from above $2\frac{1}{2}$ per cent. to below $2\frac{1}{2}$ per cent. between the years 75 and 85. By plotting the per cents. (see figure 45), it is found that the exact time falls into the year 82, that is to say, the same year when the annual soil rental culminates. Hence the wood was financially ripe in the year 82.

ONE ACRE OF SCOTCH PINE WOOD.
= 250 *shillings.*
with English prices; brushwood, under 3 inches diameter, omitted.)
at page 122.)

f	*g*	*h*	*i*	*k*	*l*
Cost of Formation = 60*s.*, with Compound Interest to Date, Shillings.	Total Yield less cost of Formation $(e-f)$ Shillings.	Value of $\frac{1.025^r-1}{.025}$.	Soil Gross Rental $\frac{g}{h}$ Shillings.	Soil Net Rental i − Annual Expenses, Shillings.	Current Forest = or Indicatin Per Cent. during every 10 Years.
126	16	43·903	0·36	−2·64	
					5·42
161	286	67·403	4·24	+1·24	
					3·86
206	589	97·484	6·04	3·04	
					4·25
264	1075	135·992	7·90	4·90	
					3·47
338	1651	185·284	8·91	5·91	
					2·86
433	2300	248·383	9·26	6·26	
					2·36
554	3007	329·154	9·14	6·14	
					2·01
709	3776	432·549	8·73	5·73	
					1·73
907	4612	564·902	8·16	5·16	
					1·48
1161	5507	734·326	7·50	4·50	

b. Notes on the Financial Rotation.

Owing to the uncertainty of the data upon which the calculation is based, the financial rotation can only be determined approximately, moreover it changes with every change of conditions. Under these circumstances it can only serve as a general guide.

Of the several items which appear in the formula for the soil rental, the rate of interest is the most important. A low rate gives a high financial rotation, and *vice versâ.* An alteration of 1 per cent. in the general per cent. p may cause the financial rotation to rise or fall by 10 to 20 years.

As has been explained on a previous occasion (page 118), the general per cent. applicable to forest finance may for

Britain at present be placed at $2\frac{1}{2}$ per cent. As the rate of interest has, for a series of years, steadily declined, it is desirable to make calculations for the future rather with a lower, than a higher rate of interest.

Of the receipts, the final yield is by far the most important item. Its present value can be easily ascertained, but forecasts for the future are of a risky nature. If, in the future, the proportion between the prices of the different classes of produce remains about the same, then a change in the financial rotation does not necessarily follow; but great changes can be

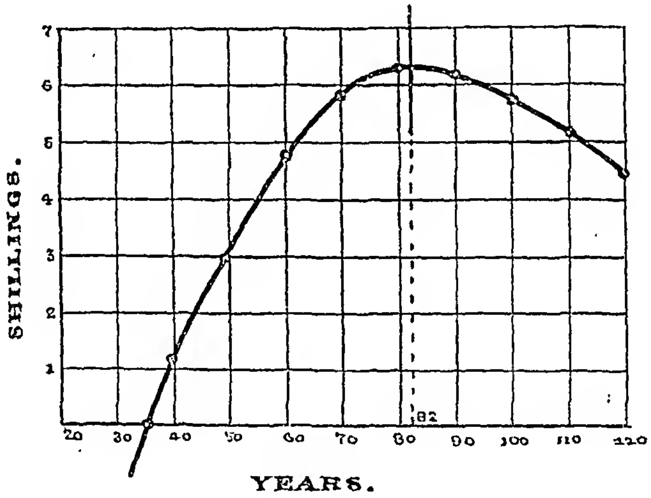


Fig. 44.

produced in the reverse case; that is to say, if for instance timber of small dimensions rises in price while that of large dimensions falls, or *vice versâ*. Such changes are difficult to foresee; the experience of the last decades shows, no doubt, that timber of large dimensions is not unlikely to rise in price; hence the selected rotation should be rather above than below the financial rotation.

The intermediate returns exercise a considerable influence upon the actual amount of the rental, but a comparatively small effect upon its culminating point; in other words, early thinnings reduce the financial rotation only to a limited

extent, and if they are made so heavy, that they reduce the value of the final return, they may even have the opposite effect.

Of the costs, the annual expenses do not affect the financial rotation, unless they alter in amount with the rotation.

The cost of formation affects the rental to a considerable extent, but its effect upon the financial rotation is small.

Taking all effects together, it may be said that the financial rotation is *low*:—

(1) in the case of forests where only firewood is saleable, that is to say, where an increase in quality per unit of volume ceases at a comparatively early age;

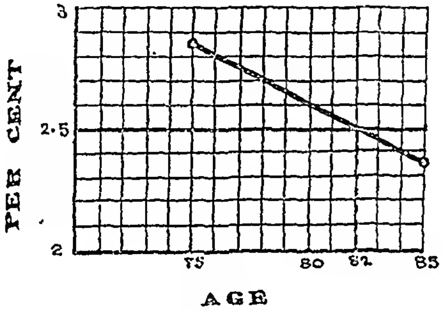


Fig. 45.

(2) if trees of small dimensions can be sold as timber, for instance in mining districts, in hop-growing countries, etc.

The financial rotation is *high*:—

(1) in localities with an unfavourable soil or climate, such as high exposed situations, where the trees take a longer time to reach marketable dimensions;

(2) in thinly populated districts, where prices generally rule low for small dimensions, while large timber can be exported to other better paying markets.

c. *Correction of the calculated Financial Rotation.*

The length of the financial rotation, as obtained by a first calculation, is subject to correction, because it is based

upon certain rates obtainable for the various classes of produce, whereas a change in the actual rotation may alter those rates. If, for instance, the calculated financial rotation is lower than that actually existing, and the former is introduced, more small and less large timber will be produced; also the proportion between timber and firewood will be altered. This may produce a fall in the average price of produce, and consequently a rise in the financial rotation. The reverse effect would be produced, if the calculated financial rotation were higher than the one actually existing. In either case it must be taken into consideration that a change in the rotation is accompanied by a change in the growing stock, and that either more or less material is brought into the market, which may be accompanied by a change in prices.

It follows, that the first calculation is generally subject to some correction in accordance with the alteration of prices which may be produced by a change in the rotation.

d. Introduction of the Financial Rotation.

Although every deviation from the financial rotation is accompanied by a financial loss, yet it is very desirable that it should be introduced, if not already followed, with great caution, because, in the first place, it can only be determined approximately, and secondly, its introduction is accompanied by a change in the existing growing stock. If a true, or assumed, surplus of growing stock has been disposed of, it would take much time to re-establish it, should further experience indicate a higher rotation than that originally calculated. Hence, it is desirable to keep always somewhat above the theoretical financial rotation.

If a change of rotation has been decided on, it can be carried out at once, provided the forest is of small extent, and the demand for produce sufficiently large to absorb the extra supply of produce thrown upon the market, without causing any appreciable change in prices. If the forest is, however,

of some extent, and the demand for produce uncertain, it is always desirable to make the change gradually, so as either to spread the extra supply of produce over a number of years, or to accumulate the extra growing stock gradually, thus disturbing the market as little as possible.

2. *Rotation of the Highest Income.*

By this is understood the rotation which yields the highest income, calculated without interest and irrespective of the time when the items of income occur. The net income is thus calculated according to the arithmetical mean of incomes diminished by the costs. All items of income and costs during one rotation are added up, and the sum of the latter deducted from the former; the difference, divided by the number of years in the rotation, represents the annual income.

Hence, the rotation in this sense is that under which the expression—

$$\text{Annual income} = \frac{Y_r + T_a + T_b + \ldots + T_q - c - r \times e}{r}$$

becomes a maximum.

As the annual expenses and the cost of formation are generally the same for differing rotations, the above expression can be reduced to the following:—

$$\text{Gross annual income} = \frac{Y_r + T_a + T_b + \ldots + T_q}{r}.$$

This rotation falls, as a rule, a number of years beyond the financial rotation.

Example.—Taking the data contained in the table at page 202, the net annual income amounts to:—

For a rotation of:—

Shillings.

$$70 \text{ years} = \frac{1592 + 4 + 36 + 67 + 86 + 91 - 60 - 70 \times 3}{70} = 22{\cdot}94.$$

$$80 \text{ years} = \frac{2130 + 4 + 36 + 67 + 86 + 91 + 95 - 60 - 80 \times 3}{80} = 27{\cdot}61$$

For a rotation of—

Shillings.

$$90 \text{ years} = \frac{2695+4+36+67+86+91+95+94-60-90\times 3}{90} = 31\cdot 53$$

$$100 \text{ years} = \frac{3273+4+36+67+86+91+95+94+103-60-100\times 3}{100} = 34\cdot 89$$

$$110 \text{ years} = \frac{3862+4+36+67+86+91+95+94+103+106-60-110\times 3}{110} = 37\cdot 76$$

$$120 \text{ years} = \frac{4450+4+36+67+86+91+95+94+103+106+96-60-120\times 3}{120} = 40\cdot 07$$

It will be observed that the annual income still rises under a rotation of 120 years, and will continue to do so, until the volume- and quality increment become so much reduced, that they will no longer cover the increase in the expenses. At the same time a rotation of 120 years would involve a financial loss, because interest on the invested capital has been altogether omitted. This can easily be seen by a reference to column k of the financial yield table at page 203. The net soil rental under a rotation of 120 years comes to 4·50 shillings, and under one of 80 years to 6·26 shillings.

3. *Rotation of the Greatest Production of Volume.*

This is the rotation under which a forest yields the greatest quantity of material per unit of area; it coincides with the year in which the mean annual volume increment culminates, that is to say, the year when the volume increment per cent. is equal to $\frac{100}{r}$ in the case of final yield only, or equal to $\frac{100+t}{r}$, in the case of final and intermediate returns (see page 187).

Let volume of final yield be $= V_r$
Volume of thinning in the year $a = v_a$
" " " $b = v_b$
etc.,
" " " $q = v_q$

then the rotation of the greatest production is that in which the value $\frac{V_r + v_a + v_b + \ldots v_q}{r}$ becomes a maximum.

The calculation can be made for timber and firewood, or for timber only.

Example.—Taking the data for total yield in the table at page 189, for timber only, the rotation of the greatest production would fall about into the year 80, which is approximately the financial rotation.

For timber and firewood (page 191) the rotation would fall into the year 60, which is considerably below the financial rotation; in this case a financial loss would be incurred.

4. *The Technical Rotation.*

By this is understood the rotation, under which a forest yields the most suitable material for a certain fixed purpose; for instance for construction generally, shipbuilding, railway sleepers, telegraph or hop poles, mining props, tanning bark, fuel, etc.

As the objects of management and the purposes for which the material is required differ very much, the technical rotation may fall into any age, either before, after, or into the age of the financial rotation. The loss occasioned by following it depends on the difference between the technical and financial rotations.

5. *The Physical Rotation.*

By the physical rotation is understood that age which is most favourable for the natural regeneration of a species, taking into consideration the conditions of the locality and the sylvicultural system. It cannot be lower, in the case of high

forest, than the age when the trees have commenced to bear good seed in sufficient quantity, nor as high as the age when the production of good seed has ceased; the best period being that towards the end of the principal height growth.

In the case of coppice woods the age must be below that at which the trees cease to produce good healthy shoots when cut over.

Sometimes a second physical rotation is mentioned as that which coincides with the natural lease of life of the trees. It is only of interest in the case of protection forests, parks, etc.

6. *Choice of Rotation.*

The choice of rotation, or the age at which a wood is to be cut over, is one of the most important questions in forest management. Many and varied are the arguments which have been brought forward in favour of the one or other rotation.

One party maintains that the financial aspect should decide the choice of rotation, since forests represent capital, which should yield the highest possible interest. Another party brings the general usefulness more into the foreground, and maintains that other considerations are more important to the general community than purely financial results, especially in the case of State forests.

In the author's view, the "objects of management" should determine the rotation. These frequently demand deviations from the financial rotation. For instance, to begin with an extreme case, for protection forests generally a very high rotation is indicated; where a nation considers it necessary to produce timber fit for naval construction, a rotation which lies far beyond the financial rotation is necessary; where hop-poles are wanted, a very low rotation would be called for; in cases where land is scarce and yet a certain quantity of wood is wanted for existing industries, the rotation of the highest production of produce is indicated; if a proprietor wishes to invest capital so as to obtain the highest annual income,

irrespective of the rate of interest, he would choose the rotation under which that income culminates, etc.

There may be good reasons in all these cases for adopting the one or other rotation. At the same time the proprietor should know what financial sacrifice he brings for the realization of his special object. Hence, the general procedure in fixing the rotation may be described as follows:—

In the first place the financial rotation should be determined, as it alone gives a true expression of the economic value of the management; then it should be ascertained in how far the objects of management demand a departure from the financial rotation; lastly, the financial loss involved in such a departure should be determined, so that the proprietor may have a clear conception of the payment which he is called upon to make in order to realize his special object.

It need hardly be pointed out that the above procedure suits all possible cases which may come under consideration.

CHAPTER III.

THE NORMAL AGE CLASSES.

IT has been stated at page 176, that by a normal distribution of age classes is understood a series of age gradations so arranged, that at all times when cuttings are to be made, mature woods of the normal age are available, and so situated that no obstacles to their cutting exist. This means that each age class must be of the proper extent, and that the several age classes must be properly grouped, or distributed, over the forest.

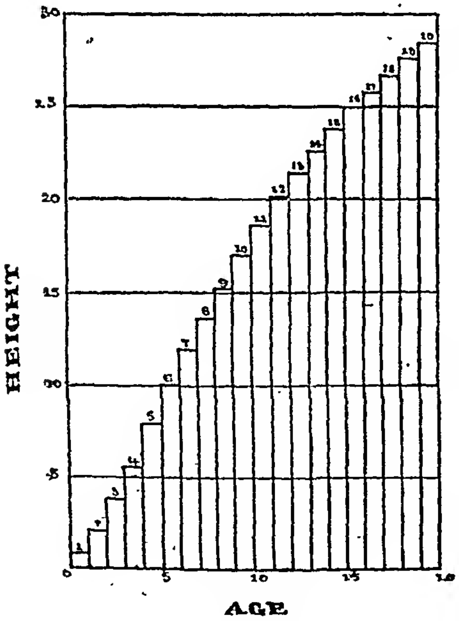


Fig. 46.

If a forest is to be managed according to the system of a sustained annual yield, it must contain a series of age gradations equal to the number of years in the rotation; the oldest age gradation must, immediately before cutting, have the age of the rotation, the youngest must be one year old, with a difference of one year in the age of every succeeding two gradations.

Example.—Assuming the rotation of a coppice wood to be 20 years, and the height which the oldest wood reaches in that time = 28 feet, then the 20 age gradations may be represented as in the appended figure 46.

If the annual returns are to be equal in volume, and the quality of the locality is the same throughout, then all age gradations must be of the same extent; if different qualities occur, the areas of the coupes must be in inverse proportion to the quality of the locality. A series of age gradations so arranged is called a normal working section. This subject will be again dealt with further on. For the present it is assumed that the quality of locality is the same throughout. The questions then are:—

(1) What is the area to be cut annually under the different methods of treatment?

(2) What is the size, or extent, of the age classes? and

(3) How should the age classes be distributed over the forest?

1. *The Annual Coupe, or the Area to be cut annually.*

This differs according to the method of treatment. (For a description of the latter, see page 203 of Volume I.)

a. Coppice and Coppice with Standards.

The annual coupe is determined by dividing the total area of the forest, or working section, by the number of years in the rotation:—

Let total area $= A$
Rotation of the coppice $= r$,

then the annual cutting area $c = \frac{A}{r}$. This holds good for the coppice with standards system, because the annual cutting area is governed by the coppice only.

b. Clear Cutting in High Forest.

Here is again:

$$c = \frac{A}{r},$$

if each clearing is at once restocked. Frequently it happens,

however, that the cleared coupes lie fallow for one or more, say s years; in that case:

$$c = \frac{A}{r+s},$$

so that the forest consists, immediately before cutting, of a series of age gradations from 1 to r years old, and s blanks, or altogether $r + s$ coupes.

c. *The Shelter-wood Compartment System.*

Under this system the regeneration of each coupe extends over a number of years, say m; hence it is necessary to throw m annual coupes together into a periodic coupe, the crop on which is, by gradual cuttings, led over, in the course of m years, into a young wood. The size of the periodic coupe is, therefore $= \frac{A}{r} \times m$.

In this case the first of the successive cuttings towards regeneration may be made:

Either in the year r, so that the trees removed at the end of the regeneration period would be $r + m$ years old, and the mean age $r + \frac{m}{2}$ years; in other words the procedure would lead to a raising of the rotation from r to $r + \frac{m}{2}$ years;

Or, the first cutting may be made in the year $r - \frac{m}{2}$ and the last in the year $r + \frac{m}{2}$, so that the mean final age comes to r years.

In the present chapter the latter is assumed.

d. *The Selection System.*

Strictly speaking, the annual coupe is equal to the total area of the forest. For convenience sake, however, the cuttings of each year are restricted to a portion of the area, so that it takes a number of years to go round the forest, and before cuttings are again made on the same area. If that number is l, then—

$$\text{Annual cutting area} = \frac{A}{l}.$$

Example.—In the beech forests of Buckinghamshire, which are worked under the selection system, it is usual to go round once in seven years; in that case the annual cutting area would be equal to $\frac{A}{7}$.

2. *Size of the Age Classes.*

In forests of some extent, which are worked under a high rotation, and especially those regenerated naturally, it is, as a rule, impracticable to separate the annual cutting areas so that a regular series of age gradations, differing by one year in age throughout, exists. In these cases it is necessary to be satisfied with larger groups, that is to say, to join a number of age gradations into an "age class." The normal size of such an age class depends on the area of the annual coupe and the number thrown together. If a class contains n gradations, its area would be $= n \times c$. The number of age classes $= \frac{r}{n}$ is variable.

Another way is to fix the number of age classes; in that case n is variable, but this procedure is not to be recommended, as it is likely to lead to confusion.

It is usual to take for n a round number, say 10, 20, or even 30; in coppice woods n is usually taken as $=5$. The age classes are numbered. It is best to call the youngest I., the next youngest II., and so on; for instance, if $n = 20$—

First age class	I.,	contains all woods up to 20 years old.		
Second ,,	II.,	,,	,,	from 21 to 40 years old.
Third ,,	III.,	,,	,,	,, 41 to 60 ,,

and so on.

In this way the number of the age class indicates directly its age. The reverse method, of calling the oldest age class I., the next oldest II., etc., is less desirable, but unfortunately it has been largely adopted. An effort should be made to alter

this. The area of the age classes under the several methods of treatment is now as follows:—

a. Clear Cutting in High Forest.

The area of each age class, in a normal state, is:—

$$C = n \times c = n \times \frac{A}{r}, \textit{ or } C = n \times \frac{A}{r+s}$$

according as to whether each clearing is at once re-stocked, or allowed to lie fallow for s years.

Example:—

Let area A	=	1050 acres
Rotation r	=	100 years
s	=	5 ,,
n	=	20 ,,

then:

$$\text{Annual age gradation} = \frac{A}{r+s} = \frac{1050}{105} = 10 \text{ acres};$$

and the age-classes:

Blanks		=	$c \times s$	=	10×5	=	50 acres.
C_I	(1—20 years old woods)	=	$c \times n$	=	10×20	=	200	,,
C_{II}	(21—40 ,, ,,)	=	,,	=	,,	=	200	,,
C_{III}	(41—60 ,, ,,)	=	,,	=	,,	=	200	,,
C_{IV}	(61—80 ,, ,,)	=	,,	=	,,	=	200	,,
C_V	(81—100 ,, ,,)	=	,,	=	,,	=	200	,,
							A = 1050 acres.	

b. Shelter-wood Compartment System.

As already explained, under this system the old crop is gradually led over into a young wood in the course of a number of years, which has been indicated by m. There is always an area under regeneration, which contains a certain number of old trees and young growth, and this may be called the regeneration class = C_v: it wanders gradually through the whole forest, until, at the beginning of the second rotation, it is found in the original position. As regeneration sometimes

takes only a few, and in others more years, it is impossible to define its duration accurately, and least of all can m be placed equal to n, the number of years in the period. Under these circumstances the arrangement of age classes can be indicated only approximately, somewhat in the following manner:—

Cuttings in the oldest age class commence when the crop is $r - \frac{m}{2}$ years old, and the last cuttings occur when the crop is $r + \frac{m}{2}$ years old. Assuming that s years pass, after the first regeneration cutting, until the new crop is fully started, then the annual cutting area, as before, $= \frac{A}{r+s}$, and the area of the regeneration class $= \frac{A}{r+s} \times m$. The latter contains the areas as yet blank, young trees from 1 to $m - s$ years old, and the remaining old trees ranging in age from $r - \frac{m}{2} + 1$ to $r + \frac{m}{2}$ years. Now, it may happen that $m - s = n$; in that case the youngest age class does not exist by itself, but forms part of C_v. Again, it may occur, that $m - s > n$, in which case C_v contains not only the youngest age class, but also a portion, if not the whole, of the second age class. Hence the size of the several age classes may be expressed as follows, assuming five age classes:—

(1) $m-s<n$:—

$$C_I = \frac{A}{r+s}\left[n-(m-s)\right]$$

$$C_{II} = \frac{A}{r+s} \times n$$

$$C_{III} = \frac{A}{r+s} \times n$$

$$C_{IV} = \frac{A}{r+s} \times n$$

$$C_V = \frac{A}{r+s} \times n$$

$$C_v = \frac{A}{r+s} \times m.$$

Total number of annual coupes $= n-(m-s)+4\times n+m = 5\times n+s = r+s$.

(2) $m-s=n$:—

$$C_I = \frac{A}{r+s}\Big[n-(m-s)\Big]=0$$

$$C_{II} = \frac{A}{r+s} \times n$$

$$C_{III} = \frac{A}{r+s} \times n$$

$$C_{IV} = \frac{A}{r+s} \times n$$

$$C_V = \frac{A}{r+s} \times n$$

$$C_v = \frac{A}{r+s} \times m$$

Total number of coupes $= 4\times n+m = 5\ n+s = r+s$.

(3) $m-s>n$, but $m-s<2\times n$:—

$$C_I = 0$$

$$C_{II} = \frac{A}{r+s}\Big[2\times n-(m-s)\Big]$$

$$C_{III} = \frac{A}{r+s} \times n$$

$$C_{IV} = \frac{A}{r+s} \times n$$

$$C_V = \frac{A}{r+s} \times n$$

$$C_v = \frac{A}{r+s} \times m$$

Total number of coupes $= 2\times n-(m-s)+3\times n+m = 5\times n+s = r+s$; and so on.

It is obvious that for the shelter-wood system with natural regeneration the above allotment is only of an ideal character, because the duration of regeneration is so uncertain. The regeneration class, the oldest and youngest classes are subject

to modifications amongst themselves, so that they cannot easily be separated the one from the other; hence they are best thrown together. The important point in that case is, that the middle-aged classes are of the proper size. The allotment may then be represented as follows :—

$$C_{II} = \frac{A}{r+s} \times n$$

$$C_{III} = \frac{A}{r+s} \times n$$

$$C_{IV} = \frac{A}{r+s} \times n$$

$$C_V + C_v + C_I = \frac{A}{r+s}(r+s-3 \times n).$$

Or again—

$$C_{III} = \frac{A}{r+s} \times n$$

$$C_{IV} = \frac{A}{r+s} \times n$$

$$C_V + C_v + C_I + C_{II} = \frac{A}{r+s}(r+s-2 \times n).$$

Example.—As above, $A = 1050$; $r = 100$; $s = 5$; $n = 20$

$$C_{III} = \frac{1050}{105} \times 20 = 200$$

$$C_{IV} = \frac{1050}{105} \times 20 = 200$$

$$C_V + C_v + C_I + C_{II} = \frac{1050}{105}(\times 105 - 40) = 650$$

Total . . . 1050

c. Coppice Woods.

As the rotation of coppice woods is short, it is usually possible to mark the annual coupes on the ground, so that grouping in age classes is not necessary. If the latter should, nevertheless, be considered desirable, generally not more than five gradations are thrown together, so that C_I comprises the 1 to 5-years-old gradations, C_{II} those from 6 to 10 years, etc.

Example:—

$$\text{Area} = 200 \text{ acres}$$
$$r = 20 \quad ,,$$
$$n = 5 \quad ,,$$

The arrangement of age classes would be normal, if—

$$C_I = \frac{200}{20} \times 5 = 50 \text{ acres.}$$
$$C_{II} = \frac{200}{20} \times 5 = 50 \quad ,,$$
$$C_{III} = \frac{200}{20} \times 5 = 50 \quad ,,$$
$$C_{IV} = \frac{200}{20} \times 5 = 50 \quad ,,$$
$$\text{Total} \quad 200 \text{ acres.}$$

d. Coppice with Standards.

Here each coupe contains coppice (underwood) and standards (overwood). As far as the underwood is concerned, the arrangement is exactly the same as in the case of simple coppice; the annual age gradation is $= \frac{A}{r}$, and the age class $= \frac{A}{r} \times n$.

The distribution of the overwood, in its normal condition, is somewhat peculiar, which may usefully be explained here, though it is only of a theoretical value.

In the first place it should be remembered that cutting in both the under- and overwood on the same area must be made at the same time, or rather those in the overwood must be made immediately after the underwood has been cut over, and before the new coppice shoots appear; hence the rotation R of the overwood must be a multiple of the rotation r of the underwood, say $R = r \times t$.

In each annual coupe, when cutting comes round to it, a certain portion of the underwood (chiefly seedling trees), is

left standing to form the youngest age gradation of the overwood. That portion would occupy an area $= \frac{A}{R}$, assuming that each age gradation of the overwood occupies the same extent of ground. The area of the youngest age class of overwood comes to $= \frac{A}{R} \times n$.

Assuming now that the youngest overwood class 1 to r years old, though still forming part of the underwood, is already counted as belonging to the overwood, then there are t overwood classes. The latter are not separated according to area, as in the case of clear cutting or coppice, but t gradations are standing mixed on each annual coupe, so that each of the latter contains $\frac{1}{r}$th part of each overwood class.

Immediately before cutting, the arrangement would be as follows:—

Underwood, Age in Years.	1	2	3	...	...	$r-1$	r
Overwood Age Class C_I age	1	2	3	...	...	$r-1$	r
" " " C_{II} "	$r+1$	$r+2$	$r+3$	...	...	$2r-1$	$2 \times r$
" " " C_{III} "	$2r+1$	$2r+2$	$2r+3$	...	...	$3r-1$	$3 \times r$
⋮							
C_r ...	$(t-1)r+1$	$(t-1)r+2$	$(t-1)r+3$	...	...	$t \times r-1$	$t \times r$

It will be seen, that a normal coppice with standards forest must have an overwood which consists of $m \times r = R$ age gradations ranging from 1 year up to R years old.

Example.—A forest of 200 acres worked under a rotation of 20 years for the underwood, and 100 years for the overwood, has $\frac{100}{20} = 5$ overwood classes. On the 10 acres which are about to be cut, will be found:—

Underwood = 20 years old
Overwood = 100, 80, 60, 40 and 20 years old.

The next oldest coupe contains—

Underwood = 19 years old
Overwood = 99, 79, 59, 39 and 19 years old.

The youngest coupe contains—

Underwood = 1 year old
Overwood = 81, 61, 41, 21 and 1 years old.

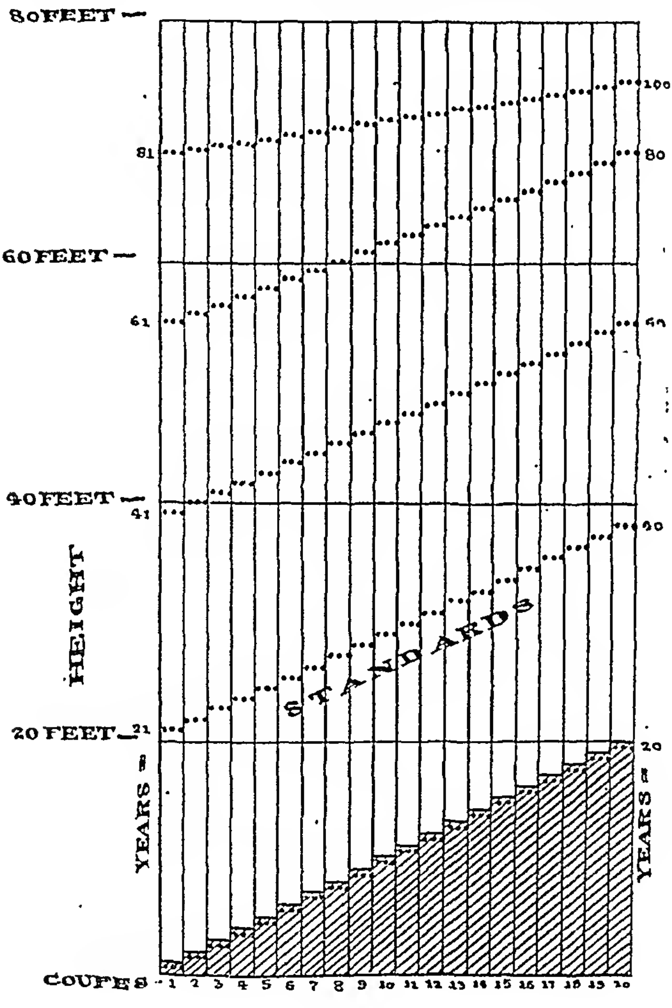


Fig. 47.

The appended figure 47 illustrates the distribution of the several age gradations over the area.

The area occupied by each overwood class can only be determined by assuming that each gradation occupies an equal area of ground; hence the youngest gradation will have most trees, and the oldest least. Imagining now that the age classes of the overwood were not intermixed, but that the trees of each class were brought together on separate areas, then the overwood, apart from the coppice, would form an open high forest resembling a selection forest. The areas to be allotted to the several classes may, therefore, be considered as equal. The youngest would contain the standards from 1 to r years, the next those from $r+1$ to $2r$ years, and so on. By degrees, the youngest class passes through all the intermediate stages, until it becomes the oldest and is cut over in the course of r years. At each annual cutting, therefore, an equal area must be cut over, on which the new, that is the youngest, gradation is started, either naturally or artificially.

The annual coupe is $c = \frac{A}{r}$ and $A = c \times r$.

The number of overwood classes is $= \frac{R}{r} = t$, hence—

Area of each age class on each annual coupe $= \frac{A}{R} = \frac{A}{t \times r} = \frac{c}{t}$.

As the whole forest consists of r coupes, each overwood class, consisting of r gradations, contains, in a normal forest, $\frac{c}{t} \times r = \frac{A}{t}$ units of area. This shows that, theoretically, the proportion of the age classes is the same as in high forest, although the distribution is different.

Example.—Data as before:—

$A = 200$; $R = 100$; $r = 20$, number of overwood classes $t = 5$.

Normal annual cutting area $c = \frac{A}{r} = \frac{200}{20} = 10$ acres.

On each coupe each age gradation of overwood occupies . . $\left.\right\}\ \frac{c}{t} = \frac{10}{5} = 2$ acres.

The area and distribution of the several age classes is as follows :—

Coupe No. 1, *oldest :*

Underwood	= 10 acres	=	20 years old.	
Overwood	2 ,,	=	20 ,,	,,
,,	2 ,,	=	40 ,,	,,
,,	2 ,,	=	60 ,,	,,
,,	2 ,,	=	80 ,,	,,
,,	2 ,,	=	100 ,,	,,

Coupe No. 20, *youngest :*

Underwood	= 10 acres	=	1 year old.	
Overwood	2 ,,	=	1 ,,	,,
,,	2 ,,	=	21 ,,	,,
,,	2 ,,	=	41 ,,	,,
,,	2 ,,	=	61 ,,	,,
,,	2 ,,	=	81 ,,	,,

If now the underwood is arranged into four classes of 5 years each, and the overwood into five classes of 20 years each, the following areas are obtained for each class :—

Underwood :

C_I = 1— 5 years old = 5 × 10 = 50 acres.
C_{II} = 6—10 ,, ,, = ,, = 50 ,,
C_{III} = 11—15 ,, ,, = ,, = 50 ,,
C_{IV} = 16—20 ,, ,, = ,, = 50 ,,

Total A = 200 acres.

Overwood :

C_I = 1— 20 years old = 20 × 2 = 40 acres.
C_{II} = 21— 40 ,, ,, = ,, = 40 ,,
C_{III} = 41— 60 ,, ,, = ,, = 40 ,,
C_{IV} = 61— 80 ,, ,, = ,, = 40 ,,
C_V = 81—100 ,, ,, = ,, = 40 ,,

Total A = 200 acres.

It will be seen that the total area has been distributed amongst the underwood, and a second time amongst the overwood.

By allotting the overwood to the four underwood classes, the following four combination classes, called "coppice with standard classes," are obtained :—

Class I., the youngest.

Underwood,	1— 5 years old		=	$10 \times 5 = 50$	acres
Overwood,	1— 5	,, ,,	=	$5 \times 2 = 10$	,,
,,	21—25	,, ,,	=	,, $= 10$	,,
,,	41—45	,, ,,	=	,, $= 10$	,,
,,	61—65	,, ,,	=	,, $= 10$	,,
,,	81—85	,, ,,	=	,, $= 10$	,,

Total . . { Underwood = 50 acres
Overwood = 50 ,,

Class II.

Underwood,	6—10 years old		= 50	acres
Overwood,	6—10	,, ,,	= 10	,,
,,	26—30	,, ,,	= 10	,,
,,	46—50	,, ,,	= 10	,,
,,	66—70	,, ,,	= 10	,,
,,	86—90	,, ,,	= 10	,,

Total . . { Underwood = 50 acres
Overwood = 50 ,,

Class III.

Underwood,	11—15 years old		= 50	acres
Overwood,	11—15	,, ,,	= 10	,,
,,	31—35	,, ,,	= 10	,,
,,	51—55	,, ,,	= 10	,,
,,	71—75	,, ,,	= 10	,,
,,	91—95	,, ,,	= 10	,,

Total . . { Underwood = 50 acres
Overwood = 50 ,,

Class IV.

Underwood,	16— 20 years old	= 50 acres
Overwood,	16— 20 ,, ,,	= 10 ,,
,,	36— 40 ,, ,,	= 10 ,,
,,	56— 60 ,, ,,	= 10 ,,
,,	76— 80 ,, ,,	= 10 ,,
,,	96—100 ,, ,,	= 10 ,,

Total . . { Underwood = 50 acres
Overwood = 50 ,,

The normal state of the age classes in the case of coppice with standards is of a still more ideal character than in the case of the shelter-wood compartment system; it can only serve as a mathematical guide for the treatment of such woods. More especially, it gives some idea of the relative number of trees which should be found in each class or gradation. As each should occupy about the same area, the youngest class must contain a large number of trees, which is gradually reduced to a comparatively small number in the oldest age class. The actual proportion in these numbers depends on the species and the quality of the locality.

e. The Selection Forest.

Here the age classes are intermixed, as in the case of the overwood in coppice with standards, or even more so. The number of age classes will, theoretically, be equal to $\frac{r}{l}$.

Let A = 1,000 acres; r = 100; l = 20; then each annual cutting area $= \frac{A}{l} = \frac{1,000}{20} = 50$ acres, and the distribution would be as follows:—

Coupe No. 1 (*youngest*).

1	year	old	trees	=10	acres
21	,,	,,	,,	=10	,,
41	,,	,,	,,	=10	,,
61	,,	,,	,,	=10	,,
81	,,	,,	,,	=10	,,
			Total	=50	acres

Coupe No. 2.

2	year	old	trees	=10	acres
22	,,	,,	,,	=10	,,
42	,,	,,	,,	=10	,,
62	,,	,,	,,	=10	,,
82	,,	,,	,,	=10	,,
			Total	=50	acres

Coupe No. 19.

19	years	old	trees	=10	acres
39	,,	,,	,,	=10	,,
59	,,	,,	,,	=10	,,
79	,,	,,	,,	=10	,,
99	,,	,,	,,	=10	,,
			Total	=50	acres

Coupe No. 20 (*oldest*).

20	years	old	trees	=10	acres
40	,,	,,	,,	=10	,,
60	,,	,,	,,	=10	,,
80	,,	,,	,,	=10	,,
100	,,	,,	,,	=10	,,
			Total	=50	acres

Each year the 100 years old trees in the oldest coupe would be cut, which cover an area equal to one-fifth of the coupe, or equal to 10 acres, thus cutting once the whole area of the forest in 100 years. It is needless to add, that such regularity is never reached in practical forest management.

3. *Distribution of the Age Classes over the Forest.*

Under a normal distribution of the age classes is understood that which admits of a proper succession of cuttings, so that each wood is cut at the proper age, and that external dangers can be successfully resisted.

It has already been explained that every deviation from the normal age interferes with the full realization of the objects of management; hence the age classes should be so distributed that no such deviations are called for. The latter are generally caused by threatening dangers, such as strong winds, dry air currents, danger from frost, fire, insects, &c., sometimes by considerations for a successful regeneration.

Strong winds or gales are a most important consideration.

Their prevailing direction must be ascertained, and cuttings must proceed against it. Assuming that the strong winds generally blow from the west, the youngest age class should, at the commencement, be situated at that side, and the oldest on the east, so that the cuttings proceed gradually from east to west. (See diagram, fig. 48.)

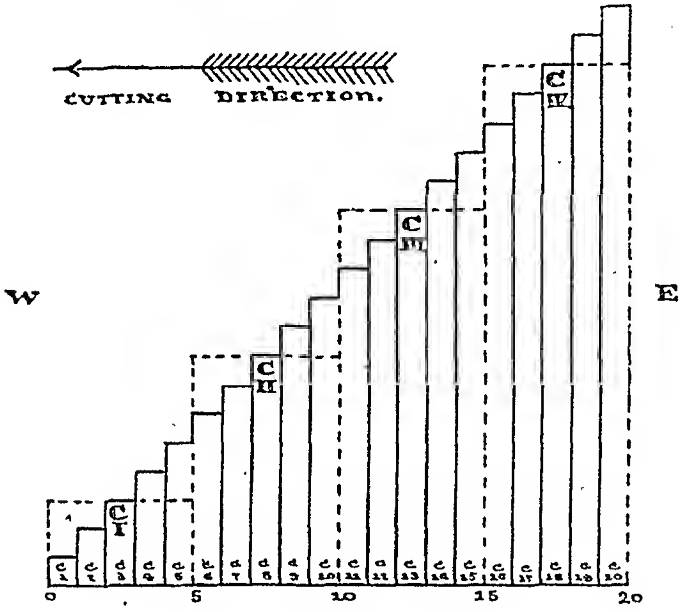


Fig. 48.

In determining the prevailing wind direction it must not be overlooked, that it is frequently changed in hilly and mountainous tracts according to the direction of the valleys and hill ranges.

Dry winds may frequently blow from a direction differing from that of strong winds; in that case the forester must decide which is the more important consideration of the two, and determine the cutting direction accordingly. Frequently the seeds of trees fall under the effect of a dry wind, so that the cleared areas, which are to be naturally regenerated, must be situated to the leeward of the seed-bearing trees.

Large clearings in one place are generally objectionable because the soil is liable to dry up, and damage by frost is

more likely to occur: hence in extensive forests cuttings must be made in several localities in each year so as to clear only a small area in one and the same locality.

Insects and fire are most injurious when several annual cuttings adjoin each other, because the former wander from one coupe to the next, while fire spreads more rapidly in young woods than if the area is interrupted by older woods.

These circumstances demand in many cases, and especially where clear cutting is practised in coniferous woods, that a second cutting should not be made in any locality until the first coupe has been successfully restocked. This leads to the splitting up of a working section or a series of age gradations, into several sub-divisions which are called "cutting series." Supposing, in a forest worked under a rotation of 20 years, it was considered necessary not to cut in the same locality more frequently than once in every 4 years, the series of age gradations would be divided into 4 cutting series, of which each would comprise 5 coupes. (Figure 49.)

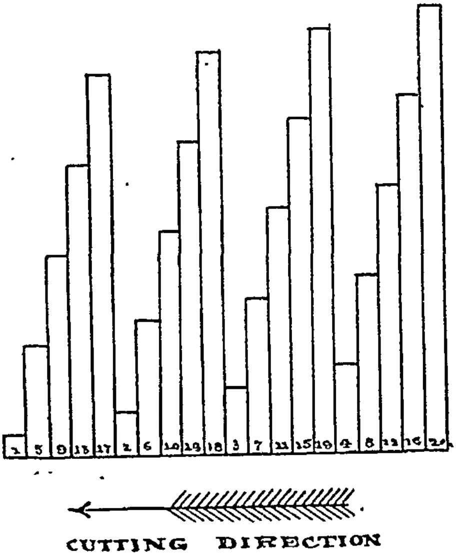


Fig. 49.

Cutting Series		
A	would comprise the coupes now old	20, 16, 12, 8, 4 years.
B	,, ,, ,, ,, ,, ,,	19, 15, 11, 7, 3 ,,
C	,, ,, ,, ,, ,, ,,	18, 14, 10, 6, 2 ,,
D	,, ,, ,, ,, ,, ,,	17, 13, 9, 5, 1 ,,

As a general rule, a careful distribution of the age classes

over the area of the forest is of special importance in the case of species which are easily thrown by wind, liable to attacks by insects, to danger from fire or frost, and also those which are difficult to regenerate naturally. In all these cases a distribution must be aimed at which allows the cutting of each wood when mature, without thereby endangering on the one hand the adjoining woods, and on the other the successful regeneration of the cleared area.

The above considerations must specially guide the forester in the case of forests worked under the systems of clear-cutting and of the shelter-wood compartment system. They are of less importance in coppice, coppice with standards, and selection forests; but even here the cutting direction should be carefully determined.

At the same time the forester should not go to extremes, as there is something to be said on both sides.

Reasons for adjoining the annual coupes are:—

(1) Best security against damage by storms.
(2) Reduction to a minimum of damage by overhanging trees.
(3) Reduction of the cost of transport of forest produce.

Reasons against adjoining the annual coupes are:—

(1) Increase of danger through fire, insects, and dry winds.
(2) Defective protection of young growth against raw winds.

The subject will again be referred to in Part IV. when dealing with the division and allotment of areas.

CHAPTER IV.

THE NORMAL GROWING STOCK.

It has been stated at page 176 that under the normal growing stock is understood that present in a forest which has a normal proportion of age classes and a normal increment. This being so, the forester need only see that the age classes and increment are normal, and the normal growing stock will be present as a natural consequence.

It happens, however, that, as far as quantity is concerned, the normal growing stock may be present, if neither the normal age classes or increment have been established; for instance, if the deficit in one age class is made good by a surplus in another. If in such a case an annually equal quantity of wood were cut, it would lead to a deviation from the normal final age, and consequently to loss. Indeed, the normal growing stock, according to quantity, might be present, if the whole forest consisted of only one uniform age class of about half the normal final age. In that case no ripe wood at all would be found in the forest, and cuttings would have to be suspended for a considerable number of years.

Under these circumstances the normal growing stock by itself is of subordinate importance in determining the yield of a forest, and yet it is useful to look at its determination for the following two reasons :—

(1) Because the yield taken out of a forest in the course of a rotation consists partly of the growing stock which was present at the beginning of the rotation, and partly of increment added to that growing stock during the rotation.

(2) Because several methods base the calculation of the yield upon the difference between the normal and real growing stock.

The amount of the normal growing stock is proportional to the length of the rotation; the higher the latter, the greater the former.

In calculating the normal growing stock only the principal part of the woods which give the final yield are taken into account, because, as previously explained, the determination of a sustained yield is, in the first place, based upon the final yield.

The normal growing stock can be looked at from the volumetric or the financial point of view.

1. *Calculation of the Normal Growing Stock as regards its Volume.*

The calculation can be made either by means of yield tables or the mean annual increment. The former is the only correct method, but the latter must be explained, as it is used by several methods of calculating the yield. The calculation differs for the several methods of treatment.

a. Clear Cutting in High Forest.

(1) *Calculation from Yield Tables.*—If a yield table is available for a forest, which gives the final yields from year to year, the normal growing stock is equal to the sum of all yields in that table from the year 1 to the year r (the rotation); that sum would represent the normal growing stock of r units of area, and for the season when the annual growth has been completed, but before the annual cuttings are made; in Europe this would be autumn.

If the yield table, and this is generally the case, gives the volumes only from period to period, say for every n years, then the approximate amount of the normal growing stock can be calculated (according to Pressler), by assuming that the

volumes rise within each period of n years according to an arithmetical series, that is to say, by adding the same amount of volume each year.

Let be volume in the year	0	$= 0$	cubic	feet,
,, ,, ,,	n	$= a$	,,	,,
,, ,, ,,	$2 \times n$	$= b$	,,	,,
,, ,, ,,	$3 \times n$	$= c$	,,	,,
,, ,, ,,	$4 \times n$	$= d$	,,	,,

then :—

Total volume from year o to year n minus $a = (0+a) \times \frac{n+1}{2} - a$

,, ,, n ,, $2 \times n$ minus $b = (a+b) \times \frac{n+1}{2} - b$

,, ,, $2 \times n$,, $3 \times n$ minus $c = (b+c) \times \frac{n+1}{2} - c$

,, ,, $3 \times n$,, $4 \times n$ $= (c+d) \times \frac{n+1}{2}$.

Total volume from o to $4 \times n$ years $=$

$$\frac{n+1}{2}\Big(0+2a+2b+2c+d\Big)-\Big(a+b+c\Big)$$

$$=\Big(n+1\Big)\Big(a+b+c+\frac{d}{2}\Big)-\Big(a+b+c\Big),$$

$$\textbf{Normal Gr. Stock, } G_n = n\left(a+b+c+\frac{d}{2}\right)+\frac{d}{2}.$$

This is the calculation for autumn.

If the oldest age gradation is cut away during winter the normal growing stock in spring must be d cubic feet less than that in autumn, that is to say :—

$$G_n \text{ in spring} = n\left(a+b+c+\frac{d}{2}\right)-\frac{d}{2}.$$

In spring the growing stock consists only of a series of gradations running from 0 to $(r-1)$ years old. In the course of the summer (the growing season), the series is brought up again to one running from 1 to r years old. If, therefore, the calculation is made for the middle of summer, it may be

assumed that one half of the annual increment has been laid on; in other words that the growing stock is then equal to the arithmetical mean of those in spring and autumn:—

$$\text{Summer's middle } G_n = n\left(a+b+c+\frac{d}{2}\right).$$

Example.—A forest of 100 acres, to which the data given in the Table at page 190 apply, worked under a rotation of 100 years, has the following normal growing stock:—

$$\text{In autumn } {}^{100}G_n = 10\ (510+1290+2140+2900+3530+4060+4530+4950+5300+2790)+2790 = 10\times 32000+2790 = 322{,}790 \text{ cubic feet.}$$

$$\text{In spring } {}^{100}G_n = 10 \times 32000 - 2790 = 317{,}210 \quad ,, \quad ,,$$

$$\text{In summer } {}^{100}G_n = 10 \times 32000 = 320{,}000 \quad ,, \quad ,,$$

The same forest, if worked under a rotation of 80 years, would, for summer, have the following growing stock:—

$${}^{80}G_n = 10\ (510+1290+2140+2900+3530+4060+4530+2475)\ \frac{100}{80}$$

$= 214{,}350 \times \dfrac{100}{80} = 267{,}937$ cubic feet, which is considerably less than if the area is worked under a rotation of 100 years.

(2) *Calculation with the Mean Annual Increment.*—Assuming that the current annual increment is the same throughout the rotation and equal to the final mean annual increment, then the volumes of all normally stocked gradations, from the youngest to that r years old, would form an ascending arithmetical series, the sum total of which would represent the normal growing stock.

Let i = volume of the 1 year old gradation, then
,, $2 \times i$ = ,, ,, 2 years ,, ,, ,,
⋮
,, $r \times i$ = ,, ,, r ,, ,, ,, ,,

and the sum of all gradation is:—

$${}^{r}G_n = (i + r \times i)\frac{r}{2} = \frac{r \times i}{2} + \frac{r \times ri}{2}.$$

Now $r \times i$ = volume of oldest age gradation and also = to increment of all gradation in one year, which may be placed = I, then :—

$$^{r}G_n = \frac{I \times r}{2} + \frac{I}{2}.\quad \text{This is for autumn.}$$

For spring there would be :—

$$^{r}G_n = \left[o + (ri - i)\right] \times \frac{r}{2} = \frac{r \times ri}{2} - \frac{ri}{2} = \frac{I \times r}{2} - \frac{I}{2}.$$

For summer, the arithmetical mean of the two :—

$$^{r}G_n = \frac{I \times r}{2}.$$

The normal growing stock is, therefore, equal to the volume of the oldest age gradation multiplied by half the number of years in the rotation, or equal to the total increment of one year multiplied by half the number of years in the rotation.

Example.—Data as above; rotation = 100 years, then :—

For autumn $G_n = \frac{5580 \times 100}{2} + \frac{5580}{2} = 281{,}790$ cubic feet.

" spring $G_n = \frac{5580 \times 100}{2} - \frac{5580}{2} = 276{,}210$ " "

" summer $G_n = \frac{5580 \times 100}{2} = 279{,}000$ " "

The forest treated under a rotation of 80 years would have :—

For summer $^{80}G_n = \frac{4950 \times 80}{2} \times \frac{100}{80} = 247{,}500$ cubic feet,

or less than above.

It will be seen that the normal growing stock calculated by the mean annual increment is smaller than that calculated from a yield table. This is, however, by no means always the case. Taking, for instance, the data in the table at page 190, and calculating the normal growing stock for various rotations, and a number of acres equal to the number of years in the

rotation in each case, the following results are obtained for spring:—

Rotation in Years.	Normal Growing Stock.		
	Calculation from Yield Table.	Calculation from Mean Annual Increment.	Excess of Calculation from Mean Annual Increment.
30	27,630	31,030	+ 3,400
40	52,450	56,550	+ 4,100
50	84,285	86,485	+ 2,200
60	121,950	119,770	– 2,180
70	164,285	156,285	– 8,000
80	211,875	195,525	– 16,350
90	262,950	235,850	– 27,100
100	317,210	276,210	– 41,000

The above example shows that the normal growing stock, calculated with the mean annual increment, is larger than that calculated from a yield table up to the age of 50 years; between 50 and 60 years the two are equal, and after that the latter is greater than the former. Under these circumstances it is evident that, except for a rotation from 50 to 60 years, a considerable error is committed in the above case by calculating the normal growing stock with the mean annual increment of the final crop.

b. Shelter-wood Compartment System.

The normal growing stock is the same as for the clear cutting system, provided the regeneration cuttings are so arranged that one half are made before the year r, and the other half after it. Strictly speaking, this is only correct if the timber in the regeneration class is removed in annually equal quantities, and if regeneration takes place in the middle of the period. In reality this does not occur, but the deviations compensate each other in the long run; anyhow a more accurate determination is practically impossible.

c. Coppice and Coppice with Standards.

The calculation for simple coppice is the same as in the case of clear cutting in high forest.

For coppice with standards forest the calculation must be made separately for under- and overwood, and the results added together. The former is of small account, as the presence of the overwood reduces the quantity of the underwood considerably.

The calculation of the normal growing stock of overwood is a complicated and uncertain operation, and at the best only of theoretical value. It must be based upon the number of trees in each age class, and the average volume per tree in each, somewhat in the following manner:—

If the normal number of trees in each of the r, $2\ r$, $3\ r$ . . . old age gradation is known, and also the volume of the average tree in each of these gradations, then it can be assumed that the trees increase, within each class, in volume according to an arithmetical series; this makes it possible to interpolate the volume of the trees $r + 1$, $2\ r + 1$ . . . years old. In that case the normal growing stock of the first age class would be expressed by—

$$\frac{r}{2}\left(V_{r+1} + V_{2r}\right),$$

where V_{r+1} represents the volume of all trees $r + 1$ years old, and V_{2r} that of all trees $2\ r$ years old. In the same way the next age class would be represented by—

$$\frac{r}{2}\left(V_{2r+1} + V_{3r}\right),$$

and so on. Adding all positions together the normal growing stock of overwood comes to:—

$$G_n = \frac{r}{2}\left(V_{r+1} + V_{2r} + V_{2r+1} + V_{3r} + \ldots + V_{(n-1)r+1} + V_{nr}\right).$$

This amount does not comprise the youngest age class of all, which still forms part of the underwood.

Example :—

Area of a coppice with standards forests = 100 acres.
Rotation of underwood = 20 years.
" " overwood = 100 "

Number of overwood classes $= \frac{100}{20} = 5$.

Area of each coupe = 5 acres.

Age of Gradation.	Number of Trees in Gradation.	Mean Volume per Tree, Cubic Feet.	Total Volume of Gradation, Cubic Feet.
21	200	·2	40·
40	200	2·0	400·
41	130	2·65	344·5
60	130	15·	1950·
61	80	15·75	1260·
80	80	30·	2400·
81	40	31·	1240·
100	40	50·	2000·

$$G_n = \frac{20}{2}\left(40 + 400 + 344{\cdot}5 + 1950 + 1260 + 2400 + 1240 + 2000\right).$$

$G_n = 10 \times 9634{\cdot}5 = 96{,}345$ cubic feet, or per acre = 963 cubic feet.

d. The Selection Forest.

The growing stock of a normal selection forest may be placed equal to that of a forest under the clear cutting system, as all the age gradations are represented in a similar way, though differently arranged over the area; hence it can be ascertained by summing up the quantities given in a yield table. At the same time the calculation is likely to be less accurate, since the younger age gradations stand under the shade of the older trees, and it is difficult to say in how far the loss of increment of the former is covered by an increased increment of the latter.

2. *Calculation of the Financial Value of the Normal Growing Stock.*

The various methods of calculating the financial value of the normal growing stock have been explained at pages 144 to 148. It has there been shown that the same results are obtained, whether the value is calculated as the cost value, expectation value, or the capitalised rental of the growing stock, provided the expectation value of the soil is introduced into the account. In each of those cases the value is expressed, for r units of area worked under a rotation of r years, by the formula :—

$$G_n = \frac{Y_r + T_a + \ldots + T_q - (c + r \times e)}{\cdot 0p} - r \times S_e;$$

in words the normal growing stock is equal to the capitalised annual net rental minus the expectation value of the soil. To make that growing stock truly normal, it is necessary that the per cent. which the capital yields, should be exactly equal to the general per cent. p. Every deviation from this leads to loss. As it is impossible to keep a forest always in that condition, it follows that the financial normal growing stock has only a theoretical value, which assists in the comprehension of the working of the capital invested in forestry, but is of little importance in determining the yield of forests.

CHAPTER V.

THE NORMAL YIELD.

UNDER the normal yield is understood that which a normal forest can permanently give. The yield may be annual, or intermittent. Instead of determining the yield for each year, or certain intermittent years, it can be ascertained for a number of years, in which case it is called the periodic yield.

The yield is composed of the final and intermediate returns. The regulation of the yield deals principally with the former, for reasons which have been explained at p. 175.

The yield of major produce is further sub-divided according to the different classes of wood, such as timber, fagots, root-wood, &c. In order to bring them into the account, all the different classes of produce are reduced to one common standard, that is, "the solid cubic foot."

The yield can be determined by area and volume, or by its financial value.

1. *The Yield determined by Area or Volume.*

a. *Clear Cutting in High Forest.*

(1) *The Normal Final Yield* is equal to the volume which stands on the oldest age gradation.

The normal cutting area is $c = \frac{A}{r}$ or $= \frac{A}{r+s}$, according as to whether the cleared area is at once re-stocked or allowed to lie fallow for s years (see p. 213). The volume standing on c must be equal to the volume of the oldest age gradation in a normal series of age gradations, if it is to give the normal yield.

The periodic normal coupe is $= \frac{A}{r} \times n$, or $= \frac{A}{r+s} \times n$.

Example:—

Area of forest	.	$=1000$ acres
Rotation	. .	$=100$ years
Annual cutting area		$=\frac{1000}{100}=10$ acres,

if the area is at once re-stocked.

The annual yield of final returns, according to the table at p. 190, amounts to $5580 + 286$ per acre $= 5866$; for ten acres $= 5866 \times 10 = 58{,}660$ solid cubic feet. Final yield during every period of 20 years $=58{,}660 \times 20=1{,}173{,}200$ cubic feet.

(2) *Intermediate Yield.*—This consists of all the thinnings which are made. Taking the same table, the following thinnings would be made in each year:—

In the coupe 30 years old	=10 acres, each giving	343 c′	=	3430
„ „ 40 „ „	=10 „ „ „	614 c′	=	6140
„ „ 50 „ „	=10 „ „ „	572 c′	=	5720
„ „ 60 „ „	=10 „ „ „	529 c′	=	5290
„ „ 70 „ „	=10 „ „ „	443 c′	=	4430
„ „ 80 „ „	=10 „ „ „	386 c′	=	3860
„ „ 90 „ „	=10 „ „ „	328 c′	=	3280
		Total	=	32,150

(3) *Total Normal Annual Yield* $=58{,}660 + 32{,}150 = 90{,}810$ cubic feet.

b. Shelter-wood Compartment System.

The calculation of the yield is the same as under the system of clear cutting, as long as the rotation r is maintained. If regeneration is commenced later than in the year $r-\frac{m}{2}$, the rotation is increased, and the calculation must be made accordingly. Supposing the first cutting is made in the year r, and the last in the year $r+m$, then the rotation $=r+\frac{m}{2}$ and the mean annual cutting area $=\dfrac{A}{r+\frac{m}{2}}$.

Example :—

Let be $m=20$; then rotation $=110$ years. Annual cutting area $=\frac{1000}{110}=9{\cdot}09$ acres. Volume standing on an acre at the age of 110 years $= 5820 + 243 = 6063$; hence annual yield $=6063 \times 9{\cdot}09=55{,}113$ solid cubic feet.

The intermediate yields would amount to $=35{,}010$ cubic feet, or :—

Total annual yield $=90{,}123$.

The raising of the rotation has led to a reduction of the yield.

c. *Selection Forest.*

If all trees which are cut in one year were brought together on a portion of the area, the latter would be $= \frac{A}{r}$: hence the yield is practically the same as in the case of clear cutting.

Another way of looking at the matter is, to determine the area on which cuttings are made in each year; this has been placed above (p. 226), $= \frac{A}{l}$. Everything which has to be cut on this area forms the normal annual yield.

Example :—

Area of a selection forest	=	1000 acres
Rotation . . .	=	100 years
l . .	=	20 years.

Then:

$$\frac{A}{l}=\frac{1000}{20}=50 \text{ acres.}$$

On these 50 acres the following material is cut :—

(1) All trees which have reached the age of 100 years.

(2) A certain proportion of trees in the younger age classes, so as to reduce their number gradually to that number which should reach maturity at the age of 100 years.

Taking the data in the table at p. 190, the material mentioned under (1) should give $= 58{,}660$ cubic feet, that is to say,

the yield under the system of clear cutting. The material under (2) represents the intermediate yields, which should amount to 32,150 c′. Total yield = 90,810 cubic feet, as before.

d. Coppice and Coppice with Standards.

The normal yield of coppice woods is calculated in the same way as for clear cutting in high forest. In this case, the annual cutting area is $= \frac{A}{r}$ and the volumetric yield is composed of the material standing on that area, plus thinnings in the younger age gradations.

In coppice with standards, the annual cutting area is the same as in simple coppice. The normal annual yield is composed of:—

(1) The underwood on the oldest age gradation, less those trees which are left to grow into standards.

(2) The contents of the oldest, R years old, age gradation of the overwood.

(3) The thinnings amongst the younger age gradations of overwood standing on the annual coupe, and occasionally in the younger underwood gradations.

Example:—

Taking the data given at p. 238, the yield in overwood is as follows:—

40 trees	(mature)	100 years old,	each	=	50 c′	=	2000
40	,,	,,	80 ,,	,,	,,	= 30 c′	=	1200
50	,,	,,	60 ,,	,,	,,	= 15 c′	=	750
70	,,	,,	40 ,,	,,	,,	= 2 c′	=	140
					Total . . .		=	4090 c′

to which the volume of the underwood has to be added.

The normal annual yield of overwood must also be equal to the annual increment laid on by all the overwood during one year, or

$$I = 2{\cdot}0 \times 200 + (15-2) \times 130 + (30-15) \times 80 + (50-30) \times 40,$$

$$I = 4090 \text{ cubic feet.}$$

2. *The Financial Value of the Normal Yield.*

The financial value of the normal yield is that which secures interest on all capital invested in a forest exactly at the rate of the general per cent. p, at which money can be obtained for forestry, or at which money taken out of the forest can be invested with equal security as in forestry. The financial yield is realised as long as a financial equilibrium on the above lines exists in the forest, that is to say, when the forest per cent. is equal to the general per cent. p. This occurs under a rotation equal to that for which the expectation value of the soil reaches its maximum.

Example:—

Taking the data in the table at p. 202, and a rotation of 80 years:—

		shlgs.
Soil expectation value for 80 units of area =		
80×250	=	20,000
Financial value of normal growing stock .	=	68,360
Total . . .	=	88,360

Financial Normal $Y = 88{,}360 \times \cdot 025 = 2209$ shillings;

or 27·61 shillings for each acre of forest.

CHAPTER VI.

RELATIONS BETWEEN INCREMENT, GROWING STOCK AND YIELD.

Between the increment, growing stock and yield of a normal forest relations exist, which are of great importance in determining the yield. In order to bring them out clearly, the system of clear cutting in high forest will be used as an illustration; it will, in the majority of cases, also be assumed that the current annual increment is equal to the final mean annual increment.

1. *Allotment of Increment during a Rotation.*

Every normal series of age gradations contains, at the commencement of the rotation, the normal growing stock. Every year the oldest age gradation is cut over, which gives the normal annual yield, and this yield is replaced during the following growing season by the laying on of the normal increment. The latter is laid on partly on the old growing stock, and removed with it during the first rotation; but partly it accumulates on the cleared areas, forming a new growing stock, which is carried over into the second rotation. The question then is, how much of the total increment of one rotation is added to the old growing stock, and how much to the new.

Making the calculation for spring, the youngest age gradation is 0 years old, and the oldest $r-1$ years. The former grows for r growing seasons, and is cut over during the last winter of the first rotation, so that all its increment is removed during the first rotation; hence, all goes to the old growing stock, and nothing to the new stock. The gradation now one year old grows for $r-1$ years during the first rotation, when it is cut over. All the increment laid on during these years goes

to the old growing stock; but that laid on during the last year is not cut, but goes over to the second rotation, and so on. This gives the following allotment, if the increment of one age gradation during a year is called $= i$:—

Allotment of Increment to—

		Old Growing Stock.	New Growing Stock.
Gradation now	0 years old $=$	$r \times i$	0
,, ,,	1 ,, ,, $=$	$(r-1)\, i$	i
,, ,,	2 ,, ,, $=$	$(r-2)\, i$	$2 \cdot i$
,, ,,	3 ,, ,, $=$	$(r-3)\, i$	$3 \cdot i$
	$\vdots$		
,, ,,	$(r-2)$,, ,, $=$	$[r-(r-2)]\, i$	$(r-2) \cdot i$
,, ,,	$(r-1)$,, ,, $=$	$[r-(r-1)]\, i$	$(r-1) \cdot i$

The terms under each growing stock form arithmetical series, and if added up, they come to the following:—

$$G_{old} = \left\{ r \cdot i + [r-(r-1)] \cdot i \right\} \frac{r}{2} = \left\{ r \cdot i + i \right\} \frac{r}{2} = \frac{r \cdot i \times r}{2} + \frac{r \cdot i}{2}.$$

$$G_{new} = \left\{ 0 + (r-1) \cdot i \right\} \frac{r}{2} = \left\{ r \cdot i - i \right\} \frac{r}{2} = \frac{r \cdot i \times r}{2} - \frac{r \cdot i}{2}.$$

Placing now $r \times i = I$, the above become:—

$$\text{Spring, } G_{old} = \frac{I \times r}{2} + \frac{I}{2},$$

$$\text{,,} \quad G_{new} = \frac{I \times r}{2} - \frac{I}{2}.$$

If the calculation is made for autumn, then the positions of the old and new growing stocks are reversed. Of the youngest age gradation, now 1 year old, only $(r-1) \times i$ goes to the old, and i to the new stock; of the second age gradation, now 2 years old; $(r-2) \times i$ goes to the old, and $2 \times i$ to the new stock, and so on. This gives—

$$\text{Autumn, } G_{old} = \frac{I \times r}{2} - \frac{I}{2},$$

$$\text{,,} \quad G_{new} = \frac{I \times r}{2} + \frac{I}{2}.$$

Making the calculation for the middle of the growing season, summer :—

$$\text{Summer, } G_{old} = \frac{I \times r}{2},$$

$$\text{,,} \quad G_{new} = \frac{I \times r}{2}.$$

The quantity I is equal to the total increment of all gradations laid on in one year, or the increment laid on by one age gradation in course of a whole rotation. Hence the following conclusions may be drawn :—

(1) The annual increment of a series of age gradations is equal to the volume of the oldest age gradation.

(2) The total increment laid on during a rotation is equal to twice the normal growing stock, calculated for the middle of summer.

(3) Calculated for the middle of summer, the total increment laid on during one rotation is equally divided between the old and new growing stock.

(4) The growing stock must increase, if less than the normal increment is removed, and *vice versâ*.

Example :—

Area of forest = 100 acres.
Rotation = 100 years.

Data those contained in the table at page 190.

Calculation made with Mean Annual Increment :—

Annual increment of one age gradation = 55·8 cubic feet.

,, ,, ,, all gradations = $55{\cdot}8 \times 100 =$ 5580

Total increment of all gradations during one rotation

$$= 5580 \times 100 = 558{,}000$$

Increment laid on by old growing stock $= \dfrac{5580 \times 100}{2} = 279{,}000$

,, ,, ,, new ,, ,, $= \dfrac{5580 \times 100}{2} = 279{,}000$

What has been said above holds also good, if the current

annual increment is substituted for the mean annual increment, provided the calculation is made for the rotation, when the normal growing stock calculated from a yield table is equal to that calculated by the mean annual increment. For all other rotations deviations occur, as the following example will show:—

For a rotation of 100 years—	Cubic feet.
Annual increment of all age gradations . .	= 5,580
Total ,, ,, ,, in 100 years	= 558,000
Total yield in 100 years	= 558,000
Normal growing stock in middle of summer, taken from yield table	= 320,000
Increment laid on by old growing stock = 558,000 − 320,000	= 238,000
,, ,, new ,, = 558,000 − 238,000	= 320,000

2. *Allotment of Increment during the Regeneration Period.*

It is of interest to know, in the case of the shelter-wood compartment system, how much of the increment laid on during the regeneration period on the area under regeneration goes to the old growing stock, and how much to the new. For this purpose it may be assumed that the old wood is removed in the course of m years by annually equal instalments, and that the cuttings are made in the commencement of the year. Total annual increment $= I$; the allotment is then as follows:—

Allotment of Increment to—

	Old Stock.	New Stock.
During the first year	$\frac{(m-1)}{m} \times I.$	$\frac{1}{m} \times I.$
,, ,, second ,,	$\frac{(m-2)}{m} \times I.$	$\frac{2}{m} \times I.$
	$\vdots$	
,, ,, n^{th} ,,	$\frac{m-m}{m} \times I = 0.$	$\frac{m}{m} \times I = I.$

Hence—

Increment on old stock=

$$\left\{\frac{m-1}{m}\times I+0\right\}\frac{m}{2}=\frac{(m-1)\times I}{2}=\frac{m\times I}{2}-\frac{I}{2}.$$

Increment on new stock=

$$\left\{\frac{1}{m}\times I+I\right\}\frac{m}{2}= \qquad =\frac{m\times I}{2}+\frac{I}{2}.$$

If the first cutting is made at the end of the first year, then the position is reversed, and—

$$\text{Increment on old stock}=\frac{m\times I}{2}+\frac{I}{2}$$

$$\text{,, ,, new ,,}=\frac{m\times I}{2}-\frac{I}{2}.$$

For the middle of the growing season—

$$\text{Increment on old stock}=\frac{m\times I}{2}$$

$$\text{,, ,, new ,,}=\frac{m\times I}{2}.$$

For all practical purposes the same result is obtained, provided m is not too long, by adding to the existing old growing stock half the increment which it would have laid on in the course of m years if no cuttings were made, in other words placing the expected yield equal to the volume of a wood, the final age of which is increased by $\frac{m}{2}$ years.

What has been said above, enables the forester to calculate the annual yield which a wood under regeneration gives during the period of m years, namely :—

$$\text{Annual yield}=\frac{G_{old}+\frac{m\times I}{2}}{m}=\frac{G_{old}}{m}+\frac{I}{2}.$$

Again, if the yield $= Y$ has been fixed, the number of years during which the wood under regeneration will give that yield, can be calculated :—

$$Y = \frac{G_{old}}{m} + \frac{I}{2}$$

and out of this—

$$Y - \frac{I}{2} = \frac{G_{old}}{m}$$

and

$$m = \frac{G_{old}}{Y - \frac{I}{2}}$$

Example:—Table at page 190.

A wood of 10 acres, now 80 years old, is to be regenerated during the next 20 years; what will be its annual yield during that period?

$$Y = \frac{49500}{20} + \frac{350}{2} = 2650 \text{ cubic feet.}$$

$$2650 \times 20 = 53{,}000, \text{ or per acre} = 5300,$$

which is the same as the wood should have had when 90 years old, if no cuttings had been made.

Again, supposing the yield has been fixed at 2,650 cubic feet a year, the 10 acres in question will furnish that yield for—

$$m = \frac{49500}{2650 - \frac{350}{2}} = 20 \text{ years.}$$

3. *Relation between Normal Yield and Normal Increment.*

The normal final yield is equal to—

(*a*) The volume of the oldest age gradation;
(*b*) The mean annual final increment of all gradations;
(*c*) The total current annual increment of all gradations.

Example:—Table at page 190.

Area of a normal forest	=	100 acres
Rotation	=	100 years.

		Cubic feet.
Every year a wood 100 years old is cut over, giving		= 5580
Normal annual yield		= 5580
Total mean annual increment	$= 55{\cdot}8 \times 100$ . .	= 5580
,, current ,, ,,	$= 10(51+78+85+76+63+53+47+42+35+28) = 558 \times 10$ . .	= 5580

4. *Relation between Normal Yield and Normal Growing Stock.*

If the normal yield (Y_n) is divided by the normal growing stock (G_n) and the quotient multiplied by 100, the result is called the "utilization per cent."

$$\text{Utilization per cent.} = \frac{Y_n}{G_n} \times 100.$$

It gives the units of yield for every 100 units of growing stock, just as the increment per cent. gives the units of increment for every 100 units of growing stock. As the increment of a whole series of age gradations is equal to the yield of the same, it follows that—

Utilization per cent. = increment per cent. of the whole series of age gradations.

Placing the current annual increment equal to the mean annual increment, and calculating G_n for summer $= \frac{r \times I}{2}$, the utilization per cent. for the rotation of the maximum volume production is always equal to twice the increment per cent. of the oldest age gradation. It has been shown above (page 187) that for the year in which the mean annual increment culminates (rotation of maximum volume production), the—

$$\text{Increment per cent.} = \frac{100}{r}.$$

For the same rotation—

$$\text{Utilization per cent.} = \frac{Y_n}{G_n} \times 100 = \frac{I \times 100}{\frac{r \times I}{2}} = \frac{200}{r}.$$

The utilization per cent. must fall with the increase of r, just as the increment per cent. has been shown to fall.

Example :—Table at page 190.

The greatest volume production occurs under a rotation of 40 years, hence :—

$$G_n = \frac{r \times I}{2} = \frac{40 \times 2900}{2} = 58{,}000 \text{ cubic feet.}$$

$$\text{Utilization per cent.} = \frac{I_n}{G_n} \times 100 = \frac{2900 \times 100}{58000} = 5\% = \frac{200}{40}.$$

$$\text{Increment per cent. of oldest age-gradation} = \frac{72 \times 100}{2900} = 2{\cdot}5\%.$$

A somewhat different result is obtained if G_n is calculated from the positions of a yield table. For summer—

$$G_n = 10\ (510 + 1290 + 2140 + 1450) = 53{,}900 \text{ cubic feet.}$$

and—

$$\text{Utilization per cent.} = \frac{2900 \times 100}{53900} = 5{\cdot}38\%, \text{ or more than } \frac{200}{40}.$$

Calculating the per cent. for various rotations the following data are obtained :—

Rotation.	Utilization per cent. By Mean Annual Increment.	Utilization per cent. From Yield Tables.
30	6·67	7·46
40	5·	5·38
50	4·	4·10
60	3·34	3·27

In this case the two are equal some time between 50 and 60 years, after which the per cent. calculated from yield tables is smaller than that calculated by the mean annual increment.

CHAPTER VII.

THE REAL FOREST COMPARED WITH THE NORMAL FOREST.

Having drawn a picture of the normal, or ideal, forest, it remains to compare it with what is found in reality. A forest which is absolutely, and in every respect, in a normal condition does not exist, especially in the case of extensive areas treated under high rotations; and if an area should ever get into that state, greater or smaller deviations are sure to occur again. The great value of the normal forest consists in its serving as a standard, towards which the forester must endeavour to lead the forest under his management. How this is done is laid down in forest working plans.

Forests which are worked for quantity or quality of produce only may be abnormal in respect of—

(1) The increment.
(2) The size and distribution of the age classes.
(3) The growing stock.

From a financial point has to be added:—

(4) There may be woods which work with a forest per cent. smaller than the general per cent. p.

Either one, more, or all these conditions may be in an abnormal state.

In determining the method by which the abnormal conditions are to be removed, it must be specially noted that the increment alone renders the growing stock an active capital; it replaces year by year that quantity of the growing stock which has been removed by fellings. Hence, it must be the forester's first care to bring the increment up to its normal amount. This is accomplished by regulating the cuttings in

a suitable manner, followed by efficient regeneration and tending of the growing woods. More especially as regards the regulation of cuttings, care must be taken that all woods which have a poor increment are cut over at an early date, and replaced by vigorous young woods. Next, a proper proportion and distribution of age classes must be aimed at, so that each wood can be cut over when ripe, without endangering thereby other adjoining woods. Only in this way is it possible to avoid loss of increment in the future, due to the premature cutting over of vigorous woods, or to the retarded cutting over of incompletely stocked or diseased woods.

The establishment of a normal proportion amongst the several age classes (or normal series of age gradations) fully insures a regular sustained yield, provided the increment is not interfered with. With these two conditions in the normal state, the third, or growing stock, must also be normal. The latter in its numerical aspect is valuable as a means to judge the capacity of a forest to yield a fixed return for a certain period of time; but it seems a procedure of doubtful expediency to begin by establishing the numerically normal state of the growing stock, because it can be reached while the forest is in other respects highly abnormal.

A forest consisting of a normal series of age gradations, and worked according to the system of a sustained annual yield, is, after all, nothing else but a number of age gradations, each of which is worked under the system of intermittent yields; by adding together the intermittent yields of the several age gradations, the sustained annual yield of the whole series is obtained. It stands, therefore, to reason that the best method of regulating the management of a forest is that which considers first the special requirements of each wood, and then adds up the cuttings which have been determined on during this process. In this way a healthy treatment can be insured to every part of the forest, leading to a healthy treatment of the whole. How this can be accomplished will be shown in Part IV. of this volume.

PART IV.

PREPARATION OF FOREST WORKING PLANS.

PREPARATION OF FOREST WORKING PLANS.

INTRODUCTORY.

A forest working plan has for its object to lay down the entire management of a forest, so that the objects for which the forest is maintained may be as fully as possible realised. In order to be of any use, it must be based upon an exact and detailed examination of the actual state of the forest in all its component parts; next, the forest must be divided into divisions of workable size; the leading principles of management must be indicated, and the yield calculated. The whole material is then brought together in a working plan report. Finally, arrangement must be made to control the execution of the plans, and to collect additional information, so that every succeeding working plan may be more accurate, and the management may become more and more exact.

The subject may, therefore, be divided into the following chapters:—

I. Examination of the Forest, or Collection of Statistics.

II. Division and Allotment of the Area.

III. Determination of the Method of Treatment and General Lines of Management.

IV. Determination of the Yield.

V. The Working Plan Report.

VI. Control of Execution and Renewal of Working Plans.

The subjects coming under I., II., and III. are not easy to

separate, because these chapters overlap to some extent. In practice they are dealt with simultaneously, more especially Chapters I. and II., but in dealing with them here they must be taken one after the other. It is not possible to put the statistics together in proper order, without having divided the forest into a number of divisions; nor is it possible to divide and allot the area to its several uses, without having previously ascertained what each part of the forest contains. Again, the division and allotment of areas cannot be finally arranged, until the method of treatment and the general lines of management have been provisionally laid down. It is for this reason that the division and allotment have been placed between the collection of statistics and the determination of the method of treatment.

At one time it was the practice to prepare working plans of high forests for long periods of time, even as much as a whole rotation. Such a procedure is to be strongly deprecated, because the conditions which govern the working of a forest change from time to time. Although the general lines of action must be determined for some time ahead, so as to secure continuity of action, the detailed prescriptions for the management should only be laid down for a short period; say 10 or perhaps 20 years. This is especially desirable where a working plan is prepared for the first time, and where the data upon which it is based are as yet defective. It is desirable, in such cases, to revise the existing arrangements in the light of the experience gained during the actual working of the forest for a limited period.

CHAPTER I.

COLLECTION OF STATISTICS.

The collection of statistics is of the first importance, because the whole fabric of the working plan rests upon the data which have been collected as regards the actual state of the forest, and the notes on the treatment which should be applied to each part. The statistics to be collected must refer, on the one hand, to each wood which forms part of the forest, and, on the other hand, to the general condition in and around the forest as a whole, which are likely to influence the management.

The data to be collected may therefore be arranged under the following heads:—

I. Survey and determination of areas.
II. Description of each wood or compartment.
III. Past receipts and expenses.
IV. General conditions in and around the forest.
V. The statistical report.

The data under II. must be collected separately for each unit of working or compartment; those under III. may be given for each compartment, or each working section, or for the whole forest, according to circumstances.

Section I.—Survey and Determination of Areas.

The survey yields the necessary data from which maps can be prepared and the area of the whole forest, as well as of its several divisions, ascertained. It is not intended to describe here the various methods of surveying, as this work must be

done by professional surveyors; the following remarks refer only to those points in which the forester must participate.

Before the survey is commenced, various preliminary matters must be attended to, such as:—

(1) Regulation and demarcation of the boundaries of the forest, and of those parts which are subject to servitudes.
(2) Demarcation of all areas which are not destined for the production of wood, such as fields, meadows, pastures, swamps, rocky parts and other areas unfit for growing woods.
(3) The laying out of a suitable system of roads and rides, in so far as it can be done without a map, or with the help of a sketch map. What cannot be done in this respect before the commencement of the survey, should, if possible, be done during its progress, that is to say, as soon as the necessary data become available. If any part cannot be done until a map becomes available, an additional survey will be necessary.
(4) Demarcation of the boundaries between woods consisting of different species, or different ages, or different quality classes. The latter is only necessary in very valuable forests.

The method of survey depends on the value of the forest, as represented by its returns; the higher the latter, the more accurate should be the survey. Generally speaking, all main lines, such as the boundaries of the property and of the areas subject to servitudes, the roads and principal rides, should be surveyed with the theodolite and chain or measuring staff. The details, such as the limits of woods and of sub-compartments, may be done with the plane table or prismatic compass.

The area of the whole forest and its main parts should be ascertained by the method of co-ordinates; the area of the

compartments or woods may be ascertained with the planimeter, or a network of squares, each of which represents a fixed area.

Whenever practical the survey should be based upon a previous triangulation.

The preparation of the maps will be dealt with in the last section of this chapter. Frequently general maps of the area are already available. If they are on a sufficiently large scale and reliable, only the additional details required for the management of the forest need be added.

SECTION II.—DESCRIPTION OF EACH WOOD OR COMPARTMENT.

The description of each wood, compartment, or other unit of working, is of the first importance, because it gives information on which depends the whole management, viz. :—

(1) The selection of species to be grown in the future.

(2) The method of treatment of each wood and the determination of the rotation.

(3) The degree of ripeness of each wood.

(4) The yield capacity of each wood and of the whole forest.

The minuteness of the investigation depends on the value of the forest and the intensity of management. Where these are high, a detailed examination and record are called for; where the returns are likely to be small, a summary procedure may be indicated. The forester must in each case determine the actual procedure which he considers to be in keeping with the interests of the owner of the forest.

1. *The Locality.*

By locality is understood the soil (and subsoil) and the climate, which depends on the situation. The agencies which are at work in the soil and the overlying air determine the yield capacity or "quality" of the locality.

The details regarding locality in relation to forest vegetation will be found in Volume I. of this Manual, pages 104—157.

From what has been said there it will be easily understood that a description of the soil and climate must form part of the basis upon which a working plan rests.

In describing the climate and soil the following points deserve attention :—

a. Climate.

(1) The geographical position of the locality, as indicated by latitude and in many cases also the longitude, especially where the vicinity of the sea, large lakes, or high mountains, are likely to influence the climate.

(2) The local peculiarities of the locality, such as altitude, aspect, slope, temperature, moisture in the air, rainfall, exposure to strong, cold, or dry winds, susceptibility to late or early frosts, &c.

(3) The surroundings of the locality, in so far as they are likely to affect the local climate.

b. Soil.

(1) The underlying rock.

(2) The mineral composition of the soil.

(3) The organic admixtures of the soil.

(4) The depth of the soil.

(5) The degree of porosity.

(6) The degree of moisture.

(7) The surface covering of the soil.

In forests situated on level ground the above data may be the same over a considerable portion or the whole of the area, but in the hills they have frequently to be determined for each compartment, or even portions of one compartment, especially if it shows considerable differences of altitude, aspect, or slope.

All these factors combined produce a certain quality or yield capacity of the locality. How this is determined has been explained at page 150 of Volume I. and in Forest Mensuration. Some further remarks on the subject will be found in the last part of this section.

2. *The Growing Stock.*

The growing wood, or the crop produced on an area, represents the results of the activity of the locality under a certain treatment. All points which have influenced the quantity and quality of the results must be ascertained, to enable the forester to judge of the merits of the treatment hitherto followed and the advisability or otherwise of any changes in it.

a. Method of Treatment, or Sylvicultural System.

The different methods of treatment have been described at p. 203 of Vol. I.

In this place the forester must ascertain the system under which the wood has actually been managed in the past.

b. Species.

Pure woods are indicated by giving the species. In the case of *mixed* woods, the degree of mixture must also be given; this can be done either by adjectives, such as "some," "a few," or by decimals, placing the whole as 1. These decimals should have reference to the area occupied by each species.

Example:—The following description—

Beech = ·5
Oak = ·3
Ash = ·2
Maple = a few,

would mean that $\frac{1}{2}$ of the area is occupied by beech, ·3 by oak, and ·2 by ash, with a few maples.

In the case of very valuable trees, such as old oak, or teak trees in Burma, it may be desirable to give their actual number. The manner of admixture is expressed as "in single trees," "in groups," "in strips," or "irregularly distributed."

It is also necessary to state whether the mixture is permanent or temporary, whether it is of special sylvicultural or

financial importance, such as a shelterwood (or nurses) over another tender species, or a soil protection wood, standards of valuable species, &c.

The undergrowth, shrubs, herbs, &c., should also be described.

c. Density of the Growing Stock.

To every method of treatment, as determined by the objects of management, corresponds a normal density of the growing stock. Deviations from that density are expressed by such terms as over-crowded, under-crowded, open, very open, interrupted, irregular, &c. Such terms are indefinite, and subject to different interpretations. It is better to place the normal density as equal to 1, and express the actual stocking in decimals of 1. The degree of density can be determined by ocular estimate, or more accurately by comparing the basal area of the stems with that of a normally stocked wood, or still more accurately by comparing the volume of the wood with that of a fully stocked wood of the same age.

When the density of stocking is insufficient, it should be stated whether the wood is generally open, or whether the deficiency is due to greater or smaller blanks.

Under a blank is understood an area which, though it belongs to the wood producing area, has no trees on it, or so few that its complete re-stocking is necessary. Areas which are not destined for the production of trees are not included here, as they form part of the areas set aside for other purposes, such as fields, meadows, &c., or are altogether unfit for the production of trees, such as bare rocks, boulder drifts, swampy ground which cannot be drained, &c. As regards the latter, it is not always easy to draw the line between actual blanks and woodland, as they frequently have a thin stocking, which may give a small return from time to time.

d. Age.

The methods of determining the age of trees and woods have been given at pp. 72 to 77.

An absolutely accurate determination of the age is only necessary when the data are required for the preparation of yield tables or other scientific purposes. Fairly approximate data suffice for the purpose of working plans.

In the case of even-aged or nearly even-aged woods, one or more sample trees are examined.

If considerable differences of age exist in a wood, the limits should be given, and the wood placed into that age class to which it belongs according to its economic character. If some older or younger groups exist, which are not of sufficient extent to be classed as separate woods, this should be mentioned. The same holds good for a limited number of standards which are to be held over for a second rotation, or for young growth which has sprung up in an old wood.

A minute calculation of the mean age is rarely called for.

In the case of woods which have been kept back in their development, the economic and not the actual age must be given. For instance, a young wood, which has stood under heavy shelter and is now 30 years old, but of a development which is ordinarily reached in 10 years, must be entered as 10 years and not as 30 years old.

In the regeneration class the age of the overwood and underwood must be given separately.

In selection forests it suffices to give the limits of the age gradations, which are frequently determined by the number of years during which cuttings go once round the forest.

In coppice with standards the ages of the overwood and underwood are given separately; for the former the limits of the existing gradations are given.

The age of coppice can generally be easily ascertained from the time when the last cutting occurred.

c. *Origin and Past Treatment.*

Whenever the necessary data can be ascertained, a short history of each wood should be prepared, giving the method of formation, whether by natural or artificial means, planting

or sowing, the manner in which the wood has been tended, cleanings, thinnings, pruning, natural phenomena which have affected the development, etc. Such a history is very useful in judging the results of the past method of treatment, and in determining the future treatment.

f. Volume.

All methods of determining the yield in material require a measurement of the volume, but to a different extent. For some it is necessary to measure all woods, excepting only those which are very young, and which are estimated, either direct, or with the assistance of yield tables. For other methods only those woods require to be measured, which will come under the axe during the immediate future of, say, 10 to 20 years.

Where a fine financial management is followed, all woods which are close to ripeness, or of which the ripeness is doubtful, must be accurately measured, so as to calculate the per cent. with which the capital is working.

For the determination of the capital value an accurate measurement of the volume is indispensable.

The volume should be given separately for the different species, if their value per unit of measurement differs considerably. It is useful to give all volumes in solid measure, as solid cubic feet. The proportion between the different classes of produce need only be given for each working section; best, according to local proportionate figures, if such are available.

The different methods, according to which the volume can be measured, have been described at pages 43 to 71. The choice of the method of measurement depends on the circumstances of each case.

g. Increment, Capital Value, and Forest Per cent.

These matters have already been dealt with in full detail.

The determination of the quantity increment is required for the calculation of the yield. It must be done for all woods,

if the yield is fixed for a whole rotation, or when the increment forms the principal basis for the determination of the yield. In the latter case both normal and real increment must be ascertained. When the yield is fixed for only one, or at the outside two periods, the current increment must be ascertained for that number of years, or the mean annual increment of the past is substituted for it.

For financial questions the quantity-, quality-, and price-increment must be determined, as well as the capital invested in the forest, so as to calculate the indicating or forest per cent. The latter is necessary only for woods, the financial ripeness of which is doubtful, that is to say, for woods which are approaching the normal final age, and woods which have suffered by injurious agencies, such as wind, snow, fire, insects, damage by game, etc.

3. *Determination of the Quality of each Wood.*

a. General.

Under the quality of a wood or compartment is understood its yield capacity, as expressed by the quantity of produce which can be derived from it.

The yield capacity depends in the first place on the locality; but injurious influences may have interfered with the full development of the producing factors of the locality, so that abnormal conditions may be the consequence. The forester distinguishes therefore between normal and abnormal quality. The quality is normal, if no extraordinary injurious influences have affected the development of the wood.

A further distinction must be made between the quality of the "locality" and of the "growing wood" or standing crop. Either of the two can be normal or abnormal. The quality of the locality may be abnormal in consequence of a variety of causes, such as the continued removal of litter, or excessive exposure to the effects of sun and air currents which have impoverished the soil; or in consequence of unfavourable

natural phenomena, for instance, if the ground has become swampy, temporarily denuded, or covered with moving sand.

An abnormal condition of the growing wood may be produced by faulty treatment, by injurious external agencies, such as drought, frost, wind, fire, insects, diseases of the trees, etc.

For the preparation of working plans, only the actually existing, or real, quality of the locality should be taken into account, because the restoration of the normal quality is generally a slow process, if it is at all effected. As regards the growing stock both values are required, because the normal quality represents the real quality of the locality, and the real quality of the growing stock forms the basis for the calculation of the yield which the forest can give.

On page 150 of Volume I. it has been said that the quality of the locality can be ascertained—

(1) By an assessment according to the several factors of the locality; or

(2) By an assessment according to a crop of trees produced on the area in question, or on a similar soil in the vicinity.

It has also been stated that the first of these two methods, however carefully carried out, is always subject to grave errors, because an examination of the chemical composition and the physical properties of the soil, and a determination of the climate do not indicate the yield capacity of the locality for forestry with any degree of certainty; hence it should be used only as an auxiliary of the second method, or when the latter is not available.

Thus it will be seen that the determination of the quality of the locality depends practically on an examination of the growing wood which it has produced. In fact, a normal growing stock is the true expression for the real quality of the locality; the same investigation gives both the quality of the locality and of the existing crop.

For the purpose of obtaining an actual figure, which repre-

sents the quality, the best way is to ascertain the volume of the growing stock and the number of years in which it has been produced. In dividing the volume by the age of the wood, the mean annual increment is obtained. Both volume and mean annual increment depend on the locality and the past treatment of the wood.

It is evident that in reality a multitude of different qualities exist, but for practical work they are grouped into a few, generally not more than five quality classes, which are numbered I. to V. Of these I. should represent the lowest and V. the best quality, but unfortunately the reverse numbering has been largely introduced. A still more convenient way is to represent the best quality by 1 and the others in decimals of 1. Each of these quality classes represents a distinct yield capacity, which differs with the species and method of treatment.

The quality can be determined with the help of yield tables, or the final mean annual increment.

b. Determination of the Quality with the help of Yield Tables.

The preparation of yield tables has been explained in Forest Mensuration (page 96). Such tables represent the progress of increment, or volume, throughout life for each quality class; hence, assessing the quality means, in this case, the selection of the proper yield table. The difficulty is that for every species and sylvicultural system a different set of yield tables is required. It may even be desirable to have different sets for different localities, so-called local yield tables; but such a procedure is likely to lead to confusion, as different standards of the quality classes are introduced into the account. Hence, general yield tables are to be preferred, even if the same degree of accuracy is not obtained as in the case of local tables. The difference is, however, not considerable, as experience has shown that, within reasonable limits, general tables give sufficiently accurate data for the preparation of working plans.

It has, for instance, been proved that the general yield tables

for the Scotch pine prepared for Germany may safely be used in the south of England. The fact is, that the sources of inaccuracy inherent to the best methods of measuring the volume of a standing crop are greater than those caused by using general yield tables for any particular locality.

The general yield tables given at page 386, Appendix D, are used in Saxony for the determination of the quality class.

The quality of young woods cannot be judged by their volume, since the factors of the locality may not yet have found full expression in the volume; here the quality must be estimated by the general condition of the crop, and especially its height growth. Indeed, the latter may be used even in older woods, as long as height growth has not ceased.

The determination of the quality from yield tables in the case of clear cutting in high forest and in coppice is a simple matter, as previously shown. The regeneration area under the shelter-wood system gives some trouble, because it is no longer fully stocked, so that the volume does not represent the quality; here the determination must be based upon an investigation of the quality of the locality combined with the condition of the shelter-wood and young growth. A similar procedure is followed in the case of coppice with standards, and in selection forests. The quality of blanks is estimated from the soil and climate, or from that of adjoining woods which have been produced on soil of a similar description.

c. Determination of the Quality by the Final Mean Annual Increment.

This method has sometimes advantages, especially if the areas are to be reduced to one common quality; but it has the great disadvantage that the final age of each wood must be fixed, since the mean annual increment changes with the age at which the wood is cut over. Moreover it is almost impossible to fix the final mean annual increment without the help of yield tables.

d. Reduction to One Quality.

Several methods of regulating the yield demand a reduction of the areas of the several woods, or working sections, to one quality, so as to have to calculate only with areas of equal yield capacity.

Such a reduction to one quality may be made as regards the locality, or the growing wood; in each case as regards the normal or real quality. The method of procedure is the same in each case.

The calculation becomes most simple; *either* if yield tables are used, in which the yields of the several classes are indicated the best by 1 and the others in decimals of 1; *or*, if the reduction is made with the final mean annual increment or yield.

Again, the reduction can be made under one of the two following conditions:—*Either* the total of the several reduced areas shall be equal to the actual area of the working section; in other words, the calculation is made with the mean quality of the area; *or*, the above equality is not required, in which case any quality can be used as the standard, frequently that being chosen which exists over the greater part of the area.

i. Reduction based upon the Final Mean Annual Increment.

(*a*) *Calculation with the Mean Quality.*—Under mean quality is understood that which, if it existed throughout the working section, would produce the same total yield as that produced by the several existing qualities in different parts of the working section.

Let a_1, a_2, a_3 . . . be the several areas,
,, y_1, y_2, y_3 . . . the corresponding annual yields per unit of area,
,, Y . . . . . . the mean yield per unit of area, then

$$a_1 \times y_1 + a_2 \times y_2 + a_3 \times y_3 + \ldots = a_1 Y + a_2 Y + a_3 Y + \ldots$$
$$= Y\,(a_1 + a_2 + a_3 + \ldots),$$

and

$$Y = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3 + \ldots}{a_1 + a_2 + a_3 + \ldots} = \frac{\text{total annual yield}}{\text{total area}}.$$

Example :—

A working section of 1000 acres contains :—

Block	(1)	200 acres	with	60 c'	average	increment	
,,	(2)	100 ,,	,,	50 ,,	,,	,,	
,,	(3)	200 ,,	,,	40 ,,	,,	,,	
,,	(4)	500 ,,	,,	30 ,,	,,	,,	then—

Mean quality

$$Y = \frac{200 \times 60 + 100 \times 50 + 200 \times 40 + 500 \times 30}{1000} = 40\ c'.$$

By reduced area is now understood that which would produce, with a uniform quality $= Y$, the same total yield as the actually existing areas with their varying qualities. It is obtained by applying, in each case, the inverse proportion of that which exists between the actual and the mean quality :

$$Y : y = a : x \text{ and } x = \frac{a \times y}{Y}.$$

In the above example :

Block (1) The proportion is 60 : 40 ; hence the reduced area x is obtained by means of the equation—

$$40 : 60 = 200 : x \text{ and } x = \frac{200 \times 60}{40} = 300 \text{ acres}$$

$$,,\quad (2)\ 40 : 50 = 100 : x \quad ,, \quad x = \frac{100 \times 50}{40} = 125 \quad ,,$$

$$,,\quad (3)\ 40 : 40 = 200 : x \quad ,, \quad x = \frac{40 \times 200}{40} = 200 \quad ,,$$

$$,,\quad (4)\ 40 : 30 = 500 : x \quad ,, \quad x = \frac{30 \times 500}{40} = 375 \quad ,,$$

Total . . . = 1000 acres.

If now the forest is to be divided into annual coupes of equal

yield capacity, the area to be placed in each is also obtained by calculating with the inverse proportion of the qualities.

Example:—The above forest shall be divided into ten coupes of equal yield capacity; then the reduced area of each coupe comes to $= \frac{1,000}{10} = 100$ acres. The real area of a coupe in each block is calculated as follows:—

$$\text{Block (1) } 60 : 40 = 100 : x \text{ and } x = \frac{40 \times 100}{60} = 66{\cdot}67 \text{ acres}$$

$$\text{,, (2) } 50 : 40 = 100 : x \text{ ,, } x = \frac{40 \times 100}{50} = 80{\cdot}00 \text{ ,,}$$

$$\text{,, (3) } 40 : 40 = 100 : x \text{ ,, } x = \frac{40 \times 100}{40} = 100{\cdot}00 \text{ ,,}$$

$$\text{,, (4) } 30 : 40 = 100 : x \text{ ,, } x = \frac{40 \times 100}{30} = 133{\cdot}33 \text{ ,,}$$

or:

Coupe No. 1 =	66·67 =	66·67 acres	Taken from block No. 1.
,, ,, 2 =	66·67 =	66·67 ,,	
,, ,, 3 =	66·66 =	66·66 ,,	
,, ,, 4 =	80·00 =	80·00 ,,	From block No. 2.
,, ,, 5 =	20 + 75 =	95·00 ,,	Partly from No. 2 and partly No. 3.
,, ,, 6 =	100 =	100 ,,	From block No. 3.
,, ,, 7 =	25 + 100 =	125 ,,	Partly from No. 3 and partly No. 4.
,, ,, 8 =	133·33 =	133·33 ,,	From block No. 4.
,, ,, 9 =	133·33 =	133·33 ,,	
,, ,, 10 =	133·34 =	133·34 ,,	
Total . .		1000·00 acres.	

(*b*) *Calculation with any Suitable Quality.*—In this case any quality can be used, whether it exists on the area or not.

The total reduced area is obtained by multiplying the several qualities by the corresponding areas and dividing the product by the selected standard quality. The total reduced area may

be greater, equal or smaller than the actual area, according to the size of the standard quality:—

$$\text{Reduced } A = \frac{a_1 \times y_1 + a_2 \times y_2 + a_3 \times y_3 + \ldots}{Y'}.$$

The reduced areas of the several parts are obtained by the inverse proportion of their qualities to the standard quality; thus:—

$$\text{Reduced } a_1 = \frac{a_1 \times y_1}{Y'},$$

$$\text{Reduced } a_2 = \frac{a_2 \times y_2}{Y'},$$

etc.

Example, as above: Let the standard quality $= 50\ c'$, then Total reduced area $=$

$$\frac{200 \times 60 + 100 \times 50 + 200 \times 40 + 500 \times 30}{50} = 800 \text{ acres,}$$

and for the several blocks:—

$$(1)\ \frac{200 \times 60}{50} = 240 \text{ acres}$$

$$(2)\ \frac{100 \times 50}{50} = 100 \quad ,,$$

$$(3)\ \frac{200 \times 40}{50} = 160 \quad ,,$$

$$(4)\ \frac{500 \times 30}{50} = 300 \quad ,,$$

Total . . 800 acres.

$$\text{Reduced area of annual coupe} = \frac{800}{10} = 80 \text{ acres,}$$

and the size of coupes in the several blocks:—

$$(1)\ 60 : 50 = 80 : x \text{ and } x = \frac{50 \times 80}{60} = 66{\cdot}67$$

$$(2)\ 50 : 50 = 80 : x \quad ,, \quad x = \frac{50 \times 80}{50} = 80{\cdot}00$$

$$(3)\ 40 : 50 = 80 : x \text{ and } x = \frac{50 \times 80}{40} = 100 \cdot$$

$$(4)\ 30 : 50 = 80 : x \quad ,, \quad x = \frac{50 \times 80}{30} = 133 \cdot 33\,;$$

as before.

It is obvious, that the last mentioned method is the more convenient of the two.

ii. Reduction based upon Yield Tables.

The reduction is based upon yield tables, of which the best is indicated by 1 and the others by decimals of 1.

(*a*) *Calculation with the Mean Quality.*—The mean quality is obtained by multiplying the several areas by their quality figures, and dividing the product by the total area.

The reduced area of each part is obtained by calculating it with the inverse proportion which exists between its own and the mean quality.

Example: as above, but let—

Quality of Block No. 1 = ·6
,, ,, ,, ,, 2 = ·5
,, ,, ,, ,, 3 = ·4
,, ,, ,, ,, 4 = ·3.

$$\text{Mean quality} = \frac{200 \times \cdot 6 + 100 \times \cdot 5 + 200 \times \cdot 4 + 500 \times \cdot 3}{1000} = \cdot 4.$$

Reduced areas of the several blocks:—

$$(1)\ \cdot 4 : \cdot 6 = 200 : x \text{ and } x = \frac{200 \times \cdot 6}{\cdot 4} = 300 \text{ acres}$$

$$(2)\ \cdot 4 : \cdot 5 = 100 : x \quad ,, \quad x = \frac{100 \times \cdot 5}{\cdot 4} = 125 \quad ,,$$

$$(3)\ \cdot 4 : \cdot 4 = 200 : x \quad ,, \quad x = \frac{200 \times \cdot 4}{\cdot 4} = 200 \quad ,,$$

$$(4)\ \cdot 4 : \cdot 3 = 500 : x \quad ,, \quad x = \frac{500 \times \cdot 3}{\cdot 4} = 375 \quad ,,$$

Total . . = 1000 acres

The reduced annual coupe would be $=\frac{1000}{10}=100$ acres.

Area of a coupe in each block :—

(1) $\cdot 6 : \cdot 4 = 100 : x$ and $x=\frac{100 : \cdot 4}{\cdot 6}=$ 66·67 acres.

(2) $\cdot 5 : \cdot 4 = 100 : x$,, $x=\frac{100 \times \cdot 4}{\cdot 5}=$ 80 ,,

(3) $\cdot 4 : \cdot 4 = 100 : x$,, $x=\frac{100 \times \cdot 4}{\cdot 4}=100$,,

(4) $\cdot 3 : \cdot 4 = 100 : x$,, $x=\frac{100 \times \cdot 4}{\cdot 3}=133\cdot 33$,,

as before.

(*b*) *Calculation with any Standard Quality.*—In this case the several areas are multiplied by their respective quality figures, and the sum of these products represents the reduced area; in other words the standard quality is placed equal to 1.

Example: as above :—

(1) $200 \times \cdot 6 = 120$ acres
(2) $100 \times \cdot 5 = 50$,,
(3) $200 \times \cdot 4 = 80$,,
(4) $500 \times \cdot 3 = 150$,,

Total reduced area $=400$ acres.

Reduced area of annual coupe $=\frac{400}{10}=40$ acres.

Size of coupe in each block :—

(1) $\cdot 6 : \cdot 1 = 40 : x$ and $x=\frac{40}{\cdot 6}=$ 66·67 acres.

(2) $\cdot 5 : \cdot 1 = 40 : x$,, $x=\frac{40}{\cdot 5}=$ 80 ,,

(3) $\cdot 4 : \cdot 1 = 40 : x$,, $x=\frac{40}{\cdot 4}=100$,,

(4) $\cdot 3 : \cdot 1 = 40 : x$,, $x=\frac{40}{\cdot 3}=133\cdot 33$,,

as before.

Under this method the size of the annual coupes in the several blocks is obtained by dividing the reduced area of the annual coupe, by the real quality figure of the wood, a method which is the simplest of all.

4. *Notes regarding Future Treatment.*

While drawing up a description of each wood it is very desirable to note down any observations which may strike the forester regarding the future treatment. Such notes are, of course, only of a preliminary nature, because a final decision on the future treatment to be followed can be arrived at only after the management of the whole forest, or working section, has been laid down. Nevertheless they are a great help during the progress of the work.

It is not possible to give a complete list of the points which should be attended to, as they differ according to circumstances; the following may, however, be enumerated:—

(*a*) Filling up the existing wood; if so, the area to be treated and the species to be grown should be given; also the method of sowing, planting, or other cultural operations.

(*b*) Cleanings, thinnings, or prunings during the period for which the working plan is prepared; the volume to be removed should be estimated.

(*c*) Degree of ripeness of the principal or final crop, taking into consideration the objects of management; if the latter are financial the forest per cent. should be calculated. If it appears advisable that final cuttings should be made, the method of cutting should be given and an estimate of the volume to be removed.

(*d*) Method of regeneration to be followed and the species to be grown, if this should occur during the period for which the working plan is prepared.

(*e*) Measures to be taken for the protection of the wood against threatening dangers, especially fire.

(*f*) Other works to be undertaken, such as construction of roads, draining, irrigation.

(*g*) Utilization of enclosures and improvement of boundaries where practicable.

(*h*) Proposals regarding the formation of sub-compartments, or the abolishment of those which exist, with reasons for such proposals.

Section III.—Past Receipts and Expenses.

There is no surer basis in estimating future returns than those of the past; hence it is of importance to ascertain and note down the yield in material, the cash receipts and costs for as many years as the available data admit. These data will, however, only be forthcoming if records have been kept for some time past.

As far as may be practicable, past yields and costs should be given for each unit of working, that is to say each wood or compartment. If the records have not been kept in sufficient detail, the data for each working section should be given; the latter may also be quite sufficient where the management is as yet in a backward condition, or where the receipts are as yet small.

The following notes indicate the class of information which may be required:—

1. *Yield of Wood or Major Produce.*

The yields should be given separately:—

(*a*) For the principal species.

(*b*) For the different classes of timber and firewood, according to size or value.

(*c*) For final and intermediate yields.

(*d*) Of cash receipts should be given the total, and the mean price of the several classes of material, separated according to species.

(*e*) The areas over which cuttings extended should, if

possible, also be given separately for final and intermediate cuttings.

2. *Minor Produce.*

Under this heading should be given the quantity of each article of minor produce which has been removed and the cash receipts obtained for it.

Receipts derived from areas not used for the production of wood, such as fields, meadows, &c., should be separately recorded.

3. *Expenses.*

These should be recorded separately for :—

(*a*) Cost of administration and protection.

(*b*) Taxes, rates, &c.

(*c*) Formation of woods.

(*d*) Tending and amelioration.

(*e*) Maintenance of boundaries.

(*f*) Construction of roads, drainage, irrigation, and other works.

(*g*) Cost of harvesting, separated according to major and minor produce.

4. *Generally.*

For forests worked on financial lines the receipts and expenses should be so arranged that it is possible to ascertain:—

(*a*) The current forest per cent. of each wood whenever desirable.

(*b*) The forest rental, being the difference between all receipts and expenses. It is used to determine the per centage which the forest capital has yielded.

(*c*) The capital value of the forest, being the sum of the value of the soil plus value of the growing stock. For the preparation of working plans based upon financial principles, these values may be ascertained as follows : the data of receipts and costs collected

for the several woods will enable the forester to calculate a series of expectation values of the soil for the different qualities of locality; besides, all available data referring to actual sales of land similar to the forest lands in question should be carefully noted. By combining these data it will be possible to determine the value of the soil with sufficient accuracy for the purpose of working plans. The capital value of the growing stock is then determined as the cost value calculated with the value of the soil as determined above, or as the utilization value in the case of woods which are near maturity.

Section IV.—General Conditions in and around the Forest.

The management of a forest depends not only on the state of its several parts, but also on the general conditions which exist in and around it. The latter must, therefore, be ascertained at this early stage, and they should be used for a general description of the forest, to be incorporated into the working plan report.

The field of enquiry here indicated is of considerable extent; the following matters may be mentioned:—

1. Name and situation of forest, giving the latitude and longitude where necessary.
2. Description of boundaries and names of the adjoining properties and their owners.
3. Topographical features of the locality.
4. General description of the geology and climate.
5. Former and present proprietors, financial position of present proprietor, whether the funds for formation, tending, administration, amelioration, &c., are available; whether specially heavy cuttings must be made to meet the demands of the proprietor.
6. Nature of proprietorship, whether full and unfettered

property, or whether servitudes and privileges rest on it; in the latter case their extent should be recorded.

7. Rights enjoyed by the proprietor of the forest elsewhere, such as rights of way or floating, or rights over other lands, &c.
8. Requirements of the surrounding population, and condition of the market for forest produce generally; special industries in the vicinity which require forest produce, such as mines, smelting works, saw mills; imports which compete with the local supply; substitutes for wood available in the vicinity.
9. Extent of forest offences, their causes, effect upon the forest; suggestions for their prevention.
10. Labour available in the vicinity; rate of wages.
11. Past system of management; changes introduced from time to time; prescriptions of former working plans and their effect upon the forest.
12. Natural phenomena which have affected the condition of the forest, such as storms, snow, frost, fire, insects, fungi, &c.
13. Conditions of game and its effect upon the forest.
14. Past seed years of the more important species.
15. Opportunities for consolidating the property, either by exchange or purchase; conversion of fields, meadows, &c., into forest, or the reverse.
16. The staff of the forest, its organization and efficiency.

Section V.—The Statistical Report.

The data which have been collected in the manner indicated in the previous four sections, must be brought together in a statistical report, accompanied by maps to illustrate it. The form of this report depends entirely on the circumstances of each case. In one instance it will be necessary to go into minute details, in another a more summary treatment is indicated. The following documents will ordinarily form part of the report.

1. *Register of Boundaries.*

This should give :—

(*a*) The boundary marks in consecutive numbers.
(*b*) The angles backwards and forwards at each point.
(*c*) The horizontal distance between every two boundary marks.
(*d*) The nature of the boundary line, whether a road, water-course, waterparting, ditch, cleared line, etc.
(*e*) The names of adjoining properties and of their owners.

The value of the register of boundaries is considerably enhanced, if its correctness has been acknowledged by the adjoining owners before the proper court of law.

DESCRIPTION OF

LOCALITY.			AREA, IN ACRES.			Boundaries.	Locality.
Working Section or Block.	Compartment.	Sub-compartment.	Stocked	Blank.	Total		
Cæsar's Camp	1	*a*	24	2	26	*North & East:* Sir J. Hayter's land. *South:* Compartments 13 and 12. *West:* Roman road.	*Elevation:* 420 feet above sea-level, sloping towards the east, with moderate gradient, down to 380 feet. *Geological Formation:* Middle Bagshot sands. *Soil:* Loamy sand, fairly good in upper part and good in lower part; no pan to 4 feet depth.

2. *Table of Areas.*

The following form of this table is given as an illustration; it serves as a summary of all areas, and shows how each part is utilized:—

Locality.			Area used for the Production of Wood, in Acres.	Area not used for the Production of Wood, in Acres.					Grand Total, in Acres.
Working Section or Block.	Compartment.	Sub-compartment.		Roads and Rides.	Fields.	Meadows	Water, etc.	Total.	
Cæsar's Camp . .	1	*a*	26	·4	2·2	—	·6	3·2	29·2
		Etc.							

Remarks.—The use of the fields forms part of the forester's emoluments.

Compartments. (See next page.)

Growing Stock.					Quality of		Remarks, and Notes regarding Future Treatment.
Sylvicultural System.	Species.	Age, in Years.	Mean Height in Feet.	Volume in solid Cubic Feet.	Locality.	Growing Stock.	
High Forest	Oak = ·4 Chestnut = ·2 Beech = ·1 Scotch pine = ·3	70	Oak, Chestnut & Beech = 54 Scotch pine = 60	Broad-leaved = 34,000 Scotch pine = 28,800 62,800	3	·8	The mixture of species is uneven; the Scotch pine is chiefly found in the Northern part, and the broad-leaved trees in the South; Northern part is well stocked, Southern part open; most of the oaks and chestnuts are frost-cracked. *Future Treatment:* Southern part is so increment-poor, that it should be cut over during the next ten years, leaving the best oaks and chestnuts, and underplanted or sown, with beech (or spruce). In the Northern part only dead or dying trees should be removed during the next ten years.

3. *Description of Compartments.*

This description may be drawn up in a tabular form, or otherwise; the former is preferable, as it presents a more intelligible picture of the forest, and gives greater security that nothing has been overlooked. It is quite impossible to recommend any particular form for this table, but by way of illustration a form based upon those recommended by Judeich and Heyer is given. (See form on previous pages.)

In this table the quality of locality indicates that which corresponds to the normal quality of the growing stock. The real quality of the growing stock is given in decimals, the normal quality being placed equal to 1.

If the forest is worked on financial principles, further columns must be added for the quantity and quality increment, wherewith to calculate the forest per cent. and the forest capital.

4. *Table of Qualities of Locality.*

Locality.			Species and Sylvicultural System.	Quality Classes of Locality. Area in Acres.					Total Area.	Remarks.
Working Section.	Compartment.	Sub-compartment.		I. Lowest.	II.	III.	IV.	V. Best.		
Cæsar's Camp	1	*a*	Scotch pine with some broad-leaved trees here and there.			26			26	
"	1	*b*					14		14	
"	2				32		6		38	
"	3		High forest	17					17	
"	4					16		15	31	
"	5			12	17		4		33	
"	6					25	6		31	
"	7						20	7	27	
	Etc.									
			Total......	29	49	67	50	22	217	

It is always useful to prepare this table, as it enables the forester to calculate the total yield capacity of the area. In this table each working section must be recorded separately, as the yield capacity depends on the species, sylvicultural system and rotation.

The Saxon Tables (p. 388, App. D) give the following mean production per acre for Scotch pine woods, under a rotation of 80 years:—

For the V. Class	(lowest)	= 19	cubic feet.
IV. ,,		= 44	,,
III. ,,		= 70	,,
II. ,,		= 96	,,
I. ,,	(best)	= 122	,,

The mean annual increment, or the yield capacity, of the area shown in the above table, would therefore be:—

$$\text{Yield capacity} = 29 \times 19 + 49 \times 44 + 67 \times 70 + 50 \times 96 + 22 \times 122 = 14{,}881 \text{ cubic feet,}$$

$$\text{or average yield capacity per acre} = \frac{14881}{217} = 69 \text{ cubic feet.}$$

This figure represents the normal yield; the real, or actual, yield depends on the quality of the growing stock and the ages of the several woods. Assuming that the age classes are represented in normal proportion, and that the mean quality of all woods was equal to ·7, the actual yield would be equal to

$$14{,}881 \times \cdot 7 = 10{,}417 \text{ cubic feet,}$$

or 48 cubic feet per acre and year.

5. *Table of Age Classes.*

This table is of great importance, as it gives a correct idea of the proportion of the different age classes, which affects the determination of the yield in the immediate future. It may be prepared in the following form:—

Table of Age Classes.

Locality.			Present Mean Age.	Age Classes, in Acres.						
Working Section.	Compartment.	Sub-compartment.		I. 1—20.	II. 21—40	III. 41—60	IV. 61—80	V. Over 80.	Blanks.	Regeneration Class.
Cæsar's Camp	1	*a*	70				24		2	
	1	*b*	35		14					
	2		95					32	6	
	3		54			17				
	4		16	31						
	5		46			33				
	6		24		31					
	7		86					27		
			Total...	31	45	50	24	59	8	—

Table of

Material Cut in Past Years,

Year.	Conifers.									Broad-		
	Final.			Intermediate.			Total.			Final.		
	Timber	Firewood.	Total.	Timber	Firewood.	Total.	Timber.	Firewood.	Total.	Timber.	Fire-wo'd	Total.
1881	7,200	1,750	8.950	4,700	1,300	6,000	11,900	3,050	14,950	—	—	—
1882												
.												
.												
.												
1890												
Total in 10 Years ..	77,600	16,400	94,000	36,400	12,200	48,600	114,000	28,600	142,600	2750	2600	5350
Annual Average ..	7,760	1,640	9,400	3,640	1,220	4,860	11,400	2,860	14,260	275	260	535

Remarks.—The area set aside for the production of wood amounted, in the beginning of 1881, to 217 acres.

6. *Table of Past Yields.*

This table should give the past yields in produce for as many years as possible, and the mean annual yield calculated from these data.

The data should be separated, where practicable, according to final and intermediate returns, and according to the principal species, and the different classes of produce.

The accompanying table illustrates this.

Where specially valuable timber has been cut, like oak standards, teak, etc., it can be entered separately.

7. *Maps.*

It is most useful to represent on maps the data required for the preparation of a working plan as far as this can be done. Such maps give at a glance a clear picture of the forest, which

PAST YIELDS.
in solid Cubic Feet.

Leaved Species.						Total.								
Intermediate.			Total.			Final.			Intermediate.			Total.		
Timber.	Fire-wo'd	To-tal.	Timber.	Fire-wo'd	To-tal.	Timber	Fire-wood.	Total.	Timber	Fire-wood.	Total.	Timber.	Fire-wood.	Total.
750	1650	2400	750	1650	2400	7,200	1,750	8,950	5,450	2,950	8,400	12,650	4,700	17,350
1350	2100	3450	4100	4700	8800	80,350	19,000	99,350	37,750	14,300	52,050	118,100	33,300	151,400
135	210	345	410	470	880	8,035	1,900	9,935	3,775	1,430	5,205	11,810	3,330	15,140

The annual yield was fixed at 15,000 solid cubic feet: or 150,000 for the period of 10 years; hence the average cuttings exceeded the fixed yield by 140 cubic feet annually.

impresses itself more readily on the mind of the forester than a lengthy description. As it is not possible to represent everything on one map, it is usual to prepare different sets, such as the—

(*a*) Topographical map.
(*b*) Detailed map on a large scale.
(*c*) Map showing the nature and age of the growing woods, called the stock map.
(*d*) Geological map.
(*e*) Soil map.
(*f*) Map showing the working sections and cutting series.
(*g*) Detailed road map.

There is, however, no need for so many separate maps, as several of them can be combined into one.

Ordinarily three maps suffice, namely :—

a. The Geological Map.

This map should show the geological formation of the upper layers, on which the nature of the soil depends. In it can also be shown the general topography of the area; the various qualities of locality can be entered by lines of a distinguishing colour into which the quality figure is written.

b. The Detailed Map.

The scale of this map depends on circumstances. In India the ordinary scale is 4 inches = 1 mile. In a few cases maps on a scale of 8 inches = 1 mile, and in others of 2 inches = 1 mile have been prepared.

The map should show, amongst other items :—

(1) Name of forest and year of survey.
(2) Boundaries, all boundary marks being indicated on the map and numbered; boundaries between free property and parts subject to servitudes.
(3) Names of adjoining properties and their owners.
(4) Area, total, as well as of the main divisions.
(5) Areas not used for the production of wood.

(6) Contour lines, or height curves.
(7) The system of roads and rides, watercourses and other natural lines, with their names.
(8) The boundaries of working sections, blocks, compartments and sub-compartments, with their names and numbers.

c. The Stock Map.

This has for its principal object to give a picture of the manner in which the area is stocked with wood; a smaller scale than 4 inches = 1 mile generally suffices for it. The map should contain, apart from the necessary details, a representation of the existing species, sylvicultural systems and distribution of the age classes. This can be done in a variety of ways, as for instance in the following:—

In *high forest* the principal species are shown by different washes; the age classes by different shades of the same wash, the youngest being given the lightest, and the oldest the darkest shade; the regeneration class receives some distinguishing mark.

Mixed woods may receive a separate wash, or they may be distinguished by the addition of small trees or marks of various colours.

Coppice woods may receive a separate wash, if shown on the same sheet.

Coppice with standards may be distinguished from coppice by the addition of miniature trees.

Selection forest may be indicated by colouring it with the wash of the principal species, and indicating other species by special marks.

Blanks remain uncoloured.

The stock map should be renewed whenever a new working plan is prepared; if this is done, it gives, in the course of time, an excellent representation of the history of the forest.

The two following illustrations will further explain what has been stated above:—

REPRESENTATION OF TWO CUTTING SERIES (THE UPPER CONSISTING OF 10 COMPARTMENTS, THE LOWER OF 5), AS PRESCRIBED IN SAXONY.

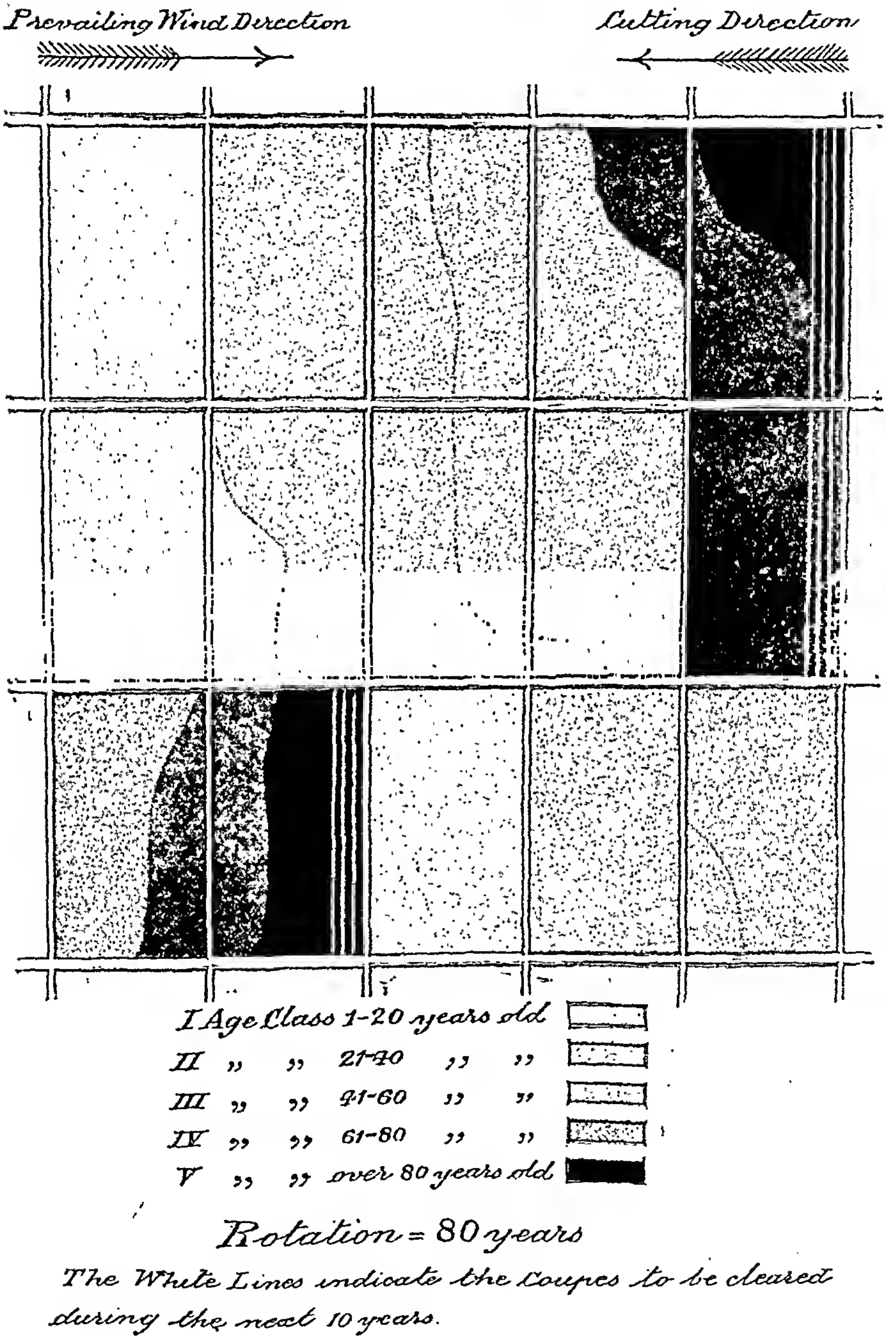
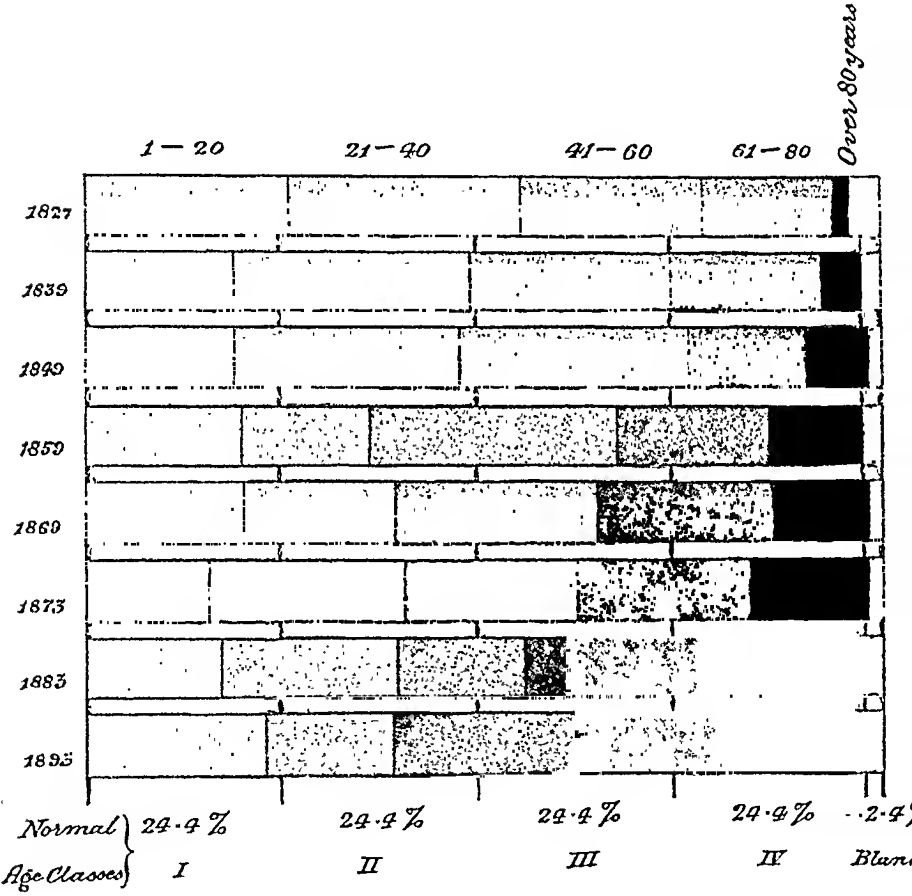


DIAGRAM SHOWING THE PROPORTIONS OF THE AGE CLASSES IN THE ANTONSTHAL RANGE, SAXONY, FROM THE YEAR 1827 TO THE YEAR 1893, WITH THE NORMAL PROPORTION INDICATED AT FOOT OF DIAGRAM.



CHAPTER II.

DIVISION AND ALLOTMENT OF THE FOREST AREA.

1. *The Working Circle.*

By a working circle is understood that area which is managed under the provisions of one and the same working plan.

The area of a working circle depends on local conditions. Its minimum size would be the area of a property belonging to the same owner; the maximum will ordinarily be the area forming one executive charge or range.

The division of an extensive property into ranges depends chiefly on :—

(1) The situation, and

(2) The intensity of management.

In the case of scattered blocks, in hilly country, or where means of rapid locomotion are wanting, a range will comprise a smaller area than if the property is consolidated, situated on level ground, or where railways and other means of locomotion enable the range officer to move rapidly from one part of his charge to the other.

In forests which yield a small return, the ranges may be large; where the money yield is high, it pays best to make the ranges small, so that an intense and detailed management may be possible.

In some cases one range officer may manage several working circles; for instance, if the owners of several small properties join in employing one officer for the management of their forest property. That case occurs frequently in many European States, where Government forest officers manage both the forests of the State and of Communes.

Each working circle or range, as the case may be, must be further divided. The unit of that division is the compartment. A number of compartments are grouped together into cutting series, and a number of the latter form the working circle, or a part of it called a working section. The whole of this division is effected by utilizing, in addition to the outer boundaries, interior natural lines, such as water partings, watercourses, precipices, etc., and artificial lines, as roads already constructed or projected and rides.

Although the division of the working circle depends chiefly on the system of roads and rides, it is desirable, before indicating how it should be laid out, to explain more fully what is understood by compartment, sub-compartment, cutting series and working section.

2. *The Compartment.*

By compartment is understood the unit of working; it forms, therefore, the unit of the division of the forest.

The above definition should never be lost sight of. If the boundaries of a compartment can be made to coincide with those of a wood showing a certain composition or age so much the better, but it is a mistake to insist upon such an arrangement; the main point is, that each compartment should be of a certain size, so as to fulfil its objects as the unit of working. If that area includes two or more different kinds of growing woods, they may be distinguished as sub-compartments; but the boundaries of the unit of working should never be twisted out of shape for the sake of including only one kind of growing stock in each compartment.

The formation of compartments is necessary—

(1) For general orientation, so as to enable the forester to define any particular part of the area accurately.

(2) To render all parts of the forest easily accessible, since one or more sides of the compartment are always formed by roads or rides.

(3) To assist in the prevention of fires, and to enable the forester to stop any which may have broken out.
(4) For the location of the annual or periodic coupes.
(5) To facilitate the transport of forest produce.
(6) To obviate the necessity for repeated surveys of the coupes.
(7) In some cases, to facilitate hunting and shooting.

The boundaries of compartments are formed by roads and rides, whenever natural lines are not available.

The shape of compartments depends on the configuration of the ground. In the plains, a rectangular shape (with sides 2 : 1, or 3 : 2) is most suitable. On hilly ground, such a shape is rarely practicable; but the actual shape should, as far as possible, approach that of a rectangle.

The size of compartments cannot be laid down; it depends on :—

(1) The intensity of management.
(2) The extent of danger from fire, and
(3) The size of the working circle.

3. *The Sub-compartment.*

If within the limits of a compartment considerable differences exist in respect of species, sylvicultural system, age of growing stock, quality of locality, etc., it may be divided into two or more sub-compartments; the latter may be temporary, if the differences will disappear after some time, or permanent. Sub-compartments may be marked by shallow ditches or other cheap boundary marks.

The forester should not go too far in the formation of sub-compartments, as it is accompanied by additional expenditure. As a rule, sub-compartments should be formed only if the additional income, derived from different treatment, at least covers the additional expense involved thereby.

4. *The Working Section.*

All parts of a working circle, which form one separate series of age classes, are called a working section. If a working circle consists of only one series of age classes, it is identical with a working section. In working circles of some extent, however, different conditions may demand the establishment of two or more series of age classes, or a division of the working circle into two or more working sections. The principal causes, which demand the formation of working sections, are the following:—

a. Species.

If several species appear as pure woods in a working circle, they must be placed into different working sections, if they require essentially different treatment, or if a certain quantity of material of each species has to be cut annually. If, on the other hand, the several species appear in mixed woods, such a separation is neither practicable or necessary.

b. Sylvicultural System.

Each sylvicultural system may demand the formation of a separate working section. If, for instance, part of a high forest is treated under the compartment system, and another part as a selection forest, each part must be formed into a separate working section. Coppice woods, and coppice with standards always must form separate working sections.

c. Rotation.

Even in the case of the same species and sylvicultural system, areas worked under different rotations must be placed into different working sections, whenever an even or approximately even annual yield is expected. Unless this is attended to, it will happen either that the annual yield is uneven, or, if the same quantity is cut every year, that the different rotations merge into one.

d. Servitudes.

If part of a working circle is subject to servitudes, it should be placed into a separate working section; this is necessary to protect the interests of the owner.

e. Differences in the Quality of the Locality.

Differences in the quality of the locality cause the establishment of different working sections, if they necessitate the growing of different species, or the adoption of different treatment or rotations.

f. Distribution of Cuttings.

If cuttings must be made annually in different parts of the working circle, it is often advisable to form different working sections, though this is not absolutely necessary.

g. Generally.

A working circle consisting of several working sections is said to be normal, if each separate working section is in a normal state.

Although the formation of working sections is in certain cases unavoidable, the forester should not go to extremes in this respect. A separate record must be kept for each working section, and they cause extra trouble and expense in other ways; hence moderate differences of conditions, especially in the rotation, should not induce the forester to introduce separate working sections.

The question may be asked, why a separate working plan should not be drawn up for each working section, thus making the latter always identical with a working circle. Such a procedure is not desirable, because it involves extra labour and repetitions in the working plan report. It is preferable, whenever practicable, to have one working plan for each executive charge, because the management of the different working sections can be so arranged that they supplement each other, thus enabling the forester to provide for a proper allotment

of work amongst the staff, and a proper distribution of the yield. Where the areas managed on different lines are mixed up with each other, the division of a working circle into two or more working sections becomes an absolute necessity.

5. *The Cutting Series.*

A working section in its simplest shape should consist of a series of age gradations equal to the number of years (or periods) in the rotation, so arranged that cuttings commence in the oldest age gradation and proceed steadily towards the youngest, in the direction which is determined by the circumstances of each case. It has, however, been pointed out on page 229, that such a simple arrangement is, in the case of high forest, rarely admissible, and that every working section in such a forest must be further divided into several parts, which are called "*cutting series.*" Only such a further division gives the necessary order and elasticity to the arrangement of the coupes.

Each cutting series should comprise a number of gradations, the ages of which differ by a certain number of years (see diagram on page 229); it can be regarded as a working section, in which cuttings are made periodically instead of annually; ordinarily, however, a certain number of cutting series together form one complete series of age gradations, or a working section.

The number of age gradations to be included into one cutting series depends on local circumstances. On the whole, small cutting series are preferable, as each gives a point of attack where cuttings can be made. Amongst the advantages of small cutting series the following may be mentioned:—

(1) The special requirements of each wood can be met at the right time; if a cutting is desirable at a given time, it can be made without interfering with the safety of adjoining woods.

(2) A suitable change of coupes can be arranged, so as to

protect the forest against the dangers which may make themselves felt, if two or more coupes adjoin each other.

(3) The establishment of small cutting series assists the forester in distributing the yield to meet local demands.

In order to realize these advantages, it is necessary that each cutting series should receive a shape, and be so situated that the coupes can be suitably arranged, and that cutting in one series does not interfere with the requirements of adjoining series. Where these conditions do not exist, they must be specially provided by the clearance of broad rides between the cutting series called severance cuttings.

6. *Severance Cuttings.*

By a severance cutting is understood a cleared strip of varying breadth, by which two woods are separated in the general direction of the cuttings, at a place where some time afterwards regular cuttings are to commence.

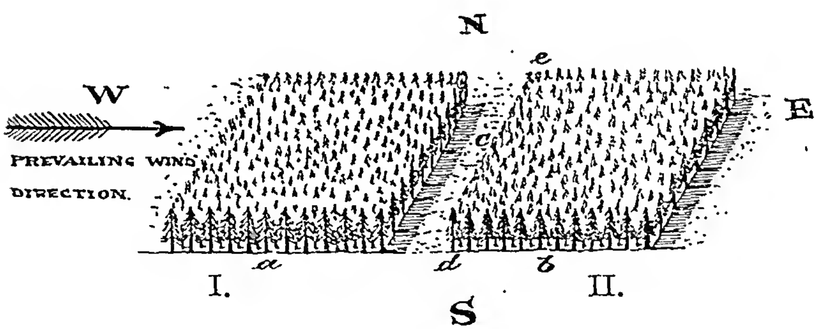


Fig. 50.

Severance cuttings are necessary, wherever an existing cutting series is too long, and where it is desirable to divide it into two or more series. Their object is to accustom the edge trees of the wood on the leeward side to a free position, so that they may develop into storm-firm trees, and be able to withstand the effects of strong winds, when the wood on the

windward side has been cut. An example will explain this. A wood comprising *a* and *b* is to be divided into two cutting series I. and II. To prevent the trees in II. being thrown by wind, when I. is cut over, a strip *c* is cleared some time before cuttings in I. are commenced, so that the edge-trees along the line *d*—*e* may become storm-firm.

Severance cuttings need not be straight; they may, if necessary, be curved, or run along two or three sides of a wood. The latter is necessary, where the prevailing wind direction is not constant, but oscillates, say, from north-west to south-west. The breadth of severance cuttings differs according to species, their height growth and the strength of threatening winds; it will ordinarily range between 30 and 60 feet.

Severance cuttings must be made while the wood to be protected is still young and capable of developing firm edge trees; such a development is generally no longer possible after the trees have passed middle age. They must be made some 15 to 20 years before the regular cuttings in the windward wood are commenced. Where danger from windfalls is great, it is desirable first to clear a narrow strip, and widen it a few years afterwards in one or more instalments, so as to gradually accustom the edge trees to the effects of strong winds. If the severance cutting is not to form a road or ride, it is at once re-stocked, so as to avoid loss of increment, and because the existence of a young wood in front of that to be protected is an additional safeguard against windfalls. When a severance cutting is made along an existing road or ride, it is of course placed to the windward of it.

If the proper time for making a severance cutting is past, and the wood to be protected is too old, it would be a dangerous procedure to make such a cutting. In that case it is better to make a series of thinnings in the strip along the edge of the wood to be protected, before cuttings in the windward wood are commenced. Whether this measure will have the desired effect is doubtful, but it is better than to risk a regular severance cutting.

7. *The System of Roads and Rides.*

As already indicated, working sections, cutting series, and compartments must be separated from each other by natural or artificial lines. Apart from suitable natural boundary lines, such as waterpartings, watercourses, precipices, fields, meadows, &c., roads are the best boundaries of compartments and cutting series, because they facilitate the transport of the produce. It is therefore desirable that, in the first instance, a suitable network of roads should be decided on and marked on the ground. Roads alone, however, rarely suffice. In some cases roads already exist which are not suitable for boundaries, in others even new roads must be so laid out that they cannot be used as boundaries, because they must lead in the direction of the places of consumption, and in hilly or swampy ground they often assume a shape and direction, which makes them unfit to serve as boundaries.

The missing division lines are provided by a system of rides, that is to say, by cleared strips of various breadths. A distinction is made between major and minor rides.

a. Major Rides.

In so far as roads or natural lines are not available, cutting series, and in many cases working sections, should be bounded by major rides. These should run in the direction of the cuttings, that is, parallel to the prevailing wind direction, whenever the configuration of the ground does not necessitate deviations.

In coppice and coppice with standards, the major rides need not be more than 6 to 8 feet broad, unless they are used as roads for the transport of the material. In high forest, they must be much broader, because they are used as severance cuttings, to induce the edge trees of adjoining woods to form wind-firm trees, and to become accustomed to other climatic influences. In the case of woods consisting of species which are easily thrown by wind, they should not be less than 30 feet

broad, and if the major ride is also used as a fire line, it may be still broader.

The edges of the woods bordering on major rides should be strongly thinned from an early age onward, so as to produce strongly developed trees.

Major rides may be utilized for stacking wood. Their area is entered as non-productive of trees; in many cases, however, they produce grass.

In young woods the major rides should be cut at once, while the edge trees are capable of producing a strong root system; in woods which are over middle age, only 6 to 8 feet broad lines should be cleared in the first instance, which are widened to the required breadth, when the adjoining woods are cut over.

b. Minor Rides.

Minor rides should run more or less at right angles to major rides; they complete the delimitation of the compartments. The annual coupes will, therefore, run parallel to the minor rides, and stand at right angles to the major rides. Minor rides need not be more than 6 to 8 feet broad, unless they are used as fire lines.

c. The Network of Rides.

Major and minor rides together form the net-work or system of rides. The laying out of it is, especially in the case of shallow-rooted species, chiefly dependent on the prevailing wind direction. In the plains, the latter can generally be determined without much trouble. In mountainous districts, the matter is frequently beset by difficulties, because the configuration of the ground may produce a local direction, which differs from the general direction. No rule can be laid down for such deviations; the question must be studied on the spot. The direction can frequently be recognised by the shape of the crowns of trees, by a slanting position of the stem, and above all, by the direction in which trees have been thrown. As regards the latter, it must not be overlooked that local storms

sometimes throw trees in a direction which differs from the ordinary direction of gales. In many cases reliable information can be obtained from local people who have lived for some time in the locality.

The laying-out of the system of rides is of great importance, because it is used in the protection of the woods against natural phenomena, and it leads to order in the management. These advantages outweigh the loss of productive area, which is, after all, very limited. Regular networks of rides, with right angles, are only practicable in the plains; on hilly ground they must accommodate themselves to the configuration of the ground. The following example (taken from Judeich) will illustrate this :—

The forest occupies a ridge, the slope of which is indicated by dotted contour lines - - - - . The top of the ridge, being much exposed, must be treated as a separate working section

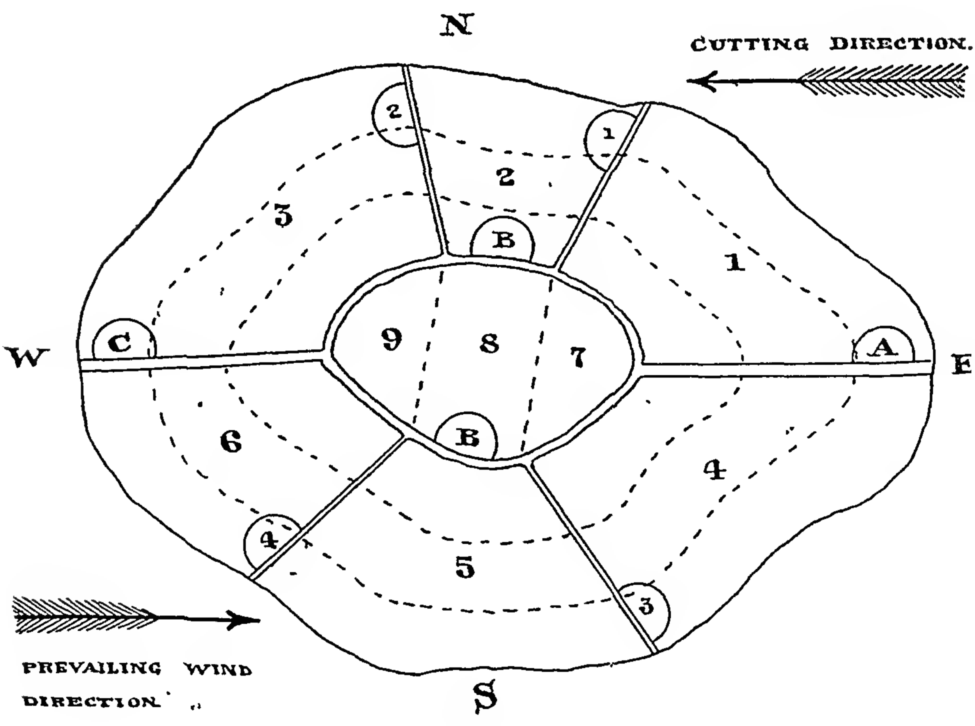


Fig. 51.

under the selection system; it is separated from the rest by a major ride (B). The slopes are treated under the compartment system, and they are divided into two parts by the major rides (A) (B) and (C). The numbers (1) (2) . . . indicate the minor rides, and 1, 2, 3 . . . . the compartments. The prevailing wind blows from the west.

The division would probably be somewhat on the following lines:—

Working Section I. = *Compartment System.*

Cutting	Series	*A*	comprises	compartments	1 & 2
,,	,,	*B*	,,	,,	3
,,	,,	*C*	,,	,,	4
,,	,,	*D*	,,	,,	5 & 6

Working II. = *Selection System.*

Comprises compartments 7, 8, and 9.

The cutting direction would be from east to west, a direction which is indicated by the numbering of the compartments.

The coupes in compartments 1 to 6 run at right angles to the major rides (A) and (C), or up and down the hill side, as it is generally objectionable to let the coupes run horizontally, even from the top gradually downwards.

8. *Demarcation of the Divisions of a Forest.*

It is generally desirable that all interior boundary lines should be demarcated by boundary marks so that they can be recognized, if they should have become obliterated in consequence of cuttings, windfalls, etc. For this purpose boundary marks may be placed at all points, where rides cross, or where they show an angle. If straight rides are very long, it is useful to place intermediate marks at suitable distances. Such marks are placed on one side of the rides, so that they may not interfere with the transport of the produce; it is useful to

choose always the same side, say the north side of the major rides, and the east side of the minor rides.

9. *Naming and Numbering of the Divisions of a Forest.*

The methods of naming and numbering the divisions differ much. Judeich recommends the following:—

(*a*) Working sections receive Roman numbers = I, II, . . .
(*b*) Cutting series receive names and slanting capital letters = *A*, *B* . . .
(c) Compartments receive Arabic numbers = 1, 2 . . .
(*d*) Sub-compartments receive small Roman letters = 1_a, 1_b . .
(*e*) Major rides receive upright capital letters surrounded by a circle = (A), (B) . . .
(*f*) Minor rides receive small Arabic figures surrounded by a circle = (1), (2) . . .

(See illustration on page 302.)

Sometimes a number of compartments are joined into a "block;" if so, the latter should receive a name.

The numbering of compartments must be consecutive throughout the working circle; it leads to confusion to have a separate series for each working section. The numbering should be done so as to indicate the cutting direction.

In the French State forests the following system of numbering the divisions is prescribed:—

Working sections are numbered = I, II, III, . . .
Periodic blocks ,, ,, = 1, 2, 3 . . .
Compartments ,, ,, = *A*, *B*, *C* . . . with the addition of the number of the periodic block; thus IV C_2 means Working Section IV, compartment *C* in the 2nd periodic block.

CHAPTER III.

DETERMINATION OF THE METHOD OF TREATMENT.

Before the yield can be calculated and the general plan of operations laid down, the method of treatment must be determined. The questions here involved have already been discussed in this and the previous volumes, and the following references will be useful :—

(1) For choice of species, see Volume II., pages 4 to 11.

(2) For choice of sylvicultural system, see Volume I., pages 226 to 232.

(3) For choice of method of formation, see Volume II., pages 169 to 180.

(4) For choice of method of tending, see Volume II., pages 189 to 228.

(5) For choice of rotation, see present volume, pages 200 to 211.

(6) The financial aspect of forestry has been explained at pages 167 to 169 of this volume.

The determination of the method of treatment depends chiefly on :

(*a*) The objects of management.

(*b*) The locality.

(*c*) The growing stock actually existing in the forest.

(*d*) The dangers to which the growing stock and the soil are exposed.

(*e*) The conditions of demand and supply of forest produce.

(*f*) The legal position of the forest, existence of rights, privileges, etc.

The nature of these determining factors shows that no general rule can be laid down, but that the method of treat-

ment and the general lines of management must be determined locally on the merits of each case. In the following lines only a few general hints are given.

1. *Choice of Species.*

The first step should always be to examine carefully how far the existing species meet the objects of management and suit the locality. If the existing species do not answer, then the forester should not hesitate to change them. On the other hand, the introduction of a new species should not be lightly undertaken, as it generally involves some loss and frequently introduces uncertainty as to future results. Only species which have been tried within a reasonable distance and succeeded, should be chosen. The cultivation of untried species is, in the first place, only justified on a small scale. Above all, personal fancies must be banished from the forester's mind.

The formation of mixed woods is of the first importance, and its advisability should always be considered. In this respect attention is invited to pages 179 to 202 of Volume I.

2. *Sylvicultural System.*

This depends, of course, on the species which exist on the area, or which have been selected for introduction.

High forest yields the greatest quantities and generally also the most valuable classes of major produce, as well as various kinds of minor produce; it is the best system for the preservation of the quality of the locality. On the other hand, it requires in the case of broad-leaved species at least fairly good soil; it also requires a greater capital than other systems, owing to its greater growing stock, accompanied, as a rule, by a lower mean annual forest per cent. The growing stock is also exposed to many dangers, such as snow, ice, wind, insects and fire. High forest must be adopted in the case of all species which cannot be regenerated by stool shoots or root suckers.

As to the different kinds of high forest, clear cutting, shelter-wood system, group system, selection forest, or their modified forms, it should be remembered that clear cutting is most easy to carry out, and that the selection system is most favourable for the protection of the quality of the locality, while the intermediate forms stand between these two extremes. Again, clear cutting exposes the growth most to dangers; at the same time it is generally obligatory in the case of light-demanding species. Its drawbacks can, to some extent, be reduced by making the coupes of small extent. Still, it should only be followed in the case of hardy species.

The question as to which of the forms of high forest gives the highest returns, has been much discussed. On the whole, a carefully carried out system of clear cutting, with a suitable species, may be expected to give the best financial results; the ordinary shelter-wood system approaches it in this respect, while even higher returns have been claimed for some of its modifications, such as the two-storied high forest, or the system of isolated trees in conjunction with an underwood.

In how far a locality can, under the clear cutting system, maintain the higher returns (if any) depends on the nature of the soil and the climate. Where these are not favourable, one of the shelter-wood systems is certainly to be preferred. The latter are absolutely essential in the case of species which are tender during early youth.

Coppice woods make smaller demands on the chemical and physical properties of the soil; they have a much smaller capital, and generally give a higher mean annual forest per cent.; owing to the shortness of the rotation, the danger of damage from outside (excepting frost and deer) is much smaller, and the whole management can be more regular and safer than in the case of high forest. On the other hand, coppice gives smaller quantities and lower quality, except in special cases as for instance where tanning bark, hop-poles, or vine-stakes are produced. Coppice is only possible in the

case of those species which reproduce well from the stool. It requires a fairly mild climate and protected position.

Pollarding gives frequently high returns, and it can be carried on in conjunction with agriculture.

Coppice with standards holds an intermediate position between high forest and coppice woods as regards the preservation of the quality of locality and the capital value of the growing stock. The comparatively free-growing standards reach a certain diameter in a shorter time than trees in high forest; on the other hand, the stems are shorter and less clean than in regular high forest, and they are more subject to frost cracks and scorching of the bark. The yield is smaller than in high forest, but larger than in coppice; financially, it generally holds a position between the two.

3. *Method of Formation.*

This depends chiefly on the sylvicultural system, the species, and the nature of the locality.

Natural regeneration is cheaper in itself, but, in the case of high forest, seed-years do not always come when they are wanted, so that a regular progress of the regeneration may be seriously disturbed, and the loss of time, and other accompanying disadvantages, may more than outweigh the smaller original outlay. Nor must it be forgotten that, as a rule, natural regeneration must be augmented by a certain amount of planting. To conduct the process of natural regeneration by seed demands the highest skill of the forester.

Artificial regeneration generally accompanies the system of clear cutting in high forest, but sowing and planting may also be done under shelter-woods; it is necessary in the case of first afforestations or where a change of species is contemplated.

Whether direct sowing or planting is preferable, depends chiefly on the species and local conditions, a subject which has been dealt with on pages 170 to 175 of Volume II.

As regards the preservation of the quality of locality, see Sylvicultural System.

4. *Method of Tending.*

This must be laid down in detail, but as the subject has been fully treated in Volume II. it would only lead to repetition to refer to it further in this place.

5. *The Rotation.*

The rotation is chiefly governed by the species, sylvicultural system and the objects of management. Though the financial aspect should never be lost sight of, there are many cases where a departure from the financial rotation is fully justified. This occurs where short rotations interfere with the preservation of the quality of locality, where considerations for the production of a certain class of produce or the dictates of political economy are of paramount importance.

CHAPTER IV.

DETERMINATION AND REGULATION OF THE YIELD.

As long as the owner of a forest is satisfied with intermittent returns, the regulation of the yield is chiefly governed by sylvicultural considerations, that is to say, every wood is cut over when it is just ripe according to the objects of management, while thinnings are made when they are necessary. If the owner demands a sustained annual yield of equal, or nearly equal, quantity and yet the forest is not in a normal state, the various cuttings may have to be made at other times. All such deviations demand certain sacrifices on the part of the owner, which differ according to the actual condition of the forest and the objects of management.

These sacrifices are due to the fact that the final cuttings must be made at an age other than the normal age as determined by the objects of management; even thinnings may have to be postponed instead of being made when the condition of the wood demands such cuttings. This state of affairs may be brought about by a surplus or deficiency of mature woods, or by their being so situated that they cannot be cut over at the proper time from consideration for the adjoining woods.

The task of the forester is, in such cases, to secure a sustained annual yield, and yet to lead the forest gradually over into the normal state as defined in Part III. of this Volume, with the smallest possible loss to the owner. Many different methods have been elaborated with the view of achieving that task, which approach the subject from various points of view; the degree of success differs however considerably. It is not proposed to describe here all methods, but only those which

deserve to be specially mentioned. These may, according to their principal characteristics, be grouped in the following manner:—

I. Division of the forest into fixed annual coupes.
II. Allotment of woods to the periods of a rotation.
   1. According to area.
   2. " " volume.
   3. " " area and volume combined.
III. Regulation of the yield according to increment and growing stock.
   1. The Austrian method.
   2. Hundeshagen's "
   3. Von Mantel's "
   4. Brandis' "
IV. Regulation of the yield according to increment and growing stock, combined with the allotment of areas to the several periods of a rotation (Heyer's method.)
V. Annual or periodic selection of woods for cutting in accordance with sylvicultural requirements and the objects of management (Judeich's method.)

SECTION I.—DIVISION OF FOREST INTO FIXED ANNUAL COUPES.

Under this method the area of the forest is divided into as many annual coupes as there are years in the rotation, and each coupe marked on the ground. Every year one coupe is cut over, giving the annual yield of final returns, to which must be added the necessary thinnings in the other coupes.

The size of each annual coupe is $= \frac{A}{r}$, if the area is at once re-stocked, or $= \frac{A}{r+s}$, if each coupe lies fallow for s years. In this case A may represent the actual or reduced area.

The merits of this method are small. It aims more directly than any other method at the establishment of a regular series of age gradations, which becomes normal after one rotation

if the division of the area is based upon the reduced area of the several parts; but it achieves this object only by heavy sacrifices, because the returns during the first rotation must be very uneven, unless at the outset a proper proportion and distribution of age classes existed. The method takes no notice of disturbances nor of the state of the market, hence it is very rigid. Above all, it neglects the fundamental principle, that the most important measure must always be the establishment of the normal increment within the shortest possible period of time.

The method is applicable to coppice woods, coppice with standards, and, with modifications, to selection forests. For all other methods of high forests it is quite unsuited, except, perhaps, for clear cutting with a very low rotation.

Section II.—Allotment of Woods to the different Periods of a Rotation.

In order to remove the great rigidity of the fixed annual coupes, and to obtain a method which is suitable for the treatment of high forests, especially those managed under the shelter-wood systems, the several woods comprising a forest are allotted to a number of periods. The latter are generally from 3 to 6 in number, and each comprises from 10 to 30 annual coupes. In this way the woods are divided into as many lots as there are periods in the rotation; during each period one of these lots is dealt with. Thus operations extend over the whole area once in each rotation. Deviations from this arrangement occur occasionally, for instance, if a sub-compartment is not cut over, or twice cut over during the first rotation, in order to make the compartment uniform.

It is evident that during the first rotation the total yield is represented by the growing stock which happens to stand in the forest at the commencement of operations plus that part of the increment which is added to it during the course of the

first rotation; it may be smaller, equal, or larger than the normal yield.

An essential part of this method of regulating the yield is the preparation of a framework or general working plan drawn up for one rotation and divided into a number of periods, showing during which period each wood is cut over. The allotment can be made according to area, volume, or the two combined, so that practically three different methods are established, which must be considered separately.

1. *The Method of Periods by Area.*

a. Description of the Method.

The woods of a forest are so allotted to the several periods of one rotation, that each contains the same, or approximately the same area, called the periodic coupe.

Where little or no difference exists in the quality of the locality in the different parts of the forest, the size of each periodic coupe will be $= \frac{A}{n}$, where n represents the number of periods in the rotation. If such differences exist, the areas must be reduced to one common quality standard, and the size of the periodic coupe becomes $= \frac{\text{red. } A}{n}$. Unless this is done the periodic yields in the second and following rotations will not be equal.

In allotting the woods to the several periods, that to be dealt with first receives the oldest woods and those with the most deficient increment, taking into consideration a suitable arrangement of the cutting series; the allotment to the other periods is made according to the age of the woods, with due consideration to a suitable grouping of the age classes. If then the totals in the several periods differ, shiftings are made by moving certain areas backward or forward, until each period contains the same, or approximately the same area.

The woods placed into the first period are measured, their

volume calculated, and the increment for half the number of years in the period, $\frac{n}{2}$, added. The total of the volume thus obtained is divided by the number of years in the period, so as to obtain the average annual yield during the first period.

For an example see Appendix A, at page 349, where the working plan for the communal forest of Krumbach, a village in Hesse-Darmstadt, has been given. This working plan is being actually followed.

b. Merits of the Method.

The method is simple, and can be applied by any intelligent manager. It establishes the normal state within one rotation, if no disturbing events occur. At the same time it may yield very uneven returns during the first rotation, though this can be avoided to some extent by suitable shiftings. Although the method is much less rigid than that of fixed annual coupes, it is often difficult to produce during the first rotation a proper grouping of the age classes; more especially, too large cutting-series are likely to be established.

Another disadvantage is that a surplus of growing stock may be dragged over a whole rotation, whereas it should be removed as quickly as possible; or, on the other hand, it may take a whole rotation to make good any deficit of growing stock.

For a financial management the method is little adapted, except in so far that it introduces order into the management. It gives only a limited latitude to the forester to hold over vigorous woods, or to cut over those which are deficient in increment.

2. *The Method of Periods by Volume.*

a. Description of the Method.

The woods of a forest are so allotted to the several periods of a rotation that each yields the same, or approximately the same, volume. In some cases only the final returns are thus

regulated; in others the intermediate returns are utilized to equalize the yields of the several periods.

The allotment is based upon the table of age classes; then shiftings are made, so as to bring woods which have a poor increment early under the axe, and establish, as far as practicable, a suitable grouping of age classes; then further shiftings are made, so as to equalize the periodic returns. The result represents the general working plan for the first rotation. It will be observed that, in the majority of cases, the areas placed into the several periods will be uneven, resulting in uneven returns during the second rotation, unless a fresh allotment is made.

Example:—In Appendix B, page 362, only the final returns have, for simplicity's sake, been equalized. The data are those of the Krumbach communal forest given in Appendix A.

As the future returns have to be estimated for a whole rotation, it is evident that yield tables must be used; accordingly the above general working plan has been based upon the data for beech high forest, given at p. 123. After making the shiftings indicated in the general working plan, the volumes allotted to the several periods stand as follows:—

	Periodic Yield.	Annual Yield.
I. period	=189,393 cubic feet.	9,470 cubic feet.
II. „	=195,474 „ „	9,774 „ „
III. „	=173,919 „ „	8,696 „ „
IV. „	=188,240 „ „	9,412 „ „
V. „	=194,845 „ „	9,742 „ „
Total	=941,871 „ „	9,419 „ „

An attempt to equalize the returns further would necessitate the cutting up of compartments, which is not desirable.

The areas placed into the several periods are:—

I. period	=	36·00 acres.
II. "	=	34·02 "
III. "	=	27·30 "
IV. "	=	29·05 "
V. "	=	33·78 "
Total	=	160·15
Mean periodic area	=	32·03

b. Merits of the Method.

The method has this advantage over the method by area, that it gives during the first rotation equal, or approximately equal, periodic returns; it considers the interests of the present generation more fully. On the other hand, the estimate of the future returns is more or less problematic, so that the equalization of the returns for a whole rotation ahead is a very uncertain operation. It shares with the method by area the disadvantage that a proper grouping of age classes is generally beset by difficulties. Again, it drags a surplus of growing stock over a whole rotation.

Whereas the method by area establishes the normal state of the forest within one rotation, the method by volume takes several rotations to accomplish this.

As regards its financial aspect, it stands on the same footing as the method by area.

3. *The Method of Periods by Area and Volume combined.*

The woods of a forest are so allotted to the several periods of a rotation, that each contains the same area and yields the same, or approximately the same, volume.

The equalization of the periodic areas and returns is effected either by adding columns for the volume to the general working plan used for the method by area, or by adding columns showing the reduced areas to the general working plan used for the method by volume. Shiftings are made until both area and yield are the same, or approximately the same, in each period.

It will easily be understood that such an equalization is a difficult operation, especially in a very abnormal forest; hence more than approximate results cannot be attempted.

The method enjoys some of the advantages and disadvantages of the two previous methods, of which it is a combination. Its principal disadvantage is, that a suitable grouping of age classes is still more difficult than in the case of the two previous methods.

In practice various modifications of the above three methods have been evolved, which sometimes partake more of the one, and sometimes more of the other method.

Section III.—Regulation of the Yield according to Increment and Growing Stock.

The methods belonging to this section calculate the yield by means of a formula which is based on the increment laid on, and any difference which may exist between the real and normal growing stock. Having thus determined the yield, the woods for cutting are selected from time to time in accordance with sylvicultural considerations. There is no necessity for drawing up a general working plan for a longer period than suits the special requirements of each case.

A number of different methods come under this section, of which, however, only the following need be mentioned here, as the others are of little practical importance.

1. *The Austrian Method.*

(Die Oesterreich'sche Cameral Taxation.)

a. Description of the Method.

In the year 1788 (during the reign of the Emperor Joseph II., one of the most enlightened sovereigns known in history) the Austrian Government issued instructions regarding the assessment of forests for the purpose of taxation. In these instructions reference was made to the difference which may exist between the real and normal growing stock of a forest. This led to the knowledge that a forest, which is expected

to give permanently an annually equal return of the normal age and amount, must contain the normal growing stock corresponding to the rotation and method of treatment. Foresters speedily applied this principle to the regulation of the yield of forests by saying that, in order to lead an abnormal forest over into the normal state, it is necessary to establish the normal growing stock, in other words, to remove a surplus or to save up any deficit, as the case might be. The method developed upon this basis is called the Austrian assessment method. Authors differ as to the details of the original method, but a general survey of the literature on the subject gives the following rule for determining the yield:—

"If the normal growing stock is present in a forest, then the actual, or real, increment must be utilized; if the real growing stock is greater than the normal, more than the real increment must be removed; if the real growing stock is smaller than the normal, less than the real increment must be utilized, until the deficiency has been made good."

In carrying this excellent idea into effect, however, errors were introduced, which are still upheld by some foresters of the present day.

The procedure is described as follows:—

(1) The increment is calculated as the mean annual increment of a series of years.

(2) The normal growing stock is placed equal to the final mean annual increment corresponding to the normal rotation multiplied successively by the ages of all age-gradations; the sum of all these products gives the value $G_n = I \times \frac{r}{2}$ (see p. 235). Here I represents the normal annual increment of all age gradations, which is equal to the volume of the oldest age gradation.

(3) The real growing stock is obtained by multiplying the real final mean annual increment by the present age of each age gradation. For this purpose it is necessary

to determine the real final age and the volume at that age for each wood.

(4) The difference between the real and normal growing stock is removed during such period as the owner, or forester, may determine according to the circumstances of each case.

(5) The general formula for calculating the yield, if the deficiency or surplus of growing stock is to be removed in the course of a years, runs as follows :—

$$\text{Annual Yield} = \text{real Increment} + \frac{\text{real Gr. Stk.} - \text{norm. Gr. Stk.}}{a}$$

$$Y = I_r + \frac{G_r - G_n}{a}.$$

b. Merits of the Method.

The method was the first which based the calculation of the yield upon a knowledge of the increment and the growing stock. It has the advantages over the previously-described methods that :—

(1) It teaches the proportion between the real and normal growing stock, and enables the owner to remove any surplus or deficiency at his pleasure.

(2) It assures to the owner the utilization of the full real increment, whenever the normal growing stock is present.

(3) It distinguishes in the yield between increment and growing stock; in other words, between the removal of genuine annual increment and that of surplus capital.

On the other hand, the method, as above described, has serious drawbacks :—

(1) The calculation of the real and normal growing stock, based upon the final mean annual increment, is not correct and not even safe. As, however, both are calculated in the same manner, and one is deducted

from the other, the error is to a great extent eliminated.

(2) As the yield is determined by a formula, the method, if applied rigidly, may lead to absurd results: for instance, it may happen that a full increment takes place, that the real growing stock is equal to the numerical normal growing stock, and yet there may not be a single mature wood in the forest fit to cut.

If the method is applied judiciously, that is to say, if:

(1) the real growing stock is taken as that actually existing in the forest, and the normal growing stock calculated from a suitable yield table;

(2) the yield as calculated with the formula is modified to suit the special conditions of each forest;

then the method is one of considerable merit. It enables the forester to arrange the grouping of the age classes in the most desirable way, and to do justice to all other sylvicultural requirements, since it leaves him an entirely free hand in the selection of the woods to be cut.

The method is applicable to all sylvicultural systems, but the determination of the increment and growing stock involves much labour. Under it a forest is gradually led over into the normal state, though perhaps not for a considerable period of time; the difference between the real and normal state will, after the first rotation is passed, be so small that it can be neglected.

The sample working plan given in Appendix C, p. 364, has been based upon the formula of this method.

2. *Hundeshagen's Method.*

a. Description of the Method.

Hundeshagen's method of determining the yield is based upon the idea that the real yield must bear the same proportion to the real growing stock as that existing between the normal yield and normal growing stock; he thus obtains the equation:

$$Y_{real} : G_{real} = Y_{norm.} : G_{norm.}$$

and

$$Y_r = G_r \times \frac{Y_n}{G_n}.$$

In words, the real yield is equal to the real growing stock multiplied by the normal yield and divided by the normal growing stock. Hundeshagen calls the quotient $\frac{Y_n}{G_n}$, by which the real growing stock is multiplied, the "utilization per cent." (More correctly this indicates only the rate of utilization, whereas the utilization per cent. is $\frac{Y_n}{G_n} \times 100$.)

The normal yield is placed equal to the normal increment, or equal to the contents of the oldest age gradation in a normal series of age classes. The normal growing stock is obtained by adding up the volumes given in a suitable yield table; under the real growing stock Hundeshagen understands that which is actually standing in the forest.

In applying the method, Hundeshagen does not ask for a general working plan, except for a limited number of years; he is satisfied with determining the species, sylvicultural system, general lines of management, the rotation and general rules for the grouping of the age classes; he leaves it to the manager to select the woods for cutting from time to time, say every five or ten years.

As the yield is determined by the growing stock which happens to exist, and as this practically changes from year to year, it would, theoretically speaking, be necessary to remeasure the growing stock every year, but as the changes are slow, Hundeshagen considers it sufficient if the re-measuring is done once every 20 or 30 years.

Hundeshagen determines, in the manner above described, only the final returns; he adds the intermediate returns, estimated in a summary manner, or calculated according to average data obtained locally.

b. *Merits of the Method.*

The principal assumption of Hundeshagen is not quite correct; at any rate there is no justification for maintaining that the real yield bears the same proportion to the real growing stock as the normal yield to the normal growing stock, because the rate of increment is not determined by the quantity of growing stock which stands in a forest. On the contrary, a large growing stock consisting of defective old woods may give a small increment, while a small growing stock consisting of vigorous young woods may show a large increment.

The method, if applied rigorously, may lead to absurd measures, just in the same way as the Austrian method; it prescribes a definite annual yield, while not a single mature wood may be present; or it prescribes too small a yield, whenever a considerable portion of the area is stocked with decrepid old woods, which ought to be cut over as quickly as possible, and replaced by vigorous young woods. In all such cases, the yield, as fixed by the formula, must be modified in accordance with the requirements of each case.

The method does not distinguish in the yield between increment and growing stock, and in this respect, it stands below the Austrian method. Moreover, it may drag a surplus of growing stock over even more than one rotation.

Hundeshagen assumes that, with the yield calculated according to his method, the normal growing stock will be established naturally, as the yield bears a fixed proportion to the real growing stock; if the latter is greater than the normal amount, more than the increment will be removed, and *vice versâ*. This is ordinarily the case, but not under all circumstances. If, for instance, both the increment and growing stock are deficient, the yield may be greater than the increment, so that the growing stock is still further reduced, at any rate for a time; hence the establishment of the normal state may be considerably delayed.

On the other hand, Hundeshagen's method has this great advantage, that the increment need not be determined, such a determination being at all times beset by difficulties and uncertainty. All that the method requires is a suitable yield table, and the measurement of the growing stock actually standing in the forest. Hence, the method is by no means to be despised, if a general plan is added indicating the grouping of the age classes to be aimed at. For the rest, it leaves a free hand to the manager to shape the management in accordance with the requirements of each case, as long as the volume determined by the formula need not be rigorously cut. It may reasonably be assumed that Hundeshagen himself expected this.

3. *Von Mantel's Method.*

Von Mantel, in arranging for the determination of the yield of certain forests in Bavaria, laid it down that this should be done according to the formula—

$$\text{Annual Yield} = \frac{\text{Real Growing stock of the forest}}{\text{Half the number of years in the rotation}} = \frac{{}_{real}G}{\frac{r}{2}}$$

This formula rests upon the same basis as Hundeshagen's method, if for the latter the normal growing stock is calculated with the final mean annual increment. Hundeshagen's formula—

$$Y = {}_{real}G \times \frac{Y_n}{G_n},$$

goes in that case over into—

$$Y = {}_{real}G \times \frac{I_n}{I_n \times \frac{r}{2}} = \frac{{}_{real}G}{\frac{r}{2}}.$$

The cutting of the yield according to Von Mantel's formula will gradually lead to the establishment of the normal growing stock, as the following considerations will show:—

Supposing the real growing stock (under which Von Mantel

understands that actually present in the forest) is equal to the normal growing stock, then his formula goes over into—

$$Y = \frac{\frac{r \times I}{2}}{\frac{r}{2}} = {}_{norm.}I.$$

The formula gives, therefore, the correct yield, provided the increment is normal.

If the actual growing stock is smaller than the normal, say ${}_{real}G = \frac{r \times I}{2} - x$, then the

$$\text{Yield} = \frac{\frac{r \times I}{2} - x}{\frac{r}{2}} = I - \frac{2 \times x}{r} = I - \frac{x}{\frac{r}{2}},$$

which means, that less than the increment is cut.

Supposing that the real growing stock is greater than the normal: ${}_{real}G = \frac{r \times I}{2} + x$; then,

$$Y = \frac{\frac{r \times I}{2} + x}{\frac{r}{2}} = I + \frac{x}{\frac{r}{2}};$$

more than the increment will be cut, so that the surplus of growing stock will gradually disappear.

All these assumptions depend, however, on the supposition that the normal increment is laid on. If the increment is deficient, the abnormal state may be further increased, until the increment has reached its normal size.

The merits of the method are approximately those of Hundeshagen's method. It introduces an additional inaccuracy by being based on the assumption that the normal growing-stock is $= \frac{I \times r}{2}$. On the other hand, the normal growing stock and normal yield need not be determined; in other words, the method can do without yield tables. It is only

necessary to measure the growing stock, and to determine the rotation.

The method is very simple, and it is specially suited for determining the yield of selection forests.

4. *Brandis' Method.*

The method to be described under this head will be better understood by indicating the circumstances which lead to its elaboration.

Doctor (now Sir Dietrich) Brandis, on being appointed Superintendent of the Pegu forests in Burma in 1856, found himself confronted by enormous areas of teak forests in danger of being heavily overworked. These forests contained teak in varying proportions, but on the whole to a limited extent, which has since been ascertained to amount to perhaps 10 per cent., while about 90 per cent. of the growing stock consisted of species, which at that time had no market value. Indeed, the latter were allowed to be removed free of charge without let or hindrance. Moreover, even teak trees required to be of a certain size to make their extraction really remunerative. At that time it was considered desirable that no teak tree should be removed, unless it had reached a circumference of 6 feet, or roughly a diameter of 2 feet, measured at 6 feet from the ground. Trees of that girth and above were called trees of the first class.

Under these circumstances, Brandis' object was to ascertain, as quickly as possible, the number of first-class teak trees which might be removed annually, without at any rate exposing the forests to deterioration. For this purpose he designed a method, by which he ascertained—

(1) the number of first class trees in the forests;

(2) the time which it takes to replace them.

By dividing the number of first class trees ascertained under (1) by the number of years ascertained under (2), he ascertained the maximum number of trees which it was permissible to cut annually.

It will thus be seen that the volume of marketable growing stock was ascertained, and that this was removed at such a rate as at least to maintain it; in other words, the maximum yield was fixed at that quantity of marketable timber which reached maturity every year, thus at least maintaining the mature growing stock in the forest, and utilizing the actual increment. With the view of utilizing an excess of mature material, it was laid down that if the proportion of first class trees appeared excessive as compared with the younger classes, extra cuttings might temporarily be made, and *vice versâ*, hence the method is one based on increment and growing stock. Various safe-guards were added, such as an allowance for trees which it did not pay to extract; where few second and third class trees existed, some first class trees were left standing, to provide seed for regeneration; immediately along the banks of streams cuttings were made very sparingly, &c.

For the rest, the method leaves a free hand to the forester, who arranges the cutting with due regard to sylvicultural requirements and a proper succession of the different coupes.

The number of trees of the several size classes were originally ascertained by measuring or counting them along narrow strips, generally 100 feet broad, laid through the forest along the line of march (so called "valuation surveys"). From the contents of these sample strips (or plots), the contents of the blocks, or forest, were calculated. The rate of increment was determined by counting the concentric rings on a sufficient number of stumps, thus ascertaining the average number of years which a teak tree takes to reach the limits of the several size classes.

The original method was subsequently further elaborated, so that the sample plots are now systematically arranged over the area, with the view of obtaining correct data for the number of trees in the several blocks of the forest. The cuttings, based on these data, were also localized: in other words, an area check was added to the calculated yield, so as to guard against overcutting.

The method does not claim to be theoretically quite correct, because it does not accurately define the extent to which the growing stock of marketable material may be reduced, or ought to be increased, but it is correct enough wherever large areas have to be dealt with in a short time. It works expeditiously, and prevents a deterioration of the forest, if judiciously applied. Had it not been for this method, the valuable teak forests of Lower Burma might have been exhausted, before their sustained yield capacity had been ascertained. It is a method to be strongly recommended for adoption in countries where systematic forest administration is in its earlier stages.

Section IV.—Regulation of the Yield according to Increment and Growing Stock, combined with the Allotment of Areas to the several Periods of a Rotation.

This method was originally elaborated by Carl Heyer, and classed under Section III., as it rested on the Austrian method. Subsequently it was further developed, especially by Gustav Heyer, until it became the combination indicated in the above heading: it is generally known as "Heyer's method."

1. *Principle of the Method.*

The principle of Heyer's method is as follows:—

(*a*) To arrange all woods into a general working plan according to periods, so that each period contains the same, or approximately the same, area. The object of this arrangement is, to prevent loss of increment during the second and subsequent rotations.

(*b*) To equalize the real and normal growing stock, if any difference should exist, in such manner and within such time as may be indicated in each case and approved by the owner.

(*c*) To utilize the real increment, calculating the mean for a

series of years, plus or minus the quota of growing stock determined under (*b*).

It is obvious that these objects can only be realized by a complicated procedure, and even then only approximately, because changes in one direction disturb the balance in another.

2. *Practical Application of the Method.*

(*a*) The first step is to allot, by means of the table of age classes, all woods to the several periods and to equalize the areas by suitable shiftings, as indicated under the method of periods by area; care being taken to allot the woods with due consideration to sylvicultural requirements, and a proper distribution of age classes, as far as this is practicable.

(*b*) The real increment is placed equal to the real final mean increment, for which purpose it is necessary to determine the final age of each wood (which may differ from the normal final age), and its probable volume at that age; the latter divided by the former gives the mean annual increment. In order to avoid having to calculate the increment year by year, it is generally calculated for a number of years, which may be called a'. If an abnormal wood is cut over during the a' years at an age differing from the normal, and a normal wood grows up in its place, the increment must be calculated separately for each part of a' years.

(*c*) The normal and real growing stocks are calculated as for the Austrian method; the former is placed $= \frac{I \times r}{2}$, where I represents the normal final mean increment; the latter is obtained by multiplying the real final mean increment of each wood by its age. The difference between the real and normal growing stock is removed as may be approved by the owner, say, in equal amounts in the course of a years.

(*d*) The theoretical yield is then fixed by the formula—

$$Y = \frac{\text{Real Increment of } a' \text{ years}}{a'} + \frac{{}_{real}G - {}_{norm.}G}{a}.$$

If a' is placed equal to a, that is to say, if the real increment is calculated for the number of years during which any difference between the real and normal growing stock is to be removed, the above expression goes over into:—

$$Y = \frac{{}_{real}G + {}_{real}\,Ia - {}_{norm.}G}{a}.$$

(*e*) The next step is to ascertain whether the woods preliminarily placed into the several periods are sufficient to meet the yield during each period, as calculated by the formula under (*d*), or whether they contain too much or too little volume; in the latter case, suitable shiftings must be made, which necessitate, of course, fresh calculations of the increment and real growing-stock, as the final ages of some of the woods are thereby altered. This process is continued until the requirements of the method are realized, that is to say, until each period contains the same area, and at the same time the volume necessary to meet the yield as calculated under (*d*). As already indicated, the forester must in this respect be satisfied with approximate results.

(*f*) The regulation of the yield is restricted to final returns. The intermediate returns are estimated only for the first period, or part of it, by means of yield tables, or past experience, and added to the final yield.

3. *Merits of the Method.*

The method is one of great precision. On the other hand, it is very complicated, and it calculates the increment, and normal and real growing stock incorrectly, as in the case of

the Austrian method. The latter objection could be removed by using suitable yield tables, instead of the final mean annual increment, for the calculation of the increment and normal growing stock, and by measuring the growing stock actually standing in the forest. Nevertheless, the method involves great labour, and the necessary calculations are of an uncertain nature.

As regards the purely financial principle, the method stands above the methods described under Section III., though it does not do full justice to it.

Section IV.—Selection of Woods for cutting in accordance with Sylvicultural Requirements and the Objects of Management.

The method now to be described has been gradually developed since Heinrich Cotta was called to Saxony in 1811; it advanced by steps, until it was finally put into a precise form by Judeich; hence it is now known as "*Judeich's Bestandswirthschaft.*" The German word "Bestand" can be rendered into English by the word "wood," meaning part of a forest forming a unit of fairly the same description. The main character of the method lies in the fact, that first of all the requirements of each wood are considered, and that the management of the whole forest represents a summing-up of the treatment laid down for each wood. The method approaches the business, therefore, from a point which differs from that of all the previously-described methods, as the latter bring the forest as a whole into play, in order to determine and regulate the yield.

The first step under Judeich's method is a suitable division of the forest into units, called woods or compartments; next, a general plan is sketched to indicate the manner in which a normal arrangement of the age classes, both as regards size and grouping, is to be effected; then the treatment of each wood is determined for a limited number of years; the cuttings

thus indicated are added up, and they represent the yield during the first, say, 10 years, unless considerations for a sustained yield in the future demand certain modifications.

It will thus be seen that the centre of the method lies in the establishment of a full increment and a proper arrangement of the age classes. Not a word is said about the normal growing stock, because this must come of itself, if the other two conditions of the normal state are established.

Finally, an important part of the method is, to fix the working only for a limited number of years, and to have revisions at regularly recurring periods when the requirements of each wood are reconsidered, with due regard to the introduction of a suitable arrangement of the age classes, and especially the introduction of small cutting series. The latter point is of great importance, because it secures a perfectly free hand to the manager to take each wood in hand at the right moment.

In describing the method, the author of this manual has, however, introduced a few changes. Judeich bases the ripeness of each wood upon financial considerations; he ascertains the forest- or indicating per cent., and calls a wood ripe, if that per cent. has sunk below the general per cent. p. This the author considers too narrow a view to take; the ripeness should, in his opinion, be determined by the "objects of management," as laid down by the owner of the forest, of which the financial ripeness may, or not, be one.

1. *Application of Judeich's Method to Clear Cutting in High Forest and the Shelter-wood Compartment System.*

a. The General Plan of Operations.

This is represented by a plan which gives the division of the forest into units of working or compartments. The latter are marked off by natural or artificial boundary lines, as described in Chapter II. This plan enables the forester to determine, in a general way, the order and direction in which

the cuttings should proceed, and the grouping together of compartments into suitable cutting series. The latter may be a definite arrangement, but in many cases temporary cutting series must be designed, which will, at the time of subsequent revisions, be gradually led over into more permanent groupings.

b. Determination of the Final Yield.

The first step is to determine the rotation in accordance with the objects to be aimed at, as laid down by the owner of the forest. How this is done has already been explained.

The financial rotation is ascertained by calculating the soil rental, and the forest per cent. for a number of characteristic woods; in this way it is possible to determine the financial rotation within 10 or 20 years. The rotation actually decided on, as determined by the objects of management, may then be compared with the financial rotation, with a view of bringing out the financial sacrifice involved in a departure from the latter.

The next step is to select, with due consideration of the desired cutting direction and the establishment of suitable cutting series, the woods where final cuttings are called for during the period, for which the working plan is to be prepared, say, for the next 10 years. Special care is taken not to put down for cutting any wood, the removal of which would expose the adjoining woods to windfall; or where difficulties of transport would be encountered. Subject to the modifications caused by these considerations, the following areas would be selected :—

(1) All areas which must be cut to meet sylvicultural or protective necessities, such as the establishment of severance cuttings, woods which must be sacrificed in order to work up to a proper grouping of age classes and arrangement of cutting series.

(2) All decidedly ripe or over-ripe woods; the ripeness to be determined by the objects of management. In the

case of a financial management, this would comprise all woods the current forest per cent. of which has sunk below the general per cent. p.

(3) All woods, the ripeness of which is more or less doubtful, and which may be situated in the direction of the cuttings. This includes the woods which will become ripe during the working plan period.

The sum total of the cuttings indicated under these three headings represents the final yield to be assigned to the period, for which the working plan is prepared.

For small forests, or those where a sustained annual or periodic yield is not called for, nothing further is required. It is different in the case of extensive areas, especially those where considerations for a steady annual income, for the regular supply of markets, or the occupation of the staff and workmen, necessitate an approximately even annual outturn. Here the yield, as determined above, must be subjected to a modifying regulator, either as regards the area to be cut or the volume to be removed during the working plan period.

This regulator can take any suitable shape, such as the size of the mean annual or periodic coupe, the yield as calculated by the Austrian method, Hundeshagen's, or Heyer's methods. Judeich prefers the mean annual coupe as obtained by dividing the total area by the fixed rotation. If a forest has an area of 2,000 acres and is worked under a general rotation of 100 years, the mean annual coupe would be equal to $\frac{2000}{100} = 20$ acres. During a working plan period of 10 years the normal cutting area for it would amount to $20 \times 10 = 200$ acres. In other words, during a period of 10 years 200 acres should be cut over, and the areas selected for cutting should be brought within that limit. This, however, is only desirable if the proportion of age classes is fairly normal. In all cases where considerable deviations from it exist, such a narrow limit cannot be drawn, because in some cases it is highly desirable to cut more than the normal area, if for

instance too large a proportion of old or defective woods exist, and in others the cuttings should be below the normal area, if for instance the area of mature woods is deficient. Hence the regulator should give merely the maximum and minimum area to be dealt with. In the above case the area might be given as 150 to 250 acres.

As long as the total area as determined above under (1) to (3) falls between these limits, it may be accepted as the area to be dealt with during the first ten years. If it is larger than the maximum, then some of the most suitable areas enumerated under (3) should be held over until the next period; if smaller than the minimum, then possibly some further woods may be found which could be added to those already placed under (3). In extreme cases the yield may be kept for a number of years below the proper minimum.

c. *The Intermediate Yields.*

The limit between final and intermediate yields is not always very clear. In a general way it may be said that—

1. Final yields comprise—

(*a*) All returns obtained from woods which are put down for regeneration during the first period.

(*b*) All returns from other woods, which in consequence of unforeseen causes are so large that the regeneration of the woods becomes necessary, whether the final cutting over is done during the working plan period or later on.

2. Intermediate yields comprise all other returns derived from—

(*a*) Cleanings.

(*b*) Ordinary thinnings.

(*c*) Pruning, cutting of standards, etc.

(*d*) Accidental cuttings, such as dry wood cuttings, windfalls, etc., in so far as they do not occur in the areas put down for regeneration during the first period.

The quantity of intermediate yields is best estimated according to past local experience, with due consideration to the condition of the several woods. Where the necessary local data are not available, the most suitable average data obtained elsewhere must be used. The woods to be cleaned and thinned are put down according to their areas.

The question, whether the regulation of the yield should refer to the final cuttings only, or include the intermediate cuttings, has been much discussed. There can be no doubt that the systematic working of a forest should, in the first place, be regulated by the final cuttings. At the same time the intermediate yields may be utilized to equalize any unavoidable inequalities of the final yield. Under any circumstances both classes of yields must be estimated so as to ascertain the probable quantities of produce which will be placed upon the market, and to prepare the annual budgets.

d. Separation of Yield into Classes of Produce.

The yield should be separated according to classes of produce, as it is brought into the market, say as timber and firewood, or large timber, poles, mining props, fagots, etc., each being given in solid cubic feet. This separation should be based upon locally obtained proportional figures.

It is also desirable to give the yield of the important species separately, as for instance oak, other broad leaved species, larch, other conifers, etc. In India, teak, sál, deodar, and some other valuable species, should always be given separately.

2. *Application of the Method to other Sylvicultural Systems.*

a. Coppice.

In the first place the rotation must be determined. By dividing the total area (real or reduced) by the rotation, the size of the annual coupe is obtained.

Next the area is divided into as many coupes as the rotation contains years, taking into consideration all matters influencing

a proper arrangement of the age gradations, more especially the requirements of transport.

If a coppice forest is so extensive that it is desirable to cut in several places in each year, while the rotation remains the same throughout equal to r, it is first divided into a corresponding number of working sections, and then each of the latter into r annual coupes.

If these several working sections are treated under different rotations, a separate account must be kept for each; for instance, oak coppice worked for bark alongside of coppice of an entirely different nature, such as alder coppice.

In order to obtain as far as practicable equal annual returns, the calculations should be made with reduced areas, though it is not necessary to go into very minute calculations. The different coupes should be marked on the ground.

The final yield is ascertained by estimating the returns which may be expected from the areas to be cut over during the working plan period.

Intermediate returns consist of all cuttings made on areas not put down for cutting over during the working plan period. As a rule they are not of much importance. Their amount should be estimated according to local average figures.

b. Coppice with Standards.

The first step is to lay down a division into annual coupes in the same way as for simple coppice; this division regulates the yield of the underwood.

The normal yield in overwood, as given on page 243, can only serve as a very general guide; in reality, the management of the overwood partakes of the character of forest gardening or selection fellings. Hence this sylvicultural system offers considerable difficulties if the areas are extensive. Any but a very elastic method of fixing the yield would be out of place.

The determination of the final yield in overwood is effected by estimating, on the areas to be dealt with during the working

plan period, the probable amount of material to be taken out. In doing this, the forester is guided by sylvicultural considerations, and the degree of ripeness of the several standards.

The sum total of the quantity of overwood thus ascertained, and of the underwood, makes up the expected yield during the working plan period. The executive officer should, however, not be forced to abide absolutely by that estimate, but be permitted to modify it within certain limits in accordance with requirements, as they may become manifest in the course of the period for which the working plan is drawn up.

Intermediate cuttings occur on the areas not put down for final cuttings. Their amount should be estimated in a summary way on the basis of local experience.

c. *The Selection Forest.*

The selection forest resembles the coppice with standard forest, since the several age classes are mixed on the area in a similar manner. In the case of selection forests it is desirable to go round the whole area within a moderate number of years, that is to say, to select again trees for felling over the same portion of the area after a moderate interval, thus avoiding having to cut too much at one time.

The area to be taken in hand annually is obtained by dividing the total area by the number of years, l, fixed as above. By multiplying the quotient by 10, the area to be dealt with during the next ten years is obtained. On the area thus fixed all mature trees are cut, and the necessary thinnings in the younger age classes made. The age of maturity, or the rotation, is fixed as in the case of clear cutting in high forest or the shelter-wood compartment system.

Example:—

Area of a selection forest = 600 acres,

Rotation = 120 years,

$l = 20$ years,

$$\text{Annual cutting area} = \frac{600}{20} = 30 \text{ acres.}$$

Area to be dealt with during the first ten years = 300 acres. On this area all 120-years-old trees are cut, as well as the necessary number of younger trees, so as to reduce them to a suitable number of mature trees. As it is a laborious matter to ascertain the age of the trees, it is desirable to substitute the diameter (or girth) at maturity for the age. For instance, instead of an age of 120 years, it may be laid down that trees with a diameter of, say, 2 feet at chest-height above the ground shall be considered mature.

The areas to be dealt with in each period of, say, 10 years, should be marked off on the ground; in some cases even the annual coupes may be marked.

A distinction between final and intermediate returns is very difficult in the case of selection forests.

To attempt a regulation of the expected returns by volume seems of doubtful utility. At any rate, to fix the yield in material only, whether in cubic feet or number of trees, is a risky procedure, which may lead either to over- or under-working of the forest. By far the best plan is to fix the yield by area, and to determine the minimum size of the mature trees to be cut. This area should not be exceeded. With this reservation, the probable amount of final returns to be cut may be estimated according to von Mantel's or Brandis' methods described above.

3. *Change from one Sylvicultural System into another, called a Conversion.*

The number of conversions from one sylvicultural system to another, which are conceivable, is considerable, and it is impossible to give any general rules of procedure. Whatever the nature of the conversion may be, the only sure basis for the determination of the expected yield is the annual cutting area. Hence the consideration of one specific case will bring out the essential points to be considered in each conversion :—

An irregularly-stocked forest of broad-leaved species, partly coppice, or partly coppice with standards, shall be converted

into a coniferous forest, a conversion which is indicated by the special conditions of the locality.

The first and most important step is, to divide the forest into a suitable number of compartments by laying out a system of roads and rides suitable to the locality. These compartments are then grouped into a suitable number of cutting series, without taking into consideration the present conditions of the several woods, but merely future requirements.

It would be problematic to determine the rotation to be adopted for the future coniferous forest. On the other hand, the age should be determined which the oldest coniferous wood should have reached when the conversion has been concluded, so as to have, from that moment forward, woods of sufficient age to cut and supply the market. This age determines the period during which the conversion is to be effected, called the "conversion period." The latter must not be too short, or else there would be no final cuttings for a number of years, after the conversion has been completed. Supposing 60 years were chosen for the period of conversion, then at its close the oldest coniferous wood should have an age of 60 years.

By dividing the total area by 60, the area is ascertained which should be converted annually.

In selecting the areas to be taken in hand year by year, two considerations present themselves:—

(1) A suitable arrangement of the future cutting series; and

(2) To begin with cutting over the woods which are poorest in increment.

A consideration of both, but more especially of the first, decides the allotment of the annual coupes to the several cutting series.

Example.—A coppice with standard forest of 1,200 acres shall in the course of 60 ears be converted into coniferous forest. Every 10 years $\frac{1200}{6} = 200$ acres must be taken in hand for conversion. In that case the yield during the first 10 years would consist of:—

(1) The clearing of 200 acres; and
(2) The treatment of 1,000 acres as coppice with standards.

During the second 10 years:—

(1) The clearing of 200 acres; and
(2) The treatment of 800 acres as coppice with standards.

During the third 10 years:—

(1) The clearing of 200 acres;
(2) The treatment of 600 acres as coppice with standards;
(3) Thinnings in the oldest coniferous woods, &c.

It is evident, therefore, that the returns fall off from period to period, in so far as the reduction is not made good by thinnings in the young coniferous woods. This can to some extent be modified by not making any cuttings in the 200 acres of coppice with standards which will come under conversion during the next period of 10 years; in other words, to let the material get 10 years older than it otherwise would.

The expected yield is determined by estimating the returns from the 200 acres to be converted and adding thereto the necessary cuttings on the rest of the area; the latter should be sparingly done, so as to equalize the cuttings as much as possible.

CHAPTER V.

THE WORKING PLAN REPORT.

UNDER the working plan report is understood the document which gives, in a systematic manner, all the information which has been indicated in the previous chapters, and describes the system of management in such detail as may be required in each case. No general rule can be laid down in this respect. For forests, which are of great value, and which yield high returns, very detailed plans should be drawn up; for forests which give as yet only small returns simple plans would be indicated.

By way of illustration the following arrangement is given, but it must be understood that in one case many of the headings may be omitted, while in another additional information may be required:—

Working Plan Report.

INTRODUCTION.

CHAPTER I.—GENERAL DESCRIPTION.

1. Name and situation of forest; name of proprietor.
2. Boundaries.
3. Area.
4. Configuration of the ground.
5. Rock and general character of the soil.
6. Climate.
7. Legal position of forest, rights and privileges.
8. Surrounding population and its requirements.
9. Markets, lines of export.
10. Prices of the several classes of produce.

11. Cost of extraction and of transport to markets; supply of labour.
12. General description of forest growth.
13. Injuries to which the crop is exposed.
14. Rate of growth.
15. Yield tables, volume tables, form factors, reducing co-efficients, &c., used in the calculation of the volume and increment of the woods.
16. Organization and strength of the forest staff.

Chapter II.—Detailed description of Compartments.

Chapter III.—Division and Allotment of Areas.

Chapter IV.—Description of the Method of Treatment.

1. The objects of management.
2. Choice of species.
3. ,, sylvicultural system.
4. Determination of the rotation.
5. General lines of treatment.
6. ,, ,, yield.

Chapter V.—Special Working Plans.

1. Plans of utilization.
    - *a.* Final cuttings.
    - *b.* Intermediate cuttings.
    - *c.* Minor produce.
2. Plan of formation.
3. ,, ,, other works.
4. Maps illustrating the condition of the forest and the proposed treatment.

Chapter VI.—Miscellaneous.

1. Reorganization of the forest staff.
2. Financial forecast.

3. Proposals for the control of the execution of the working plan.
4. Miscellaneous observations.

The shape of the special working plans enumerated in Chapter V. of the report depends entirely on local circumstances. Some forms will be found in the appendices of this volume, such as were considered suitable for the forests in question. On the whole it is desirable that the forms should at once provide for a suitable control of the execution of the working plan, a subject to be dealt with in the next chapter.

CHAPTER VI.

CONTROL OF EXECUTION AND RENEWAL OF WORKING PLANS.

It is not sufficient to prepare a working plan; it is also necessary to see that its provisions are carried out; and when the period for which it lays down the management of a forest has come to an end, a new plan, or rather a revised plan, must be prepared.

As the preparation of a first working plan is to some extent based upon doubtful data, it is of importance to keep a careful record during its execution, so as to eliminate in the course of time all such doubtful elements. Apart from this, changes in areas or in other respects may occur, which must be noted. The work of control and renewal comprises, therefore, three distinct operations:—

1. The record of changes as they occur.

TABLE OF RECEIPTS

Year.	Area, Acres.	Wood Sold, in Cubic Feet Solid.				Receipts, Shillings.			Ex-	
		Timber.	Fire-wood.	Bark.	Total.	From Wood.	From Minor Produce	Total.	Har-vesting of Wood.	Har-vesting Minor Produce
1891	253	12,300	7,000	100	19,400	7,600	400	8,000	1,200	100
1892										
.										
.										
1900										
Total......		130,000	70,000	1000	201,000	75,000	4000	79,000	14,000	900
Annual Average	253	13,000	7,000	100	20,100	7500	400	7,900	1,400	90

2. The record of works.
3. The preparation of revised working plans from time to time, or renewals.

1. *Record of Changes.*

(*a*) All changes in the areas must be recorded. Part of the area may be sold or exchanged, or additional areas bought; areas hitherto used for the production of wood may be set aside for other purposes, or *vice versâ*. The progress of the cuttings may cause alterations in the allotment of areas; natural phenomena may produce changes, such as floods, landslips, &c. All such changes should be noted at the close of each year, in the maps as well as in the table of areas.

(*b*) All final cuttings should be entered on the record and the maps.

2. *Record of Works.*

The record of works has for its object:—

(*a*) To give a general view of all cuttings in the forest, and their distribution over the several woods, or compartments.

AND EXPENSES.

PENSES, SHILLINGS.					NET RESULT, IN SHILLINGS.		FOREST CAPITAL, SHILLINGS.			Percentage given by Forest Capital during Year.	Remarks.
Formation & Improvement.	Administration & Protection	Taxes, &c.	Miscellaneous.	Total.	Total.	Per Acre.	Soil.	Growing Stock.	Total.		
200	100	200	100	2,200	5,800	22·92	31,100	128,700	159,800	3·63	
2,100	4,000	2,000	1000	21,000	55,000						
210	100	200	100	2,400	5,500	21·74	31,100	128,700	159,800	3·44	

(*b*) To give the means of comparing the provisions of the working plan with the execution or actual results.

The special shape to be adopted depends on local circumstances, but information on the following points is required:—

(1) Result of each cutting according to quantity and amount realized by its sale.
(2) A comparison of the estimate with the actual results.
(3) The harvest of minor produce according to receipts, and if possible, quantity.
(4) The data showing the net results of management. For a sample see table on previous page.
(5) The means of following up the history of each wood or compartment, as illustrated on page 384, Appendix C.

3. *Renewal of Working Plans.*

When the period for which a working plan has been prepared comes to an end, it must be renewed. Such a renewal may, in some cases, amount to an entirely new plan; but in the majority of cases much of the work done on the first occasion can be used again, only subsequent changes being noted.

The most important part of what remains from the provisions of the first working plan is the allotment of areas, or the order of cuttings then initiated; but even this frequently requires modification.

The task at a renewal is, strictly speaking, the same as on the first occasion, except that a good portion of the work need not be done over again, and that the experience gained during the past period makes that task a much easier one than on the first occasion. Hence it may be indicated as follows:—

(*a*) Investigation of the manner in which the provisions of the former working plan have been carried out, whether there were reasons for departing from them, and if so what they were.

(*b*) Investigation of the extent to which the provisions of the former working plan were judicious and appropriate.

(*c*) Preparation of a new working plan, based upon :—

(1) The old working plan.
(2) The corrected records and maps.
(3) The results of past yields in money and material.
(4) The account of past works of formation, tending, and improvement.

APPENDIX A.

WORKING PLAN FOR THE COMMUNAL FOREST OF KRUMBACH

IN THE

OBERFÖRSTEREI LINDENFELS, FORST WALD-MICHELBACH, IN THE PROVINCE OF STARKENBURG OF HESSE-DARMSTADT.*

(PERIOD 1888—1907.)

PRELIMINARY REPORT.

THE preparation of a working plan for the communal forest of Krumbach having been ordered by a resolution dated the 2nd May, 1887, the Grand-ducal Forstmeister in Wald-Michelbach and the Oberförster in Lindenfels have agreed that the following main points shall be attended to:—

(*a*) The forest in question shall form *one* working circle.

(*b*) The prevailing species is the beech, with which oak and Scotch pine are associated in single trees here and there. The method of treatment shall be that of high forest with natural regeneration under a shelter-wood. In order to increase the proportion of valuable timber, it is desirable that, during regeneration, an increased number of useful timber trees should be introduced, especially oak, then ash, maple, silver fir, larch and spruce. The mixture shall, especially in the case of oak, be in small groups suitably distributed over the area, the best soil being selected for the oak. On stony, shallow soil spruce, larch, and Scotch pine should be chiefly mixed into beech.

(*c*) The rotation should be fixed at 100 years.

(*d*) As the several species are not separated according to area, only one working section is required to be called "mixed

* The method of determining the yield prescribed in Hesse-Darmstadt is that of allotment of woods to the several periods of a rotation according to area. The annual yield during the first period is ascertained and fixed by volume.

broad-leaved high forest, with coniferous trees here and there."

(*e*) As the difference in the quality of the locality ranges within very narrow limits, only one quality class need be recognized.

(*f*) The road system is complete. Though many of the roads are steep and not very well laid out, it will not be necessary to alter them at present.

(*g*) The mensuration of the woods, which will be regenerated during the next 20 years (first period) is to be done according to the method of Draudt-Urich.

THE FORSTMEISTER.—THE OBERFÖRSTER.

15th June, 1887.

DESCRIPTION OF COMPARTMENTS.

PRELIMINARY—GENERAL CHARACTER OF THE LOCALITY.

THE Krumbacher communal forest, comprising an area of 164·62 acres, occupies the western and southern slopes of a mountain which rises from the valley of the river Weschnitz, and forms a range of some height. The highest part of the mountain (called Die Stotz) and of the communal forest has an elevation above the sea of 1,437 feet. The forest, commencing at the highest point, stretches down to an elevation of 853 feet, being bounded at the lower edge by other communal forests.

Geologically, the Stotzberg belongs to the granulite-syenite group of the Odenwald; more especially syenite rich in quartz with narrow strips of granulite is here the forming rock. It disintegrates easily into grit and forms a clayey or loamy soil, mixed with pieces of stones, which is generally fresh and deep. Only on exposed ridges and peaks is the soil shallow, dry and hard; in these places the broad-leaved species show only moderately good growth, whereas they exhibit great vigour in all other parts.

As laid down in the Preliminary Report, only one quality class of the locality has been recognized, which, assuming three classes, is the II. or middle class.

[*g.* = good ; *m.* = middling ; *b.* = bad.]

Block.	Compartment.	Area. Acres.	Quality of locality	Working section	Description of Compartment.	Growing Stock.			
						Age in 1888.	Growth	Density.	Quality
Farrenfeld	1	18·78	II.	I.	*Situation:* Western slope, moderately steep, with shallow depressions. *Soil:* Sandy loam, fairly deep, fresh, stony, and covered with boulders. *Wood:* Beech (·9) with oak (·1), latter mostly affected with cancer; thinned in 1878; in south-east a group = 1·73 acres 30—40 years old of not yet thinned beech.	80	g.	g.	g.
,,	2	7·98	,,	,,	*Situation:* Western steep slope. *Soil:* As in Compt. 1. *Wood:* 50—70 years old beech (·9) with oak (·1), older and younger groups changing; requires thinning.	60	m.	g.	m.
,,	3	5·76	,,	,,	*Situation:* Ridge with adjoining western slope. *Soil:* Very stony, covered with boulders. *Wood:* Beech with single oaks; requires thinning.	60	m.	g.	m.
Heimaths	4	10·68	,,	,,	*Situation:* Southern to south-eastern steep slope. *Soil:* Sandy loam, very stony. *Wood:* Beech = ·8, ash and sycamore = ·1, spruce and larch = ·1; thinned for first time in 1878—86, yielding 1377 c' solid.	40	g.	g.	g.
,,	5	9·44	,,	,,	*Situation:* Southern steep slope, partly trough-shaped. *Soil:* Fresh, deep; in the upper part somewhat stony. *Wood:* Beech regeneration coupe; on about 6·18 acres fairly complete young growth of beech 9 years old, with oaks and maples, and on the	15	g.	...	g.
		52·64							

[g. = good; m. = middling; b. = bad.]

Block.	Compartment.	Area. Acres.	Quality of locality	Working section	Description of Compartment.	Growing Stock.			
						Age in 1888.	Growth	Density.	Quality
Br. forward	...	52·64	II.	I.	stony parts spruce here and there; on 3·26 acres 20—35 years old beech; thinned in 1887.				
Heimaths	6	3·21	,,	,,	*Situation:* A narrow deep trough, sloping towards the west, traversed by a small rivulet. *Soil:* Very fresh, deep humus; rich. *Wood:* Open beechwood, 120—140 years old, with long boles, under which a good quantity of beech advance-growth up to 10 years old.	130	g.	g.	g.
,,	7	7·34	,,	,,	*Situation:* South-western slope, pretty steep. *Soil:* Sandy loam, fairly deep and fresh. *Wood:* Beech regeneration, with oak, ash and maple here and there; requires filling up in parts. Over the young growth stands an open beechwood in the final stage, 116 years old.	10	g.	...	g.
,,	8	8·95	,,	,,	*Situation:* Western steep slope. *Soil:* Stony, fairly fresh and deep. *Wood:* On 7·12 acres, beech = ·8, oak = ·2, age 50—60 years; thinned in 1875. ·69 acres and 1·14 acres are two groups on western edge containing beech = ·8, sycamore = ·1, spruce and larch = ·1, aged 27 years; thinned in 1881 for first time.	50	g.	g.	g.
,,	9	3·66	,,	,,	*Situation and Soil:* As in Compt. 8. *Wood:* Beech; thinned in 1875.	75	g.	g.	g.
Kohlwald	10	10·68	,,	,,	*Situation:* Western fairly	75	m.	g.	m.
		86·48							

[g.=good; m.=middling; b.=bad.]

Block.	Compartment.	Area. Acres.	Quality of locality	Working section	Description of Compartment.	Growing Stock.			
						Age in 1888.	Growth	Density.	Quality
Br. forward	...	86·48	II.	I.	steep slope, partly a ridge. *Soil:* Very stony, a good part shallow and dry. *Wood:* Beech=·9, oak=·1, of moderate growth, especially on the ridge; last thinned in 1884—85.				
Kohlwald	11	8·70	,,	,,	*Situation:* Western and north-western steep slope. *Soil:* Stony and covered with dèbris of rock, but mostly deep. *Wood:* Beech=·9, oak=·1; last thinned in 1884—85.	75	g	g.	g.
,,	12	12·65	,,	,,	*Situation:* Western and south-western slope, steep. *Soil:* Stony, on the ridge-shaped south-western parts shallow and dry. *Wood:* 70—80 years old beech, with some oaks and a few conifers. The covering of the soil pierced by much grass.	75	m.	m.	m.
,,	13	3·43	,,	,,	*Situation:* Western slope, moderately steep, some depressions. *Soil:* Fresh, humus-rich, deep. *Wood:* Beech, last thinned in 1880; a few oaks.	80	g.	g.	g.
Sausudel	14	6·99	,,	,,	*Situation:* South-western and southern steep slope. *Soil:* Stony, fairly fresh. *Wood:* Beech=·7, oak==·2, Scotch pine, larch and spruce=·1. In depressions also some small groups of ash.	16	g.	g.	g.
,,	15	6·82	,,	,,	*Situation and Soil:* As in Compt. 14. *Wood:* Thicket, the result of natural regeneration, containing beech=·6,	16	g.	g.	g.
		125·07							

[g. = good; m. = middling; b. = bad.]

Block.	Compartment.	Area. Acres.	Quality of locality	Working section	Description of Compartment.	Growing Stock. Age in 1888.	Growth	Density.	Quality
Br. forward	...	125·07	II.	I.	oak = ·2, Scotch pine, larch, and a few ash = ·2. Some small stony patches have been filled up with spruce.				
Laubheeke	16	6·67	,,	,,	*Situation:* Western and south-western slope, here and there ridges. *Soil:* Stony, here and there shallow. *Wood:* Beech with single oaks, 60—70 years old. In the south-eastern part of the compartment about $1\frac{1}{4}$ acres 22 years old (beech = ·5, spruce = ·2, larch = ·3), once thinned.	65	m.	m.	m.
,,	17	3·81	,,	,,	*Situation:* Gentle western slope, with depressions. *Soil:* Sandy loam, rather stony, mostly deep. *Wood:* Beech = ·9, oak = ·1, with single birches; thinned in 1876.	70	m.	m.	m.
,,	18	5·29	,,	,,	*Situation:* Western slope, moderately steep with depressions. *Soil:* Fairly fresh, with a good covering of leaves. *Wood:* Beech = ·8, oak = ·2; thinned last in 1880.	70	g.	g.	g.
,,	19	5·54	,,	,,	*Situation:* Western slope, moderately steep partly in ridges. *Soil:* Stony, here and there covered with bilberry. *Wood:* Beech = ·9, oak = ·1, with single Scotch pines; thinned in 1876—78.	75	m.	m.	m.
Grosses Köpfchen	20	10·58	,,	,,	*Situation:* Ridge sloping gently towards the west, with its northern to western steep slopes. *Soil:* Sandy loam, stony, on the top of the ridge shallow.	82	m.	m.	m.
		156·96							

[g. = good ; m. = middling ; b. = bad.]

Block.	Compartment.	Area. Acres.	Quality of locality	Working section	Description of Compartment.	Growing Stock. Age in 1888.	Growth	Density.	Quality
Br. forward	...	156·96	II.	I.	*Wood:* Beech = ·9, oak and Scotch pine = ·1. In the southern part on about 1¼ acres regeneration cuttings have been commenced, with incomplete 6 years old growth of beech ; the rest fully stocked, requiring thinning.				
Kleines Köpfchen	21	3·19	,,	,,	*Situation:* Western slope, moderately steep. *Soil:* Loamy sand, fairly fresh and deep, with few stones. *Wood:* Beech, the result of natural regeneration, with an admixture of oak = ·1, and Scotch pine = ·2. The mother trees were removed in 1883.	13	g.	g.	g.
Total area under Wood .		160·15							
Roads & Rides		4·47							
Total area of Forest . .		164·62							

GENERAL WORKING PLAN OF THE HIGH FOREST WORKING

Block.	Compartment	Area, Reduced Acres.	Age in 1888.	Quality of Locality		Allotment of Woods to					
						I. Period. 1888—1907.		II. Period. 1908—1927.		III. Period. 1928—47.	
				No.	Proportional Figure	Reduced Area. Acres.	Mean Age when Cut Over.	Reduced Area. Acres.	Age when Cut Over.	Reduced Area. Acres.	Age when Cut Over.
						Sole Working Section: Mixed Broad-leaved High					
Farrenfeld	1	18·78	80	II.	1·0	...	...	...	...	*18·78*	*130*
	2	7·98	60	...	...	...	...	...	...	7·98	110-
	3	5·76	60	...	...	...	...	...	...	...	...
Heimaths	4	10·68	40	...	...	...	...	...	...	...	...
	5	9·44	15	...	...	...	...	...	...	...	...
	6	3·21	130	...	...	3·21	140				
	7	7·34	10	...	...	...	...	...	...	...	...
	8	8·95	50	...	...	...	...	...	...	...	...
	9	3·66	75	...	...	...	...	...	...	...	...
Kohlwald	10	10·68	75	...	...	...	...	10·68	105		
	11	8·70	75	...	...	...	...	8·70	105		
	12	12·65	75	...	...	*12·65*	*85*	...	...	...	...
	13	3·43	80	...	...	*3·43*	*90*	...	...	...	...
Sausudel	14	6·99	16	...	...	...	...	...	...	...	...
	15	6·82	16	...	...	...	...	...	...	...	...
Laubhecke	16	6·67	65	...	...	...	...	...	...	*6·67*	*115*
	17	3·81	70	...	...	...	...	3·81	100		
	18	5·29	70	...	...	...	...	5·29	100		
	19	5·54	75	...	...	...	...	5·54	105		
Grosses Köpfchen	20	10·58	82	...	...	10·58	92				
Kleines Köpfchen	21	3·19	13	...	...	...	...	...	...	...	...
		160·15				*29·87*		*34·02*		*33·43*	

Remarks.—The areas which have been shifted from

SECTION OF THE COMMUNAL FOREST OF KRUMBACH.

THE SEVERAL PERIODS.

IV. Period. 1948—67.		V. Period. 1968—87.				Blanks.	Remarks.
		With Overwood.		Without Overwood.			
Re-duced Area. Acres.	Age when Cut Over.	Re-duced Area. Acres.	Age when Cut Over.	Re-duced Area. Acres.	Age when Cut Over.	Re-duced Area. Acres.	
Forest, with Conifers here and there.							
...	...	...	...	...	...	...	Shifted, because the wood is of vigorous growth and the second period is overstocked.
5·76	*130*	...	...	...	...	...	Shifted to fill up the IV. period.
10·68	110						
...	...	6·18	105	3·26	105		
...	...	7·34	100				
8·95	*120*	...	...	...	...	...	Of good growth; shifted to fill up the IV. period.
3·66	*145*	...	...	...	...	...	„ „ „
...	...	...	...	...	...	...	Shifted to provide for the I. period.
...	...	...	...	...	...	...	„ „ „
...	...	...	...	6·99	106		
...	...	...	...	6·82	106		
...	...	...	...	...	...	...	Shifted to relieve the II. period.
3·19	*83*	...	...	...	...	...	Shifted to provide for IV. period.
32·24		13·52		*17·07*			Mean area of period $= \frac{160·15}{5} = 32·03$
		30·59					

their proper age class are printed in *italics*.

I. List of Woods already under Regeneration in 1888 and Quan-

II. List of Woods Selected for Utilization and

Serial Number	Block.	Com-partment	Area, Acres. Actual.	Area, Acres. Re-duced.	Species.	Age. In 1888.	Age. At the Time of Cutting (mean).	Work-ing Section.	Trees to be Left as Standards. Species.	Trees to be Left as Standards. No.
								I. Woods already under		
1	Heimaths	5	9·44	9·44	Beech	104	107	I.	Oak	16
					Scotch pine	104	107	...	...	...
2	Heimaths	7	7·34	7·34	Beech	116	119	...	...	...
	Total .		16·78	16·78						
							II. Woods to be Cut and Regenerated			
1	Heimaths	6	3·21	3·21	Beech	130	140	I.	Oak	3
2	Kohlwald	12	12·65	12·65	Beech	75	85	...	...	...
					Oak	71	81	...	Oak	150
					Conifers	70	80	...	...	...
3	Kohlwald	13	3·43	3·43	Beech	80	90	...	Oak	11
4	Grosses Köpfchen	20	10·58	10·58	Beech	90	100	I.	Oak	54
					Scotch pine	80	90	I.	...	...
	Total .		29·87	29·87						

TITY OF OVERWOOD REMAINING IN THEM IN THE BEGINNING OF 1888.

REGENERATION DURING THE PERIOD OF 1888—1907.

Yield in Solid Cubic Feet.					Regeneration.				Remarks.
Estimate.				Actual Result.	Natural. Acres.	Artificial.			
Present Volume.	Increment.	Total.	Mean per Acre.			Manner of Formation	Species.	Area. Acres.	
Regeneration in 1888.									
5238	151	5389	596	...	5·00	Planting	Oak, Spruce	4·44	Increment $\frac{5238}{104} \times 3 = 151$
232	7	239							
8091	209	8300	1131	...	4·50	Planting	Oak, Ash, Spruce	2·84	$\frac{8091}{116} \times 3 = 209$
13,561	367	13,928			9·50			7·28	
during the Period of 1888—1907.									
19,827	1525	21,352	6659	...	2·00	Sowing	Oak	1·21	$\frac{19827}{130} \times 10 = 1525$
33,237	4432	37,669	3442	...	7·50	Sowing	Scotch pine, Larch, Spruce	5·15	
4444	626	5070							
706	101	807							
14,575	1822	16,397	4780	...	2·00	Sowing	Oak	1·00	
						Planting	Ash	·43	
36,279	4031	40,310	3957	...	6·00	Planting	Larch, Spruce, Scotch pine	4·58	
1384	173	1557							
110,452	12,710	123,162	4123		17·50			12·37	

YIELD TABLE

OF

INTERMEDIATE RETURNS FOR BEECH IN THE KRUMBACHER COMMUNAL FOREST.

In the absence of sufficient data for a local yield table, the normal yield tables of Dankelmann have been used, after determining the quality by means of the average height of the woods. Based upon the latter, it has been ascertained that the Krumbacher Communal Forest belongs to the IVth Quality of Dankelmann's yield tables for the beech.

Small modifications were introduced, especially because it is desired to increase the proportion of oak, ash, and conifers, a circumstance which necessitates a moderate increase in the yield of the thinnings as compared with pure beech woods.

YIELD-TABLE.

Age Class.	Mean height of dominant part of Wood at end of age class, in feet.	Yield of Thinnings c' solid per acre.
21— 30	16	170
31— 40	28	200
41— 50	38	230
51— 60	44	245
61— 70	49	260
71— 80	52	230
81— 90	54	200
91—100	—	155
	Total . . .	1690

This table has been used to calculate the expected yield of thinnings

during the next 20 years. The details have been omitted; the total volume amounts to 50,402 cubic feet.

CALCULATION OF YIELD FOR THE I. PERIOD, 1888–1907.

Sources of Yield.	Solid Cubic Feet.			
	Yield.		Grand Total.	Mean Annual Yield.
	Detailed.	Total.		
a. Thinnings	50,402			
b. Other intermediate yields . .	...	50,402	50,402	2,520
c. Balance in woods already under regeneration	13,928	13,928		
d. Final yield of woods to be regenerated	123,162			
* To be deducted as remaining at end of period	21,792			
e. Balance of *d* to be cut . . .	...	101,370		
f. Total of *c* and *e*	...	...	115,298	5,765
Total of all yields . . .	...	...	165,700	8,285

* The calculation is made as follows:—Regeneration period = 10 years. Mean volume per acre of woods in first period = 4123 cubic feet. Remains, when seeding cutting has been made = $4123 \times \cdot 6 = 2474$ cubic feet. It is assumed that the 2474 cubic feet are cut away in annually equal shares of $\frac{1}{10}$th, that is to say $= \frac{2474}{10} =$ 247·4 cubic feet annually; hence the ten coupes, of $\frac{160 \cdot 15}{100}$ acres of area each, will have per acre at the end of the period volumes equal to

X. Coupe.	IX. Coupe.	VIII.	II.	I.
2474	$2474 - \frac{2474}{10}$	$2474 - \frac{2474}{10} \times 2 - \ldots$	$2474 - \frac{2474}{10} \times 8$	$2474 - \frac{2474}{10} \times 9$ $= 247 \cdot 4$

$$\text{Total} = \frac{160 \cdot 15}{100} \times \frac{10}{2} (2474 + 247 \cdot 4) = 21{,}792 \text{ cubic feet.}$$

General Working Plan for the Method of Periods by Volume, and Data of the Krumbacher

Block.	Compartment.	Area, in Acres.	Age of Woods. Present, in 1888, Years.	Age of Woods. Final.	Final Yield per Acre, in Cubic Feet.	Allotment of Woods. I. Over 80 Years. 1888–1907	II. 61–80. 1908–1927	III. 41–60 1928–1947
						Volume in Cubic		
Farienfeld .	1	18·78	80	*90*	*5045*	*94,745*	...	...
	2	7·98	60	110	6045	...	...	48,239
	3	5·76	60	*130*	*6817*	...	...	...
Heimaths .	4	10·68	40	110	6045	...	...	...
	5	9·44	15	105	5809	...	...	...
	6	3·21	130	140	7117	22,846	...	...
	7	7·34	10	100	5573	...	...	...
	8	8·95	50	*120*	*6460*	...	...	...
	9	3·66	75	*145*	*7267*	...	...	...
Kohlwald .	10	10·68	75	105	5809	...	62,040	...
	11	8·70	75	105	5809	...	50,538	...
	12	12·65	75	*125*	*6638*	...	...	*83,971*
	13	3·43	80	*90*	*5045*	*17,304*	...	...
Sausudel .	14	6·99	16	106	5856	...	...	...
	15	6·82	16	106	5856	...	...	...
Laubhecke .	16	6·67	65	*115*	*6252*	...	...	*41,701*
	17	3·81	70	100	5573	...	21,233	...
	18	5·29	70	100	5573	...	29,481	...
	19	5·54	75	105	5809	...	32,182	...
Grosses Köpfchen .	20	10·58	82	92	5151	54,498	...	...
Kleines Köpfchen .	21	3·19	13	103	5715	...	...	...
Total . .		160·15				*189,393*	*195,474*	*173,919*

Remark.—The compartments which have been shifted

B.

FOR THE METHOD BY AREA AND VOLUME COMBINED, APPLIED TO THE COMMUNAL FOREST. (See page 315.)

TO PERIODS.		Remarks.	AREAS PLACED INTO THE SEVERAL PERIODS.					Remarks.
IV. 21–40. 1948–1967.	V. 1–20. 1968–1987.		I.	II.	III.	IV.	V.	
Feet.								
...	...	...	*18·78*					
...	...	...	...	...	*7·98*			
39,266	...	...	...	...	...	*5·76*		
64,560	...	...	...	...	...	10·68		
...	54,837	...	...	...	...	...	9·44	
...	...	...	3·21					
...	40,906	...	...	...	...	...	7·34	
57,817	...	...	...	...	...	*8·95*		
26,597	...	...	...	...	...	*3·66*		
...	...	...	...	10·68				
...	...	...	...	8·70				
...	...	...	...	...	*12·65*			
...	...	...	*3·43*					
...	40,933	...	...	...	...	...	6·99	
...	39,938	...	...	...	...	...	6·82	
...	...	...	...	...	*6·67*			
...	...	...	...	3·81				
...	...	...	...	5·29				
...	...	...	...	5·54				
...	...	...	10·58					
...	18,231	...	...	...	...	...	3·79	
188,240	194,845		*36·00*	*34·02*	*27·30*	*29·05*	33·78	

from their proper age classes are printed in *italics*.

APPENDIX C.

WORKING PLAN FOR A PORTION OF THE STATE FORESTS OF THE HERRENWIES RANGE

IN THE BLACK FOREST, GRAND DUCHY OF BADEN.

(Period 1884—1893.)

WITH THE RESULTS OF THE ACTUAL WORKING.

GENERAL DESCRIPTION.

1. *Area and Boundaries.*

The areas are recorded as follows :—

(*a*) Productive area	1,747 acres
(*b*) Unproductive area	nil. „
(*c*) Other areas, including fields, meadows, etc. .	2 „
Total area . . .	= 1,749 acres

Alterations in the above figures will probably become necessary when a fresh survey is made.

The outer boundaries are in order, but the internal boundaries require rectification.

2. *Locality.*

The forest here in question occupies on the whole the slopes lying between a hill range on the south and the river Schwarzenbach on the north. The highest point of the hill range, the Hoher Ochsenkopf, has an elevation of 3,465 feet above the sea, while the lowest part, near the Schwarzenbach, is only 2,000 feet above the sea, the mean elevation being placed at 2,600 feet.

The slopes, on which the forest is found, are mostly steep, level spots being only found on the summits of the hills, and towards the lower end, where granite and Bunter Sandstein meet.

The area is drained by the Schwarzenbach (a feeder of the Raumünzach) with its two feeders, the Gartenbach and Dobelbach.

The first mentioned runs from west to east, and the two latter, more or less, from south-west towards north-east. It follows that the forest in the valley of the Schwarzenbach has generally a north aspect, and in the valleys of the Gartenbach and Dobelbach a north-west aspect on one side, and a south-east aspect on the other side of the streams. All the forest areas (except those situated at the highest elevations and which are of no importance) are protected by intervening ranges against the prevailing winds.

Up to a mean elevation of 2,500 feet, granite is the principal rock, which is sometimes (though rarely) pierced by porphyry. Above the afore-mentioned elevation the granite underlies upper Bunter Sandstein (Vogesen Sandstein), and the latter accordingly prevails in the larger part of the forest area.

The granite is generally rich in orthoclase and oligoclase, and therefore decomposes readily, and furnishes mostly a deep soil rich in mineral elements. The decomposition is facilitated, and the quality of the soil improved, by the remarkably numerous springs which appear between the granite and the Bunter Sandstein. Hard slow decomposing quartzite is of rare occurrence.

The Bunter Sandstein is characterized by rapidly aud greatly changing mineral composition, consisting sometimes of readily decomposing rock yielding a deep clay soil, in other cases of hard quartz-gravel, frequently found on the surface in the numerous boulder-drifts. The Bunter Sandstein has numerous rents and fissures in all directions, so that it is rapidly drained, and the disintegration and decomposition are only rarely assisted by springs, which at the best are scanty and intermittent. It follows that the Bunter Sandstein soils, even when formed by the easily decomposed and minerally rich clay sandstone, never equal the best quality of the granite soil; moreover, they change frequently and very suddenly, and without any visible cause, into almost unproductive areas.

On the flat hill tops, layers of fine white sand (produced by the disintegration of the gravelly sandstone) frequently produces an impermeable stratum, preventing the water from percolating, thus causing bogs (or "Grinde") which often extend over considerable areas and are almost unproductive.

The quality of the soil, therefore, ranges between good and unproductive, in the following proportion :—

Good and fairly good to medium	= 78	per cent.
Medium to indifferent	= 12	,,
Indifferent to unproductive	= 10	,,

The climate is rough, and is characterized by long winters with an abundant snowfall, and by rapid changes of temperature ; at the same time it is throughout favourable for forest vegetation, especially for conifers.

3. *Species.*

The details will be found in the description of compartments. Generally speaking, the spruce and silver fir are the prevailing trees, the former being more abundant in the middle and upper parts, the latter at the lower elevations. The beech is associated with them locally and in varying proportions. Scotch pine is found in the granite region chiefly upon dry, steep, rocky slopes with a southerly aspect, and in the sandstone region, especially on dry ridges and the top of the mountains, as well as here and there in other localities. The three conifers attain a maximum height of 140 feet, with regular shaped and little tapering stems. Towards the upper limit of the area the height growth diminishes rapidly, dwindling down to 20 or even 15 feet on the high plateaux. Here the mountain pine and the birch are also found. Reproduction is generally good, except at the higher elevations. A marked difference is found between northern and southern slopes, the growth and reproduction being far more vigorous on the former than on the latter.

The silver fir is much exposed to cancer. Windfalls and snow breakage are fairly moderate, while the damage from insect attacks is very small. During the years 1874–83, the following proportion existed between the different classes of fellings :—

Cuttings caused by insect attacks	= 1	per cent. of total fellings
,, ,, snow breaks	= 12	,, ,, ,,
,, ,, windfalls	= 16	,, ,, ,,
Cancer and other diseases and injury	= 19	,, ,, ,,
Other cuttings	= 52	,, ,, ,,
Total	= 100	,, ,, ,,

4. *Method of Treatment and Rotation.*

The situation and the species necessitate the area being treated under the high forest system. The quality gradations, as indicated under 2, are so conspicuous locally that it is possible (as well as desirable in order to secure a proper idea of the condition of the forest), to group the growing stock according to its characteristics as produced by the quality of the locality, and according to the

method of treatment thereby indicated. The actual basis of this grouping is the yield, and based upon it, the net income or financial result of the management. In this sense the forest may be divided into the following three groups :—

a. Areas subjected to an intensive Management.—To this group belong all areas which, in virtue of their quality (as indicated mainly by the height growth of the trees on fully stocked areas) are capable of producing large timber; areas on which carefully conducted regeneration fellings will produce natural regeneration within a reasonable period of time, and where the cost of any artificial assistance in regeneration is commensurate with the anticipated returns. As lowest limit of this group a normal increment of 43 cubic feet per year and acre, calculated for a rotation of 120 years, has been fixed. The area thus included in the group amounts to 78 per cent. of the whole. It is with this area, and the growing stock standing on it, that the management must more especially reckon, and from which the largest possible sustained yield must be secured. With a suitable composition of the growing stock and a careful application of sylvicultural principles, that object may be obtained under an average rotation of 120 years.

As regards the sylvicultural treatment, and especially the regeneration of the woods, two different classes of forest or growing stock (corresponding with two qualities of locality) stand out prominently.

First: Forest of spruce with a strong admixture of silver fir (the latter occasionally predominating) more or less frequently interspersed with beech and more rarely with Scotch pine.

Secondly: Forest in which spruce predominates with a slight admixture of silver fir and here and there of Scotch pine, but devoid of beech.

The first class of forest occurs in the granite area and on those parts of the Bunter Sandstein (clay sandstone), which have deep easily decomposed soils fit to be classed as good. The characteristic features of this class of forest are the occurrence of beech and deep soils, rarely covered with boulders or débris, lying mostly at the lower elevations; natural regeneration can here be successfully effected in a comparatively short period of time.

The second class of forest occupies the stony slopes of the Bunter Sandstein area, and in exceptional cases the quartzite parts of the granite area. Here the soil is generally covered with loose boulders and rock débris of varying size. These areas are nearly all found at the middle to upper elevations. The conditions described demand

the maintenance of an uninterrupted canopy up to the age of maturity, and a careful execution of the regeneration cuttings spread over a prolonged period of time, or else weeds will spring up, which make regeneration very difficult, and at any rate expensive.

On the whole, however, careful management is sure to be successful in securing natural regeneration in all the areas pertaining to this group; for this purpose, as well as for the production of valuable timber, a rotation of 120 years on an average is considered of sufficient length. The length of the regeneration period differs considerably in the different parts, varying on the whole from 30 to 50 years.

b. The second group consists of woods growing on soils, which, even under the most careful management, cannot be expected to produce trees of first or even second quality. The trees here produced are of such limited height growth, that the production of valuable timber is out of the question. The woods are found in the upper, and here and there in the lower part of the Bunter Sandstein area, where the soil is covered with large masses of the débris of gravelly sandstone, which is not easily decomposed, and where the slightest interruption of the canopy overhead is followed by the appearance of a dense growth of bilberry and heather.

Nevertheless, these areas are capable of yielding timber of the inferior classes, as well as firewood, and the returns which may reasonably be expected from them, justify the application of a method of treatment which, while avoiding any interruption in the canopy and all expensive cultural operations, facilitates natural regeneration; in other words the treatment under the selection system by removing all trees which are deteriorating or incapable of increasing in value. It is difficult to fix any definite rotation, but it is estimated that the trees will take about 150 years to reach maturity.

The lowest quality limit for this group has been fixed at 7 cubic feet increment per acre and year, while the upper limit is, as already indicated, 43 cubic feet. The area comprised in this group amounts to 12 per cent. of the total area.

c. The third group comprises the so-called "Grinden," that is to say the highest parts of the ridges, which are mostly level and have a tendency to bogginess. They are covered by a dense growth of bilberry and heather, and are incapable of producing more than a stunted tree growth, which yields only a scanty quantity of firewood, frequently not covering the price of preparing it; hence financial considerations are entirely out of the question, the areas being pro-

tected merely for the sake of preserving some cover on the hill tops. The group comprises all parts which produce an annual increment per acre of 7 cubic feet and under; they amount to 10 per cent. of the total area.

In so far as the management aims at the production of valuable material, and at favourable financial results as regards outlay for artificial regeneration (where natural regeneration has failed), for improvement, tending, etc., only the areas in the first group can be considered. But in the treatment of those forests which pertain to the principal mountain region of the Black Forest, representing a certain drainage area, the task of forestry goes beyond mere financial considerations. It has in fact been recognized that it is necessary to keep areas of this class well wooded for the sake of a proper husbanding of the water supply in the streams. Accepting this further task, the forest administration has endeavoured, during the last 50 years, to afforest the poorly stocked and frequently entirely bare areas at the higher elevations of the Bunter Sandstein region. In so far as the cultural operations were confined to the boulder drifts of the Bunter Sandstein, they were moderately successful, but the cultural attempts made in the "Grinden" prior to 1870 turned out failures. Since 1873 the cultural operations in the Grinden present a more hopeful aspect, owing to the experience gained by former failures, and it seems desirable to continue them in the future.

The working plan deals in detail only with the forest area subjected to intensive management, but the group worked under the selection system has also been adequately noticed in the general provisions.

The working plan lays special stress upon the execution of improvement fellings, more particularly the removal of cancerous silver firs. For this purpose the ordinary thinnings are utilized; but over and above these, cancerous trees must also be removed from the old woods, where otherwise no further thinnings would be required. In regeneration fellings the trees to fall first under the axe must be those attacked by cancer. Even then not nearly all cancerous trees can be removed during the next ten years. This fact teaches the management that in future a sharp attack must be made on all cancerous trees at the time of the first and second thinnings, even if a temporary interruption of the canopy should thereby be caused. On the rich deep soils of the granite area, which are almost exclusively concerned in these remarks, even an interruption of the canopy extending over a somewhat lengthy period would

not be a misfortune, and preferable to the maintenance of a full canopy consisting to a considerable extent of cancerous trees. The existence of enormous quantities of such trees on the granite area was one of the reasons which led to the yield being fixed at its present rate.

5. *Utilization.*

a. Yield of Major Produce.

The actual yield during the last 40 years has been as follows :—

Compartment.	Yield, in Solid Cubic Feet.					
	1844–53.	1854–63.	1864–73.	1874–83	Total.	Area in Acres.
1. Schwarzenbronn .	213,836	122,369	149,843	79,141	565,189	65
2. Schwarzenberg .	311,518	158,778	200,733	158,955	829,984	211
3. Riesenkopf . .	12,502	47,288	206,212	65,617	331,619	76
4. Mehliskopf . .	—	—	—	—	—	34
5. Grünwinkel . .	19,742	121,629	57,423	202,252	401,046	202
6. Dobelbach . .	26,875	42,697	30,195	69,925	169,692	178
7. Hoher Ochsenkopf	—	—	—	—	—	101
8. Kleingartenkopf .	34,256	2,331	1,448	1,024	39,059	76
9. Kleingarten .	375,687	138,825	256,603	195,578	966,693	362
10. Grossgarten . .	62,544	46,688	26,417	59,118	194,767	175
11. Sachsenbronn .	34,927	47,783	111,351	106,194	300,255	95
12. Gartenbach . .	86,311	83,345	494,665	156,412	820,733	172
	1,178,198	814,733	1,534,920	1,094,216	4,622,067	1717
Average per year .	117,820	81,473	153,492	109,422	115,552	
Average per year and acre	67·44	46·64	87·86	62·63	66·14	

From the appended statistical table it will be seen that the estimated increment of the next ten years amounts to 1,086,130 cubic feet.

The actual growing stock amounts to			9,488,731	cubic feet
The normal	„	„	7,892,160	„
The surplus of	„	„	1,596,571	„

The surplus of growing stock is due to a surplus of woods over 100 years old. With favourable prices for timber, the removal of this surplus in the shortest possible time would be advisable, so as to prevent loss of increment, and take unnecessary capital out of the forest, but as prices run low at present, it appears judicious to keep the greater part of it over for a while.

A consideration of the several compartments showed that the removal of the following material during the next ten years is advisable on sylvicultural grounds :—

Final cuttings . . .	1,146,000 cubic feet
Intermediate cuttings .	154,000 „
Total . .	1,300,000 „

As this amount exceeds the expected increment by 213,870 cubic feet, equal to about $\frac{1}{7}$th of the surplus of growing stock, the yield has been fixed at 1,300,000 cubic feet, or annually :—

Final cuttings . . .	114,600 cubic feet
Intermediate cuttings .	15,400 „
Total . .	130,000 cubic feet.

If in the course of the 10 years prices should rise, there would be no objection to reduce the surplus of growing stock further by additional cuttings.

The disposal of the yield is effected as follows :—

(1) Free grant to the Roman Catholic Priest at Herrenwies	=	1,500 cubic feet
Free grant to the Roman Catholic School at Herrenwies	=	1,000 „
(2) Sale by public auction and occasionally by private sale	=	127,500 „
Total annual disposals . .		130,000 „

b. Minor Produce.

The principal items are forest pasture and the removal of litter, the utilization of which is permitted to the Herrenwies settlers, as a privilege.

According to Government orders the privilege of forest pasture may be exercised only to such extent as the condition of the forest and the requirements of regeneration may permit. The district

forest officer indicates from time to time the localities in which the privilege may be exercised. The privilege of removing litter free of charge is exercised under the same conditions. The exercise of these privileges is nowhere injurious, and may be continued during the next ten years.

The grass growing in blanks, on roads and in plantations has hitherto been sold for the benefit of the State, and, under suitable supervision, the practice may be continued.

The removal of building stones, the sale of plants, etc., is insignificant.

6. *Division into Compartments.*

The contemplated new division into compartments must be postponed until the projected road system has been completed.

DESCRIPTION OF COMPARTMENTS.

Block and Compartment.		Area in Acres.	Description of Wood.
Name.	No.		
I. Ochsenköpfe.			
Schwarzenbronn	1	65	Spruce with silver fir, some beech, Scotch pine, larch. About ·6 of area 30—50 years old, some trees older. About ·4 of area 10—30 years old. Above the road fairly complete stocking; in youngest parts still suffering from frost; below road still some blanks caused by late cutting out of old trees; in the latter part still 120—150 years old spruce and silver fir in the final stage; these show a decreasing increment. Growth on the whole fairly good.
Schwarzenberg .	2	211	*a* = 130 acres; 15—40 years old spruce and silver fir with some Scotch pine and beech; some lately planted, younger, a few up to 60 years old. About 25 acres planted. Where the shelter wood has been removed, stocking generally complete, in the rest still patchy with patches of bilberry intervening. Growth generally between good and fairly good; along Herrenwies meadows partly only fair, the spruce still suffering from frost. In the north-western part, below the road, on the Riesenkopf road, and in the south-east along Dobelbach, on about 37 acres 110—140 years old spruce and silver firs of decreasing increment are standing in the final stage.

DESCRIPTION OF COMPARTMENTS—*continued.*

Block and Compartment. Name.	No.	Area in Acres.	Description of Wood.
			b = 81 acres (in three parts), spruce and silver fir with a few beech and Scotch pine, generally 50—70 years old, but some small groups only 30—50 years old; generally well stocked, here and there somewhat thin and patchy. Growth between good and fairly good. On 3 acres on the Dobelbach, 80—90 years old spruce, cover complete and growth good.
Riesenkopf .	3	76	*a* = 47 acres; 100—130 years old spruce and silver fir, some older; on the whole cover fairly complete; towards compartment Schwarzenberg somewhat thin, but on about 10 acres with a fair young crop of silver fir and spruce up to 15 years old. Growth fairly good, on the higher part inferior. About 5 acres along the road is a windfall area, now stocked with some silver fir and spruce growth. *b* = 24 acres; 9—20 years old spruce (a few older), with some Scotch pine and larch, mostly well stocked, showing good to fairly good growth. *c* = 5 acres; Grinde, in upper part heather covered, with 100 and more years old short and stunted Scotch pine, some spruce and mountain pine. On the whole poorly stocked. Part underplanted with 20—40 years old spruce, which show very poor growth.
Mehliskopf .	4	34	50—90 years old (and more), mountain pine with some spruce, Scotch pine, birch and mountain ash; towards compartments 3 and 5 cover fairly complete, in the southern and south-western parts interrupted by larger and smaller areas of heather. Growth inferior.
Grünwinkel .	5	202	*a* = 186 acres; 110—150 years old, some older, spruce and silver fir, some beech with a few Scotch pine. In irregular final and seeding stage, in the southern part cover still fairly complete in strips. On ·4 of the area stocked with up to 30 years old silver fir and spruce and a few beech. Growth of old trees still fairly good; on some stony ridges (about 7 acres) middling and inferior; young growth mostly only middling. *b* = 16 acres on the highest part in the south and west, Grinde; heather-ground

DESCRIPTION OF COMPARTMENTS—*continued.*

Block and Compartment. Name.	No.	Area in Acres.	Description of Wood.
			with 100 and more years old crippled Scotch pine, spruce, some mountain pine and birch; in some parts up to 60 years old advance growth thinly stocked. Here and there traces of plantings, 24 years old spruce.
Dobelbach .	6	178	a = 133 acres; 100—130 years old, some up to 200 years, spruce and silver fir, some Scotch pine; on the whole cover fairly complete; only in the western third along Grünwinkel through windfalls and dry wood cuttings somewhat thin and patchy: in the thin parts as yet little, up to 15 years old, advance growth in single trees. Growth good to fairly good. (Ilex found). b = 27 acres (consisting of the upper south-eastern portion and a ridge running from it in a north-western direction to the centre of the compartment), 100—130 years old (some older), short-stemmed spruce with some Scotch pine and silver fir forming a thin, often very thin, wood; in parts younger up to 60 years old spruce, or an incomplete miserable undergrowth of 25 years old spruce and Scotch pine (experimental planting). Growth middling to inferior. c = 18 acres (uppermost part on the south) Grinde; heather-land with 100 years and more old crippled Scotch pine, some spruce, birch, thinly stocked; here and there remnants of 25 years old planted spruce and Scotch pine.
Hoher Ochsen-	7	101	70 and up to over 100 years old Scotch pine and mountain pine with spruce, some birch, sometimes forming a very thin wood of single trees, sometimes in smaller or larger groups; everywhere intersected by heather places and blanks. Growth inferior, even crippled.
Kleingartenkopf	8	76	100—120 years old, in some parts younger, some over 300 years old, spruce with Scotch pine, few silver fir, some mountain pine. In the western third and on the eastern point still fairly well stocked, some groups even well stocked; otherwise the wood is very thin and open. Growth middling to inferior; here and there an incomplete miserable under-

DESCRIPTION OF COMPARTMENTS—*continued.*

Block and Compartment. Name.	No.	Area in Acres.	Description of Wood.
			growth of 30—50 and more years old spruce and Scotch pine (planted).
Kleingarten .	9	362	*a* = 161 acres. Spruce and silver fir, some beech. Mostly 50—80 years old, in strips and single trees up to 100 years old, others only 30—50 years old. In the eastern part are about 50 acres 80—100 years old. Everywhere spruce and silver fir standards up to 150 years old, mostly showing good growth. Almost throughout rather thinly stocked, here and there patchy, in consequence of late final cuttings and removal of cancerous silver firs. Growth mostly good, only towards the southern higher part decreasing. *b* = 122 acres (in 3 places). Spruce and silver fir with some beech, $\frac{15\text{—}40}{\text{average} = 30}$ years old, some groups up to 50 years; mostly fully stocked. 120—150 years old (some older) mostly pruned spruce and silver firs in the final stage are standing almost everywhere over the above younger growth. The strip along Dobelbach is finally cleared. Growth good; of the old trees fairly good. *c* = 79 acres (upper part towards the south), 120—300 years old pruned Scotch pine and spruce, few silver fir and birch, thinly stocked, often open; on the whole poorly undergrown with 20—50 years old spruce (mostly planted), a few silver fir; the latter in some places form, with up to 100 years old spruce, the picture of a selection forest. Soil much covered with heather. Growth middling to bad; rarely fairly good. On 6 acres near compartment Dobelbach on the main path, 100 and more years old spruce, with a few Scotch pine and silver fir, form a thin canopy and show middling growth.
Grossgarten .	10	175	*a* = 108 acres; spruce and silver fir 80—110 years old, some up to 150, some beech and a few Scotch pine. Partly fully stocked, but the greater part somewhat thin, in the lower part very thin; and here spruce and silver fir advance growth up to 50 years old in single trees and groups. Growth good to fairly good;

DESCRIPTION OF COMPARTMENTS—*continued.*

Block and Compartment. Name.	No.	Area in Acres.	Description of Wood.
			in the upper parts with stones (Halde), partly middling only. b = 37 acres. (Ridge through middle of compartment and strip on south, south-west, and north-west.) 90—110, some up to 200 years old, spruce and Scotch pine, some silver firs, in the uppermost part some mountain pine in a thin, patchy, and often very thin wood; most of inferior growth; here and there traces of 30—40 years old spruce plantings. c = 30 acres (adjoining compartment Kleingarten). A wood resembling a selection forest, of spruce and silver fir with beech, the trees 30—50 years old prevailing; little quite young. The 100—120 years old and older trees appear single and in groups. Growth good; above the cattle track inferior.
Sachsenbronn .	11	95 (and 2 acres other areas.)	100—120 years old (some up to 200 years), spruce and silver fir, also some beech, namely:— On 42 acres, final stage, partly pruned, throughout with $\frac{10\text{—}30}{20}$ years old (in the western part up to 40 years old), silver fir and spruce young growth; about 25 acres in the position of the seeding stage brought about by windfalls and dry wood cuttings; on 5 acres, 80—100 years old, generally complete cover; in the thinner stocked parts is found up to 15 years old silver fir and spruce young growth; on 12 acres (south-eastern corner, near compartment Gartenbach) generally canopy complete, here and there with a little advance growth. On 10 acres (in the west), 70—90 years old, some older spruce with silver fir, fairly complete canopy. On 7 acres (western point), 12—40 years old (in groups and single up to 60 years old), mostly irregular young growth of spruce with some silver fir, forming a fairly complete stocking. Growth of old trees good to fairly good, in the pruned portions partly less good; growth of young wood fairly good.

DESCRIPTION OF COMPARTMENTS—*continued.*

Block and Compartment. Name.	No.	Area in Acres.	Description of Wood.
Gartenbach .	12	172	110—140 years old spruce, silver fir, some older, some Scotch pine, the latter prevailing in places in the upper part, few beech ; in the northern two-thirds in the final stage, partly in seeding stage. In these two-thirds about 85 acres are stocked with young growth of spruce and silver fir pretty completely, in the eastern part very fully ; in the southern third still fairly complete cover, but on the western slope, already somewhat thin, as yet little young growth. Growth in northern two-thirds good, in the southern third good to fairly good ; in the upper part, in the south-east, only middling. In the middle of the compartment are 3 windfall and 1 beetle clearing, together 12 acres ; of these, 7 acres fairly well stocked with up to 25 years old spruce and silver fir.

TABULAR STATISTICAL REPORT OF THE

Compartments.		Distribution of Age'					
		1—40 years old.		41—60.		61—80.	
Name.	Number	Cubic feet.	Acres	Cubic feet.	Acres	Cubic feet.	Acres
		I. Working-Section=Yield-capacity over					
Schwarzenbronn . .	1	48,030	41	70,630	20	...	...
Schwarzenberg . . .	2	200,945	117	169,514	40	192,117	40
Riesenkopf . . .	3	19,072	34	...	...	...	...
Grünwinkel . . .	5	21,189	38	...	...	...	...
Dobelbach . . .	6	...	...	...	...	...	...
Kleingarten . . .	9	109,479	87	201,299	37	423,787	67
Grossgarten . . .	10	26,487	24	46,617	11	25,074	5
Sachsenbronn . . .	11	28,605	51	...	...	35,316	5
Gartenbach . . .	12	13,420	48	...	...	...	...
Total . . .	...	467,227	440	488,060	108	676,294	117
Normal state under a rotation of 120 years .	...	...	...	...	...	...	...
Comparison of real and normal state . +	...	...	...	...	...	...	...
Comparison of real and normal state . −	...	...	...	...	...	...	...
		II. Working-Section=Yield-capacity from					
Dobelbach	6	1,770	...	...	...	...	...
Kleingartenkopf . .	8	10,595	25	...	...	...	...
Kleingarten . . .	9	10,948	25	...	...	...	...
Grossgarten . . .	10	...	...	...	...	...	...
Total . . .	...	23,313	50	...	...	...	...
Normal state under a rotation of 120 years .	...	...	...	...	...	...	..
Comparison of real and normal state . +	...	...	...	...	...	...	...
Comparison of real and normal state . −	...	...	...	...	...	...	...
	III.	**Working Section=Yield-capacity 7 cubic feet**					
Riesenkopf . . .	3	...	...	...	...	...	...
Mehliskopf . . .	4	...	...	...	...	18,900	34
Grünwinkel . . .	5	...	...	...	...	...	...
Dobelbach . . .	6	...	...	...	...	...	...
Hoher Oehsenkopf . .	7	...	...	...	...	...	...
Total . . .	...	...	...	...	...	18,900	34
Normal state under a rotation of 120 years .	...	...	...	...	...	...	...
Comparison of real and normal state . +	...	...	...	...	...	...	...
Comparison of real and normal state . −	...	...	...	...	...	...	...
				Summary of the Three			
Real state of forest . .	...	...	...	...	...	...	...
Normal state of forest .	...	...	...	...	...	...	...
Comparison of real and normal state . +	...	...	...	...	...	...	...
Comparison of real and normal state . −	...	...	...	...	...	...	...

HERRENWIES RANGE.

Classes.						Volume per acre, cubic feet.	Increment.			
81—100.		Over 100 years.		Total.			Annual, per acre.		Total in 10 years.	
Cubic feet.	Acres	Cubic feet.	Acres	Cubic feet.	Acres		Normal.	Real.	Normal.	Real.
		43 cubic feet per Acre annually.								
...	...	34,250	4	152,910	65	2,352	85	70	55,250	45,500
15,892	2	118,662	12	697,130	211	3,304	85	75	179,350	158,250
...	...	381,408	37	400,480	71	5,641	70	61	49,700	43,310
...	...	1,606,861	148	1,628,050	186	8,753	85	71	158,100	132,060
...	...	1,522,104	133	1,522,104	133	11,444	100	71	133,000	94,430
353,156	49	540,329	43	1,628,050	283	5,753	100	78	283,000	220,740
494,418	49	365,870	49	958,466	138	6,945	85	85	117,300	117,300
42,379	5	459,103	34	565,403	95	5,952	100	71	95,000	67,450
...	...	1,373,758	124	1,387,178	172	8,065	100	86	172,000	147,920
905,845	105	6,402,345	584	8,939,771	1354	6,603	...	76	...	1,026,960
...	...	...	...	7,456,200	...	5,507	92	...	1,242,700	...
...	...	...	...	1,483,571	...	1,096	...	...	...	...
..	...	...	...	...	...	...	...	16	...	5,740
		7 to 43 cubic feet per Acre annually.								
...	...	38,850	27	40,620	27	1,504	30	21	8,100	5,670
...	...	203,418	51	214,013	76	2,816	14	14	10,640	10,640
...	...	108,419	54	119,367	79	1,511	43	29	33,970	22,910
...	...	79,460	37	79,460	37	2,148	21	21	7,770	7,770
...	...	430,147	169	453,460	219	2,071	...	21	...	46,990
...	...	...	...	362,880	...	1,657	28	...	60,480	...
...	...	...	...	90,580	...	414	...	...	...	...
...	...	...	...	...	...	...	...	7	...	13,490
	and under per Acre annually.									
...	...	4,500	5	4,500	5	900	7	7	350	350
...	...	...	...	18,900	34	556	7	7	2,380	2,380
...	...	11,300	16	11,300	16	706	7	7	1,120	1,120
...	...	7,400	18	7,400	18	411	7	7	1,260	1,26C
...	...	53,400	101	53,400	101	529	7	7	7,070	7,070
...	...	76,600	140	95,500	174	549	...	7	...	12,180
...	...	...	...	73.080	...	420	7	...	12,180	...
...	...	...	...	22,420	...	129	—	...	...	...
	Working Sections.									
...	...	...	...	9,488,731	...	...	...	...	...	1,086,130
..	...	...	...	7,892,160	...	...	...	...	1,315,360	...
...	...	...	...	1,596,571	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...	...	229,230

SPECIAL WORKING PLAN.

Compartments.	Description of Cuttings, Cultivation, &c.	Cuttings. Final. Cubic feet.	Cuttings. Intermediate Cubic feet.	Cultivation. Acres.	Draining, ditches. Feet.	Road construction. Feet.
1. Schwarzenbronn	Final cutting in regenerated part . . .	34,000				
	Filling up blanks with spruce . . .	...	...	3		
	Thinning and cutting of cancerous silver firs .	...	10,000			
	Total . .	34,000	10,000	3		
2. Schwarzenberg .	*a.* Thinning of shelter-wood and partial final cutting	35,000				
	Filling up blanks with spruce and Scotch pine .	...	...	10		
	a & *b.* Thinning and removal of cancerous trees	...	53,000			
	Total . .	35,000	53,000	10		
3. Riesenkopf . .	*a.* Seeding cutting, and partly final cutting . .	53,000				
	b & *c.* Rest.					
	Total . .	53,000				
4. Mehliskopf . .	Rest.					
5. Grünwinkel . .	*a.* Thinning of shelter-wood, seeding cutting in the fully stocked parts by the removal of cancerous and large trees . .	318,000				
	b. Rest.					
	Total . .	318,000				
6. Dobelbach . .	*a.* Thinning and removal of cancerous trees . .	19,000	19,000			
	b & *c.* Rest.					
	Construction of an export road to meet the main road	...	...	...	...	4,900
	Total . .	19,000	19,000	...	...	4,900
7. Hoher Ochsenkopf .	Rest.					
8. Kleingartenkopf	Rest.					

SPECIAL WORKING PLAN—*continued.*

Compartments.	Description of Cuttings, Cultivation, &c.	Cuttings. Final. Cubic feet.	Cuttings. Intermediate Cubic feet.	Cultivation. Acres.	Draining, ditches. Feet.	Road construction. Feet.
9. Kleingarten .	*a.* Cutting of all old standards and cancerous trees	45,000				
	Thinning	...	3,000			
	b. Thinning of shelter-wood and partially final cutting	198,000				
	Filling up blanks with spruce	..	...	12		
	c. Cutting out of old defective trees where young growth exists . . .	14,000				
	Construction of an export road to meet the main road	...	...	...	...	9,500
	Total . .	257,000	3,000	12	...	9,500
10. Grossgarten .	*a.* Thinning and removal of cancerous trees . .	47,000	47,000			
	b. Rest.					
	c. Removal of standards and cancerous trees . .	25,000				
	Thinning	...	15,000			
	Construction of an export road	...	...	...	...	5,000
	Total . .	72,000	62,000	...	...	5,000
11. Sachsenbronn .	In the regeneration area: thinning of shelter-wood and partially final clearing; in the rest seeding cutting	163.000				
	Filling up blanks with spruce	...	...	3		
	Construction of an export road	...	...	...	...	3,500
	Total . .	163,000	.	3	...	3,500
12. Gartenbach .	Continuation of regeneration cuttings and removal of cancerous trees	195,000				
	Thinning in fully stocked parts	...	7,000			
	Filling up blanks with spruce and Scotch pine.	...	...	8		
	Construction of an export road	...	...	..	...	3,000
	Total . .	195,000	7,000	8	...	3,000

SUMMARY OF THE PROVISIONS OF THE

Compartment.	Provisions of Working Plan.					
	Cuttings.			Cultivation. Acres.	Draining. Feet.	Road Construction
	Final. Cubic Feet.	Intermediate. Cubic Feet.	Total. Cubic Feet.			
1. Schwarzenbronn . .	34,000	10,000	44,000	3	—	—
2. Schwarzenberg .	35,000	53,000	88,000	10	—	—
3. Riesenkopf . . .	53,000	—	53,000	—	—	—
4. Mehliskopf . .	—	—	—	—	—	—
5. Grünwinkel . . .	318,000	—	318,000	—	—	—
6. Dobelbach . .	19,000	19,000	38,000	—	—	4,900
7. Hoher Ochsenkopf .	—	—	—	—	—	—
8. Kleingartenkopf .	—	—	—	—	—	—
9. Kleingarten . . .	257,000	3,000	260,000	12	—	9,500
10. Grossgarten . .	72,000	62,000	134,000	—	—	5,000
11. Sachsenbronn . .	163,000	—	163,000	3	—	3,500
12. Gartenbach . .	195,000	7,000	202,000	8	—	3,000
Total	1,146,000	154,000	1,300,000	36	—	25,900

Note.—The excess was due to heavy windfalls ; it will not derange future

WORKING PLAN AND OF THE EXECUTION.

Results of Actual Work done.						Comparison of Proposed and Executed Cuttings.		Remarks.
Cuttings.								
Final. Cubic Feet	Intermediate. Cubic Feet.	Total. Cubic Feet	Cultivation Acres.	Draining. Feet.	Road Construction. Feet.	Cut too much. Cubic Feet.	Cut too little. Cubic Feet.	
33,034	12,549	45,583	4·4	—	—	1,583	—	
54,517	75,000	129,517	5·0	—	—	41,517	—	Excess due to windfalls and snow break.
132,900	—	132,900	·1	—	—	79,900	—	Excess due to windfalls and snow-break.
—	—	—	—	—	—	—	—	
177,169	—	177,169	·1	—	—	—	140,831	Held back, on account of extra fellings in other compts.
86,606	68,301	154,907	—	—	5,003	116,907	—	Excess due to windfalls.
—	—	—	—	—	—	—	—	
—	—	—	—	—	—	—	—	
342,444	21,635	364,079	8·4	—	9,679	104,079	—	Excess: windfalls and construction of road.
95,852	—	95,852	—	—	5,299	—	38,148	Thinning held over.
111,049	—	111,049	·9	—	3,691	—	51,951	Held back on account of excess in other compartments.
197,660	—	197,660	—	—	2,953	—	4,340	
1,231,231	177,485	1,408,716	18·9	—	26,625	108,716	—	

arrangements, as there is yet a considerable excess of growing stock in the forest.

SAMPLE PAGE OF THE DETAILED CONTROL BOOK.

1. *Schwarzenbronn.*

Year.	Description of Cuttings, Cultivation, etc.	Cuttings. Final. Cubic Feet.	Cuttings. Intermediate Cubic Feet.	Cultivation. Acres.	Draining Ditches Feet.	Road Construction. Feet.
Provision of Working Plan.						
	Final cutting in regenerated part	34,000				
	Filling up blanks with spruce .	...	...	3		
	Thinning and cutting of cancerous silver firs	...	10,000			
	Total....................	34,000	10,000	3	—	—
Execution.						
1884	Final cutting	14,297				
„	Dry and windfall wood . .	813				
1885	Windfalls.	665				
1886	Final cutting, thinning . .	6,166	832			
„	Windfalls	547				
1887	Windfalls	1,363				
1888	Final cutting, thinning . . .	7,759	11,717			
„	Planting	...	...	1·7		
„	Windfall	82				
1889	Dry wood, windfalls . . .	649				
„	Planting	...	...	2·2		
1890	Windfalls	693				
„	Planting	...	...	·1		
1891	Planting	...	...	·2		
1892	Planting	...	...	·1		
1893	Planting	...	...	·1		
	Total....................	33,034	12,549	4·4		

APPENDIX D.—TABLES.

THIS Appendix contains the following Tables :—

TABLE I.—Yield Tables used in the Kingdom of Saxony for the determination of the Quality of Locality.

TABLE II.—Area of Circles for Diameters ranging from 1 inch to 60 inches.

TABLE III.—Volumes of Cylinders and the Sum of the Area of Circles for Diameters ranging from 1 inch to 48 inches.

Example.—Find the volume of a log which has a diameter in the middle of 15 inches and a length of 24 feet :—

Volume of 20 feet length $= 10 \times 2{\cdot}4544$ . .	= 24·544	cubic feet.
Volume of 4 feet length .	= 4·9088	„ „
Total . .	= 29·4528	„ „

In the same way: Area of 24 circles of 15 inches diameter . . . = 29·4528 sq. feet.

TABLE IV.—Tables of Compound Interest :—

A. Amount to which a capital accumulates with compound interest in n years :—$C_n = C_0 \times 1{\cdot}op^n$.

B. Present value of a capital to be realized after n years, $C_0 = \frac{C_n}{1{\cdot}op^n}$.

C. Present value of a perpetual rental due every n years, $C_0 = \frac{R}{1{\cdot}op^n - 1}$.

D. Present value of a rental due at the end of every year, altogether n times, $C_0 = \frac{r\,(1{\cdot}op^n - 1)}{1{\cdot}op^n \times {\cdot}op}$.

If the rentals refer to the past n years, the positions in this Table must be multiplied by the corresponding values of $1{\cdot}op^n$, to be taken from Table IV., A., so as to comply with the formula :

$$C_0 = \frac{r\,(1{\cdot}op^n - 1)}{{\cdot}op}.$$

I. Yield Tables used in Saxony for the Determination

For One Acre of a normal, or fully stocked, wood in solid cubic feet, including

Oak.						Beech.					
Age, Years.	Quality classes, in solid cubic feet.					Age, Years.	Quality classes, in solid cubic feet.				
	I.	II.	III.	IV.	V.		I.	II.	III.	IV.	V.
10	290	210	200	140	70	10	330	290	210	190	100
20	740	610	490	360	200	20	800	660	510	390	200
30	1,270	1,060	840	630	330	30	1,310	1,160	900	640	330
40	1,870	1,560	1,210	930	470	40	2,160	1,760	1,360	960	470
50	2,560	2,120	1,670	1,250	640	50	3,020	2,440	1,870	1,300	630
60	3,320	2,750	2,170	1,600	810	60	3,990	3,220	2,440	1,670	800
70	4,160	3,430	2,700	1,970	1,000	70	5,070	4,070	3,070	2,070	980
80	5,060	4,160	3,250	2,340	1,190	80	6,260	5,000	3,740	2,490	1,160
90	5,990	4,990	3,800	2,720	1,360	90	7,460	5,930	4,400	2,880	1,330
100	6,960	5,660	4,360	3,070	1,520	100	8,570	6,800	5,030	3,260	1,480
110	7,950	6,430	4,920	3,420	1,670	110	9,620	7,650	5,590	3,570	1,600
120	8,950	7,220	5,480	3,740	1,800	120	10,580	8,330	6,090	3,840	1,700
130	9,900	7,950	6,000	4,060	1,930	130	11,450	8,990	6,530	4,070	1,770
140	10,830	8,670	6,520	4,340	2,040	140	12,250	9,570	6,920	4,260	1,830
150	11,700	9,350	6,990	4,620	2,140	150	12,960	10,100	7,250	4,400	1,860
160	12,560	10,000	7,450	4,890	2,240						
170	13,350	10,600	7,860	5,120	2,330						
180	14,090	11,160	8,250	5,330	2,410						
190	14,780	11,680	8,590	5,500	2,470						
200	15,330	12,100	8,870	5,660	2,530						

Example.—A fully stocked beech-wood has a volume, when 60 years old, of 3,220 cubic feet; hence *it* grows on a locality of the II. Quality. *Again*, a fully stocked oakwood, 140 years old, shows a volume of 7,600 cubic feet; hence it grows on a locality between II. and III. Quality.

OF THE QUALITY OF LOCALITY.

all wood above ground, but exclusive of roots and stumps.

Alder. Age, Years.	Quality classes, in solid cubic feet. I.	II.	III.	IV.	V.	Birch. Age, Years.	Quality classes, in solid cubic feet. I.	II.	III.	IV.	V.
10	860	690	510	340	160	10	670	530	400	260	110
20	1,800	1,440	1,090	730	340	20	1,410	1,110	820	540	230
30	2,800	2,260	1,700	1,160	540	30	2,360	1,860	1,360	860	390
40	3,820	3,070	2,330	1,580	760	40	3,370	2,660	1,940	1,230	540
50	4,790	3,860	2,930	1,990	950	50	4,300	3,390	2,470	1,560	690
60	5,730	4,630	3,520	2,390	1,130	60	5,120	4,020	2,920	1,810	790
70	6,640	5,350	4,050	2,730	1,300	70	5,740	4,490	3,230	1,970	840
80	7,530	6,050	4,560	3,060	1,460	80	6,230	4,840	3,460	2,070	870
90	8,360	6,720	5,040	3,370	1,600						
100	9,100	7,300	5,470	3,640	1,710						

Coppice of Alder, Poplar, Willow. Age, Years.	Quality classes, in solid cubic feet. I.	II.	III.	IV.	V.	Coppice of Oak, Beech, Ash, Birch. Age, Years.	Quality classes, in solid cubic feet. I.	II.	III.	IV.	V.
5	490	370	260	140	30	5	310	240	160	95	15
10	1,000	740	510	290	90	10	640	490	330	170	45
15	1,540	1,190	860	440	160	15	980	760	510	280	85
20	2,100	1,640	1,190	710	240	20	1,330	1,040	700	480	130
25	2,730	2,120	1,510	900	300	25	1,730	1,330	930	530	175
30	3,360	2,600	1,840	1,090	350	30	2,120	1,630	1,140	660	220
35	3,940	3,040	2,140	1,240	400	35	2,460	1,900	1,330	770	260
40	4,520	3,470	2,420	1,390	430	40	2,800	2,160	1,510	870	290

I. Yield Tables used in Saxony for the

For One Acre of normal, or fully stocked, wood in solid cubic

Scotch Pine.						Larch.					
Age, Years.	Quality classes, in solid cubic feet.					Age, Years.	Quality classes, in solid cubic feet.				
	I.	II.	III.	IV.	V.		I.	II.	III.	IV.	V.
10	560	460	360	260	140	10	760	610	470	330	170
20	1,500	1,210	930	640	310	20	1,890	1,510	1,140	770	360
30	2,830	2,270	1,690	1,110	500	30	3,290	2,620	1,940	1,260	570
40	4,320	3,410	2,520	1,610	740	40	4,840	3,820	2,790	1,760	790
50	*5,800*	*4,590*	*3,370*	*2,140*	*960*	*50*	*6,400*	*5,030*	*3,610*	*2,260*	*990*
60	7,220	5,690	4,160	2,630	1,170	60	7,800	6,120	4,400	2,700	1,170
70	8,550	6,730	4,920	3,100	1,370	70	9,060	7,070	5,100	3,130	1,340
80	9,750	7,670	5,600	3,520	1,550	80	10,190	7,960	5,730	3,500	1,500
90	10,830	8,520	6,200	3,890	1,710	90	11,200	8,750	6,290	3,840	1,640
100	11,760	9,250	6,730	4,220	1,860	100	12,120	9,460	6,800	4,140	1,760
110	12,520	9,850	7,170	4,490	1,980	110	12,950	10,100	7,260	4,420	1,870
120	13,100	10,300	7,500	4,700	2,060	120	13,680	10,660	7,650	4,630	1,960
130	13,490	10,600	7,730	4,830	2,120	130	14,310	11,130	7,960	4,800	2,020
140	13,690	10,760	7,830	4,900	2,160	140	14,860	11,560	8,260	4,950	2,060

DETERMINATION OF THE QUALITY OF LOCALITY.

feet, including all wood above ground, but exclusive of roots and stumps.

Spruce.						Silver Fir.					
Age, Years.	Quality classes, in solid cubic feet.					Age, Years.	Quality classes, in solid cubic feet.				
	I.	II.	III.	IV.	V.		I.	II.	III.	IV.	V.
10	460	400	330	260	140	10	430	360	290	210	110
20	1,430	1,170	930	660	330	20	1,190	970	760	540	270
30	2,790	2,260	1,700	1,160	540	30	2,520	2,000	1,500	990	460
40	4,420	3,500	2,600	1,690	770	40	4,130	3,260	2,390	1,500	670
50	6,190	4,870	3,560	2,240	990	50	5,920	4,630	3,350	2,060	890
60	8,050	6,290	4,530	2,790	1,190	60	7,800	6,090	4,370	2,640	1,120
70	9,890	7,700	5,520	3,320	1,390	70	9,730	7,580	5,410	3,230	1,360
80	11,690	9,080	6,460	3,830	1,570	80	11,690	9,090	6,450	3,820	1,570
90	13,360	10,350	7,330	4,300	1,740	90	13,550	10,480	7,400	4,340	1,760
100	14,910	11,520	8,130	4,730	1,900	100	15,350	11,850	8,320	4,840	1,930
110	16,270	12,550	8,830	5,100	2,030	110	17,030	13,120	9,200	5,300	2,090
120	17,420	13,420	9,420	5,420	2,140	120	18,610	14,310	10,000	5,690	2,220
130	18,320	14,120	9,890	5,560	2,220	130	20,010	15,350	10,690	6,030	2,320
140	18,920	14,550	10,180	5,820	2,270	140	21,240	16,250	11,290	6,300	2,390
						150	22,280	17,020	11,760	6,500	2,430

II. Area of Circles for Diameters

Diam. in inch's.	Area of circle in square ft.	Diam. in inch's.	Area of circle in square ft.	Diam. in inch's.	Area of circle in square ft.	Diam. in inch's.	Area of circle in square ft.	Diam. in inch's.	Area of circle in square ft.
1·0	0·0055	**2·0**	0·0218	**3·0**	0·0491	**4·0**	0·0873	**5·0**	0·1364
1	·0067	1	·0240	1	·0524	1	·0917	1	·1418
2	·0079	2	·0264	2	·0559	2	·0963	2	·1474
3	·0092	3	·0289	3	·0594	3	·1009	3	·1532
4	·0107	4	·0314	4	·0631	4	·1056	4	·1590
5	·0123	5	·0341	5	·0669	5	·1105	5	·1650
6	·0140	6	·0369	6	·0707	6	·1154	6	·1710
7	·0158	7	·0398	7	·0747	7	·1205	7	·1772
8	·0177	8	·0428	8	·0788	8	·1257	8	·1835
9	·0197	9	·0459	9	·0830	9	·1310	9	·1899
11·0	0·6600	**12·0**	0·7854	**13·0**	0·9218	**14·0**	1·0690	**15·0**	1·2272
1	·6721	1	·7986	1	·9360	1	1·0843	1	1·2437
2	·6842	2	·8118	2	·9504	2	1·0997	2	1·2602
3	·6965	3	·8252	3	·9648	3	1·1153	3	1·2768
4	·7089	4	·8387	4	·9794	4	1·1309	4	1·2936
5	·7214	5	·8523	5	·9941	5	1·1467	5	1·3104
6	·7340	6	·8660	6	1·0089	6	1·1626	6	1·3274
7	·7467	7	·8798	7	1·0237	7	1·1785	7	1·3444
8	·7595	8	·8937	8	1·0387	8	1·1946	8	1·3616
9	·7724	9	·9077	9	1·0538	9	1·2108	9	1·3789
21·0	2·4053	**22·0**	2·6398	**23·0**	2·8852	**24·0**	3·1416	**25·0**	3·4088
1	2·4283	1	2·6638	1	2·9103	1	3·1679	1	3·4361
2	2·4514	2	2·6880	2	2·9356	2	3·1942	2	3·4636
3	2·4745	3	2·7122	3	2·9610	3	3·2207	3	3·4911
4	2·4978	4	2·7366	4	2·9864	4	3·2471	4	3·5188
5	2·5212	5	2·7611	5	3·0120	5	3·2748	5	3·5465
6	2·5447	6	2·7857	6	3·0377	6	3·3006	6	3·5744
7	2·5684	7	2·8104	7	3·0635	7	3·3275	7	3·6024
8	2·5921	8	2·8352	8	3·0894	8	3·3545	8	3·6305
9	2·6159	9	2·8602	9	3·1154	9	3·3816	9	3·6587
40	8·7266	41	9·1684	42	9·6211	43	10·0847	44	10·5592
50	13·6354	51	14·1863	52	14·7480	53	15·3207	54	15·9043
60	19·6350								

Footnote.—The circles of full inches were calculated with logarithms

OF 1 INCH TO 60 INCHES.

Diam. in inch's.	Area of circle in square ft.	Diam. in inch's.	Area of circle in square ft.	Diam. in inch's.	Area of circle in square ft.	Diam. in inch's.	Area of circle in square ft.	Diam. in inch's.	Area of circle in square ft.
6·0	0·1963	7·0	0·2673	8·0	0·3491	9·0	0·4418	10·0	0·5454
1	·2029	1	·2750	1	·3579	1	·4517	1	·5564
2	·2096	2	·2828	2	·3668	2	·4617	2	·5675
3	·2164	3	·2907	3	·3758	3	·4718	3	·5787
4	·2234	4	·2987	4	·3849	4	·4820	4	·5900
5	·2304	5	·3068	5	·3941	5	·4923	5	·6014
6	·2376	6	·3151	6	·4034	6	·5027	6	·6129
7	·2448	7	·3234	7	·4129	7	·5132	7	·6245
8	·2522	8	·3319	8	·4224	8	·5238	8	·6362
9	·2597	9	·3404	9	·4321	9	·5345	9	·6481
16·0	1·3963	17·0	1·5763	18·0	1·7671	19·0	1·9689	20·0	2·1817
1	1·4138	1	1·5949	1	1·7868	1	1·9897	1	2·2036
2	1·4314	2	1·6136	2	1·8066	2	2·0106	2	2·2256
3	1·4492	3	1·6324	3	1·8265	3	2·0316	3	2·2477
4	1·4670	4	1·6513	4	1·8465	4	2·0527	4	2·2699
5	1·4849	5	1·6703	5	1·8666	5	2·0739	5	2·2922
6	1·5030	6	1·6894	6	1·8869	6	2·0952	6	2·3146
7	1·5212	7	1·7087	7	1·9072	7	2·1167	7	2·3371
8	1·5394	8	1·7280	8	1·9277	8	2·1382	8	2·3597
9	1·5578	9	1·7475	9	1·9482	9	2·1599	9	2·3825
26·0	3·6870	27·0	3·9761	28·0	4·2761	29·0	4·5869	30·0	4·9087
1	3·7154	1	4·0056	1	4·3067	1	4·6186	31	5·2414
2	3·7439	2	4·0353	2	4·3374	2	4·6504	32	5·5851
3	3·7725	3	4·0650	3	4·3682	3	4·6823	33	5·9396
4	3·8013	4	4·0948	4	4·3991	4	4·7143	34	6·3050
5	3·8301	5	4·1248	5	4·4301	5	4·7464	35	6·6813
6	3·8591	6	4·1548	6	4·4612	6	4·7787	36	7·0686
7	3·8882	7	4·1850	7	4·4925	7	4·8110	37	7·4667
8	3·9174	8	4·2152	8	4·5238	8	4·8435	38	7·8758
9	3·9467	9	4·2456	9	4·5553	9	4·8760	39	8·2958
45	11·0447	46	11·5410	47	12·0482	48	12·5664	49	13·0954
55	16·4988	56	17·1042	57	17·7206	58	18·3478	59	18·9859

of 7 places; the intermediate values were found by interpolation.

III. Table of the Volumes of Cylinders and of the

Length of Cylinder, or Number of Circles.	Diameter in Inches.							
	1	2	3	4	5	6		
1	0·0055	0·0218	0·0191	0·0873	0·1361	0·1963	0·2673	0·3491
2	0·0110	0·0436	0·0982	0·1746	0·2728	0·3926	0·5346	0·6982
3	0·0165	0·0654	0·1473	0·2619	0·4092	0·5889	0·8019	1·0473
4	0·0220	0·0872	0·1964	0·3192	0·5456	0·7852	1·0692	1·3964
5	0·0275	0·1090	0·2455	0·4365	0·6820	0·9815	1·3365	1·7455
6	0·0330	0·1308	0·2946	0·5238	0·8181	1·1778	1·6038	2·0946
7	0·0385	0·1526	0·3437	0·6111	0·9548	1·3741	1·8711	2·4437
8	0·0140	0·1744	0·3928	0·6984	1·0912	1·5701	2·1384	2·7928
9	0·0495	0·1962	0·4419	0·7857	1·2276	1·7667	2·4057	3·1419
	17	18	19	20	21	22	23	24
1	1·5763	1·7671	1·9689	2·1817	2·4053	2·6398	2·8852	3·1416
2	3·1526	3·5342	3·9378	4·3634	4·8106	5·2796	5·7704	6·2832
3	4·7289	5·3013	5·9067	6·5451	7·2159	7·9194	8·6556	9·4248
4	6·3052	7·0684	7·8756	8·7268	9·6212	10·5592	11·5408	12·5664
5	7·8815	8·8355	9·8445	10·9085	12·0265	13·1990	14·4260	15·7080
6	9·4578	10·6026	11·8134	13·0902	14·4318	15·8388	17·3112	18·8496
7	11·0341	12·3697	13·7823	15·2719	16·8371	18·4786	20·1964	21·9912
8	12·6104	14·1368	15·7512	17·4536	19·2424	21·1184	23·0816	25·1328
9	14·1867	15·9039	17·7201	19·6353	21·6477	23·7582	25·9668	28·2744
	33	34	35	36	37	38	39	40
1	5·9396	6·3050	6·6813	7·0686	7·4667	7·8758	8·2958	8·7266
2	11·8792	12·6100	13·3626	14·1372	14·9334	15·7516	16·5916	17·4532
3	17·8188	18·9150	20·0439	21·2058	22·4001	23·6274	24·8874	26·1798
4	23·7584	25·2200	26·7252	28·2744	29·8668	31·5032	33·1832	34·9064
5	29·6980	31·5250	33·4065	35·3430	37·3335	39·3790	41·4790	43·6330
6	35·6376	37·8300	40·0878	42·4116	44·8002	47·2548	49·7748	52·3596
7	41·5772	44·1350	46·7691	49·4802	52·2669	55·1306	58·0706	61·0862
8	47·5168	50·4400	53·4504	56·5488	59·7336	63·0064	66·3664	69·8128
9	53·4564	56·7450	60·1317	63·6174	67·2003	70·8822	74·6622	78·5394

Sum of Circles, for Diameter of 1 Inch to 48 Inches.

Length of Cylinder, or Number of Circles.	Diameter in Inches.							
	9	10	11	12	13	14	15	16
1	0·4418	0·5454	0·6600	0·7854	0·9218	1·0690	1·2272	1·3963
2	*0·8836*	*1·0908*	*1·3200*	*1·5708*	*1·8436*	*2·1380*	*2·4544*	*2·7926*
3	1·3254	1·6362	1·9800	2·3562	2·7654	3·2070	3·6816	4·1889
4	1·7672	2·1816	2·6400	3·1416	3·6872	4·2760	4·9088	5·5852
5	2·2090	2·7270	3·3000	3·9270	4·6090	5·3450	6·1360	6·9815
6	2·6508	3·2724	3·9600	4·7124	5·5308	6·4140	7·3632	8·3778
7	3·0926	3·8178	4·6200	5·4978	6·4526	7·4830	8·5904	9·7741
8	3·5344	4·3632	5·2800	6·2832	7·3744	8·5520	9·8176	11·1704
9	3·9762	4·9086	5·9400	7·0686	8·2962	9·6210	11·0448	12·5667

	25	26	27	28	29	30	31	32
1	3·4088	3·6870	3·9761	4·2761	4·5869	4·9087	5·2414	5·5851
2	6·8176	7·3740	7·9522	8·5522	9·1738	9·8174	10·4828	11·1702
3	10·2264	11·0610	11·9283	12·8283	13·7607	14·7261	15·7242	16·7553
4	13·6352	14·7480	15·9044	17·1044	18·3476	19·6348	20·9656	22·3404
5	17·0440	18·4350	19·8805	21·3805	22·9345	24·5435	26·2070	27·9255
6	20·4528	22·1220	23·8566	25·6566	27·5214	29·4522	31·4484	33·5106
7	23·8616	25·8090	27·8327	29·9327	32·1083	34·3609	36·6898	39·0957
8	27·2704	29·4960	31·8088	34·2088	36·6952	39·2696	41·9312	44·6808
9	30·6792	33·1830	35·7849	38·4849	41·2821	44·1783	47·1726	50·2659

	41	42	43	44	45	46	47	48
1	9·1684	9·6211	10·0847	10·5592	11·0447	11·5410	12·0482	12·5664
2	18·3368	19·2422	20·1694	21·1184	22·0894	23·0820	24·0964	25·1328
3	27·5052	28·8633	30·2541	31·6776	33·1341	34·6230	36·1446	37·6992
4	36·6736	38·4844	40·3388	42·2368	44·1788	46·1640	48·1928	50·2656
5	45·8420	48·1055	50·4235	52·7960	55·2235	57·7050	60·2410	62·8320
6	55·0104	57·7266	60·5082	63·3552	66·2682	69·2460	72·2892	75·3984
7	64·1788	67·3477	70·5929	73·9144	77·3129	80·7870	84·3374	87·9648
8	73·3472	76·9688	80·6776	84·4736	88·3576	92·3280	96·3856	100·5312
9	82·5156	86·5899	90·7623	95·0328	99·4023	103·8690	108·4338	113·0976

IVa. Amount to which a Capital of 1 accumulates with Compound Interest in n Years $C_n = C_o \times 1{\cdot}op^n$.

No. of Years, $= n$.	Per Cent.						
	2.	2·5.	3.	3·5.	.	4·5.	5.
1	1·0200	1·0250	1·0300	1·0350	1·0400	1·0450	1·0500
2	1·0404	1·0506	1·0609	1·0712	1·0816	1·0920	1·1025
3	1·0612	1·0769	1·0927	1·1087	1·1249	1·1412	1·1576
4	1·0824	1·1038	1·1255	1·1475	1·1699	1·1925	1·2155
5	1·1041	1·1314	1·1593	1·1877	1·2167	1·2462	1·2763
6	1·1262	1·1597	1·1941	1·2293	1·2653	1·3023	1·3401
7	1·1487	1·1887	1·2299	1·2723	1·3159	1·3609	1·4071
8	1·1717	1·2184	1·2668	1·3168	1·3686	1·4221	1·4775
9	1·1951	1·2489	1·3048	1·3629	1·4233	1·4861	1·5513
10	1·2190	1·2801	1·3439	1·4106	1·4802	1·5530	1·6289
15	1·3459	1·4483	1·5580	1·6753	1·8009	1·9353	2·0789
20	1·4859	1·6386	1·8061	1·9898	2·1911	2·4117	2·6533
25	1·6406	1·8539	2·0938	2·3632	2·6658	3·0054	3·3864
30	1·8114	2·0976	2·4273	2·8068	3·2434	3·7453	4·3219
35	1·9999	2·3732	2·8139	3·3336	3·9461	4·6673	5·5160
40	2·2080	2·6851	3·2620	3·9593	4·8010	5·8164	7·0400
45	2·4379	3·0379	3·7816	4·7024	5·8412	7·2482	8·9850
50	2·6916	3·4371	4·3839	5·5849	7·1067	9·0326	11·4674
55	2·9717	3·8888	5·0821	6·6331	8·6464	11·2563	14·6356
60	3·2810	4·3998	5·8916	7·8781	10·5196	14·0274	18·6792
65	3·6225	4·9780	6·8300	9·3567	12·7987	17·4807	23·8399
70	3·9996	5·6321	7·9178	11·1128	15·5716	21·7841	30·4264
75	4·4158	6·3722	9·1789	13·1985	18·9452	27·1470	38·8327
80	4·8754	7·2096	10·6409	15·6757	23·0498	33·8301	49·5614
85	5·3829	8·1570	12·3357	18·6179	28·0436	42·1585	63·2544
90	5·9431	9·2289	14·3005	22·1122	34·1193	52·5371	80·7304
95	6·5617	10·4416	16·5782	26·2623	41·5114	65·4708	103·0347
100	7·2446	11·8137	19·2186	31·1914	50·5049	81·5885	131·5013
110	8·8312	15·1226	25·8282	43·9986	74·7597	126·7045	214·2017
120	10·7652	19·3581	34·7110	62·0643	110·6626	196·7682	348·9120
130	13·1227	24·7801	46·6486	87·5478	163·8076	305·5750	568·3409
140	15·9965	31·7206	62·6919	123·4949	242·4753	474·5486	925·7674
150	19·4996	40·6050	84·2527	174·2017	358·9227	736·9594	1507·9775
200	52·4849	139·5639	369·3558	972·9039	2550·7498	6656·6863	17292·5808

IVB. PRESENT VALUE OF A CAPITAL OF 1 TO BE REALIZED AFTER n YEARS: $C_o = \frac{C_n}{1 \cdot op^n}$.

No. of Years, $= n$.	PER CENT. 2.	2 5.	3	3·5.	4.	4·5.	5.
1	0·9804	0·9756	0·9707	0·9662	0·9615	0·9569	·9524
2	·9612	·9518	·9426	·9335	·9246	·9157	·9070
3	·9423	·9286	·9151	·9019	·8890	·8763	·8638
4	·9238	·9060	·8885	·8714	·8548	·8386	·8227
5	·9057	·8839	·8626	·8420	·8219	·8025	·7835
6	·8880	·8623	·8375	·8135	·7903	·7679	·7462
7	·8706	·8413	·8131	·7860	·7599	·7348	·7107
8	·8535	·8207	·7894	·7594	·7307	·7032	·6768
9	·8368	·8007	·7664	·7337	·7026	·6729	·6446
10	·8203	·7812	·7441	·7089	·6756	·6439	·6139
15	·7430	·6905	·6419	·5969	·5553	·5167	·4810
20	·6730	·6103	·5537	·5026	·4564	·4146	·3769
25	·6095	·5394	·4776	·4231	·3751	·3327	·2953
30	·5521	·4767	·4120	·3563	·3083	·2670	·2314
35	·5000	·4214	·3554	·3000	·2534	·2143	·1813
40	·4529	·3724	·3066	·2526	·2083	·1719	·1420
45	·4102	·3292	·2644	·2127	·1712	·1380	·1113
50	·3715	·2909	·2281	·1791	·1407	·1107	·0872
55	·3365	·2572	·1968	·1508	·1157	·0888	·0683
60	·3048	·2273	·1697	·1269	·0951	·0713	·0535
65	·2760	·2009	·1464	·1069	·0781	·0572	·0419
70	·2500	·1776	·1263	·0900	·0642	·0459	·0329
75	·2265	·1569	·1089	·0758	·0528	·0368	·0257
80	·2051	·1387	·0940	·0638	·0434	·0296	·0202
85	·1858	·1226	·0811	·0537	·0357	·0237	·0158
90	·1683	·1084	·0699	·0452	·0293	·0190	·0124
95	·1524	·0958	·0603	·0381	·0241	·0153	·0097
100	·1380	·08465	·05203	·03206	·01980	·01226	·00761
110	·1132	·06613	·03872	·02273	·01338	·00789	·00467
120	·09289	·05166	·02881	·01611	·00904	·00508	·00287
130	·07620	·04036	·02144	·01142	·00611	·00327	·00176
140	·06251	·03153	·01595	·00810	·00412	·00211	·00101
150	·05128	·02463	·01187	·00574	·00279	·00136	·00066
200	·01905	·007165	·002707	·001028	·000392	·000150	·000578

IVc.—Present Value of a Perpetual Rental of 1, due every n Years:

$$C_o = \frac{R}{1{\cdot}op^n - 1}.$$

Number of Years $= n$.	Per Cent.						
	2	2·5	3	3·5	4	4·5	5
1	50·0000	40·0000	33·3333	28·5714	25·0000	22·2222	20·0000
2	24·7525	19·7531	16·4204	14·0100	12·2549	10·8666	9·7561
3	16·3377	13·0054	10·7843	9·1981	8·0087	7·0839	6·3442
4	12·1312	9·6327	7·9676	6·7786	5·8873	5·1943	4·6402
5	9·6079	7·6099	6·2785	5·3280	4·6157	4·0620	3·6195
6	7·9263	6·2620	5·1532	4·3620	3·7690	3·3084	2·9403
7	6·7256	5·2998	4·3502	3·6727	3·1652	2·7711	2·4564
8	5·8255	4·5787	3·7485	3·1565	2·7132	2·3691	2·0944
9	5·1258	4·0183	3·2811	2·7556	2·3623	2·0572	1·8138
10	4·5663	3·5703	2·9077	2·4355	2·0823	1·8084	1·5901
15	2·8913	2·2307	1·7922	1·4807	1·2485	1·0692	0·9268
20	2·0578	1·5659	1·2405	1·0103	0·8395	0·7084	·6049
25	1·5610	1·1710	0·9143	0·7335	·6003	·4986	·4190
30	1·2325	0·9111	·7006	·5535	·4458	·3643	·3010
35	1·0001	·7282	·5513	·4285	·3394	·2727	·2214
40	0·8278	·5934	·4421	·3379	·2631	·2076	·1656
45	·6955	·4907	·3595	·2701	·2066	·1600	·1252
50	·5912	·4103	·2955	·2181	·1638	·1245	·0955
55	·5072	·3462	·2450	·1775	·1308	·0975	·0733
60	·4384	·2941	·2044	·1454	·1050	·0768	·0566
65	·3813	·2514	·1715	·1197	·0848	·0607	·0438
70	·3334	·2159	·1446	·0989	·0686	·0481	·0340
75	·2928	·1861	·1223	·0820	·0557	·0382	·0264
80	·2580	·1610	·1037	·0681	·0453	·0305	·0206
85	·2282	·1397	·0882	·0568	·0370	·0243	·0161
90	·2023	·1215	·0752	·0474	·0302	·0194	·0125
95	·1798	·1059	·0642	·0396	·0247	·0155	·0098
100	·1601	·0925	·0549	·0331	·0202	·0124	·0077
110	·1277	·07081	·04028	·02326	·01356	·00796	·00469
120	·1024	·05447	·02966	·01638	·00912	·00511	·002874
130	·08249	·04205	·02191	·01155	·006142	·003284	·001763
140	·06668	·03255	·01621	·008164	·004141	·002112	·001081
150	·05406	·02525	·01201	·005774	·002794	·001357	·0006636
200	·01942	·007217	·002707	·001029	·0003926	·0001502	·00005783

IVd.—Present Value of a Rental of 1 due at the End of every Year, altogether n times: $C_0 = \frac{r\,(1\cdot op^n - 1)}{1\cdot op^n \times \cdot op}$.

Number of Years $= n$.	Per Cent.						
	2	2·5	3	3·5	4	4·5	5
1	0·9804	0·9756	0·9709	0·9662	0·9615	0·9569	0·9524
2	1·9416	1·9274	1·9135	1·8997	1·8861	1·8727	1·8594
3	2·8839	2·8560	2·8286	2·8016	2·7751	2·7490	2·7232
4	3·8077	3·7620	3·7171	3·6731	3·6299	3·5875	3·5459
5	4·7135	4·6458	4·5797	4·5150	4·4518	4·3900	4·3295
6	5·6014	5·5081	5·4172	5·3285	5·2421	5·1579	5·0757
7	6·4720	6·3494	6·2303	6·1145	6·0020	5·8927	5·7864
8	7·3255	7·1701	7·0197	6·8740	6·7327	6·5959	6·4632
9	8·1622	7·9709	7·7861	7·6077	7·4353	7·2688	7·1078
10	8·9826	8·7521	8·5302	8·3166	8·1109	7·9127	7·7217
15	12·8493	12·3814	11·9379	11·5174	11·1184	10·7395	10·3797
20	16·3514	15·5892	14·8775	14·2124	13·5903	13·0079	12·4622
25	19·5235	18·4244	17·4131	16·4815	15·6221	14·8282	14·0939
30	22·3965	20·9303	19·6004	18·3920	17·2920	16·2889	15·3725
35	24·9986	23·1452	21·4872	20·0007	18·6646	17·4610	16·3742
40	27·3555	25·1028	23·1148	21·3551	19·7928	18·4016	17·1591
45	29·4902	26·8330	24·5187	22·4955	20·7200	19·1563	17·7741
50	31·4236	28·3623	25·7298	23·4556	21·4822	19·7620	18·2559
55	33·1748	29·7140	26·7744	24·2641	22·1086	20·2480	18·6335
60	34·7609	30·9087	27·6756	24·9447	22·6235	20·6380	18·9293
65	36·1975	31·9646	28·4529	25·5178	23·0467	20·9510	19·1611
70	37·4986	32·8979	29·1234	26·0004	23·3945	21·2021	19·3427
75	38·6771	33·7227	29·7018	26·4067	23·6804	21·4036	19·4850
80	39·7445	34·4518	30·2008	26·7488	23·9154	21·5653	19·5965
85	40·7113	35·0962	30·6312	27·0368	24·1085	21·6951	19·6838
90	41·5869	35·6658	31·0024	27·2793	24·2673	21·7992	19·7523
95	42·3800	36·1692	31·3227	27·4835	24·3978	21·8828	19·8059
100	43·0984	36·6141	31·5989	27·6554	24·5050	21·9499	19·8479
110	44·3382	37·3549	32·0428	27·9221	24·6656	22·0468	19·9066
120	45·3554	37·9337	32·3730	28·1111	24·7741	22·1093	19·9427
130	46·1898	38·3858	32·6188	28·2451	24·8474	22·1495	19·9648
140	46·8743	38·7390	32·8016	28·3401	24·8969	22·1754	19·9784
150	47·4358	39·0149	32·9377	28·4074	24·9303	22·1921	19·9867
200	49·0473	39·7134	33·2431	28·5421	24·9902	22·2189	19·9988